Dynamic Phasors for Small Signal Stability Analysis

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Outline

1. Introduction
   - Simulation and Analysis Techniques
   - Typical Outputs
   - Modelling of Components
2. Dynamic Phasors
3. Applications
   - Interactions Between Nearby HVDC Converters
   - Torsional Interactions
4. Current Research Work

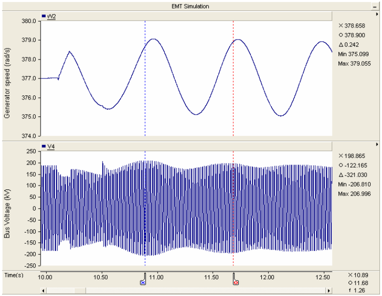
Power System Simulation and Analysis

- Electromagnetic Transient Simulation - Time Domain Technique
- Transient Stability Simulation - Time Domain Technique
- Small Signal Stability Analysis - Frequency Domain Technique

Typical Output of an EMT Simulation

Sample responses of a four-generator power system after a three phase fault

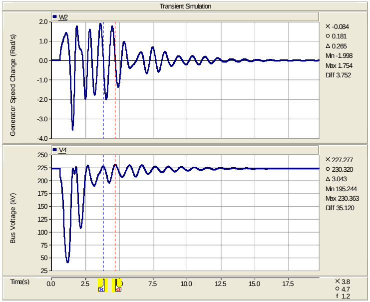
The amplitude of the 60 Hz voltage waveform is modulated by the low frequency of oscillations of the rotor.



Typical Output of a TS Simulation

Sample responses of a four-generator power system after a three phase fault

Rotor angle of generator 2 and the rms voltage of Bus 2 show low frequency oscillations around 1 Hz.



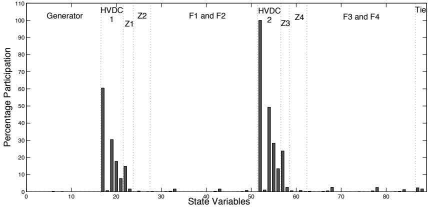
Typical Output of a Small Signal Analysis

Structural Information

- Oscillation modes (frequencies and corresponding damping).
- Mode Shapes of oscillation frequencies.
- Participation of state variables in oscillation modes.
- Observability of oscillation modes.
- Controllability of oscillation modes.
- Residues for input-output pairs.

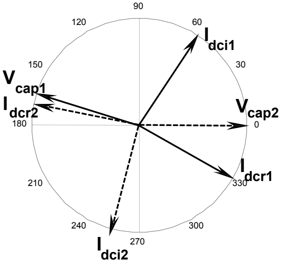
Typical Output of a Small Signal Analysis: Participation Factors

Participation Factors show the relative participation of state variables when a mode is excited.



Typical Output of a Small Signal Analysis: Mode Shape

Mode Shape shows whether the state variables are oscillating together or not.



Machine Models

Transient Stability and Small Signal Stability

- Rotor fluxes are modelled as state variables.
- Stator fluxes are NOT modelled as state variables.

Electromagnetic Transient Simulation

- Rotor fluxes and stator fluxes are modelled as state variables.

Common to both

- Dynamics of the rotor and that of auxiliary controllers are modelled using differential equations.

Transmission Line Models

Transient Stability and Small Signal Stability

- Series inductance and shunt capacitance are modelled as constant impedances (admittances) calculated at the nominal frequency ω_0.

Electromagnetic Transient Simulation

- Transmission line is modelled using differential equations (telegraphic equations).

Small Signal Stability: Frequency domain technique

- Only accurate in the vicinity of nominal frequency.
- Structural Information relevant to the system is available.

Transient Stability: Time domain technique

- Only accurate in the vicinity of nominal frequency.
- Large integration time step is used $\Rightarrow$ simulation is fast.

Electromagnetic Transient Simulation: Time domain technique

- Accurate over a wide frequency range.
- Integration time step is small $\Rightarrow$ simulation is slow.

Dynamic Phasors

Instantaneous Current Waveform

$$i_{ac} = A_m e^{j\phi} e^{j\omega_0 t} = [A_m \cos(\phi) + jA_m \sin(\phi)] e^{j\omega_0 t}$$

A_m is the magnitude of the current , ϕ is the phase of the current, and ω_0 is the nominal system frequency.

In Rectangular Coordinates

$$i_{ac} = (I_R + jI_I) e^{j\omega_0 t}$$

Modelling a Transmission Line using Dynamic Phasors

Series Branch

Series R-L circuit connected between nodes 1 and 2.

$$v_{12} = L\frac{di_{12}}{dt} + Ri_{12}$$

Using the Complex rotating phasor relationships

$$(V_R + jV_I)e^{j\omega_0 t} = L\frac{d(I_R + jI_I)e^{j\omega_0 t}}{dt} + R(I_R + jI_I)e^{j\omega_0 t}$$

Assuming that the nominal system frequency (ω_0) is constant

$$V_R + jV_I = L\frac{d(I_R + jI_I)}{dt} + (R + j\omega_0 L)(I_R + jI_I)$$

Since L is in pu, (ω_0/L) terms appear instead of $(1/L)$

$$\begin{bmatrix} \Delta \dot{I}_R \\ \Delta \dot{I}_I \end{bmatrix} = \begin{bmatrix} \frac{-R\omega_0}{L} & \omega_0 \\ -\omega_0 & \frac{-R\omega_0}{L} \end{bmatrix} \begin{bmatrix} \Delta I_R \\ \Delta I_I \end{bmatrix} + \begin{bmatrix} \frac{\omega_0}{L} & 0 & \frac{-\omega_0}{L} & 0 \\ 0 & \frac{\omega_0}{L} & 0 & \frac{-\omega_0}{L} \end{bmatrix} \begin{bmatrix} \Delta V1_R \\ \Delta V1_I \\ \Delta V2_R \\ \Delta V2_I \end{bmatrix}$$

Modelling a Transmission Line using Dynamic Phasors

Parallel Branch

$$\begin{bmatrix} \Delta \dot{V}_{1R} \\ \Delta \dot{V}_{1I} \end{bmatrix} = \begin{bmatrix} \frac{-\omega_0}{RC} & \omega_0 \\ -\omega_0 & \frac{-\omega_0}{RC} \end{bmatrix} \begin{bmatrix} \Delta V_{1R} \\ \Delta V_{1I} \end{bmatrix} + \begin{bmatrix} \frac{\omega_0}{C} & 0 \\ 0 & \frac{\omega_0}{C} \end{bmatrix} \begin{bmatrix} \Delta I_R \\ \Delta I_I \end{bmatrix}$$

Other Interpretations of Dynamic Phasors

d-q Components of Network Voltages and Currents

Network voltages and currents are represented by their d-q components which are modelled as state variables.

Fourier Components of Network Voltages and Currents

Network voltages and currents are represented by their Fourier components which are modelled as state variables.

Power System Signals as Amplitude Modulated Signals

If R and I components are constants

The instantaneous waveforms are sinusoidal.

If R and I components are oscillating at frequency ω

The instantaneous waveforms are amplitude modulated waveforms with carrier frequency ω_0. This results in two sidebands of $\omega_0 - \omega$ and $\omega_o + \omega$

Power System Signals as Amplitude Modulated Signals

Example

If $f_0 = 60$ Hz and $f = 5$ Hz, the two sideband frequencies are $f_1 = 55$ Hz and $f_2 = 65$ Hz. Both are close to 60 Hz and the constant admittance representation of transmission network is acceptable.

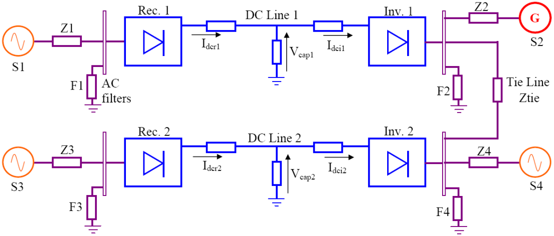
Example

If $f_0 = 60$ Hz and $f = 25$ Hz, the two sideband frequencies are $f_1 = 35$ Hz and $f_2 = 85$ Hz . Both are significantly different to 60 Hz and the constant admittance representation of transmission network is NOT acceptable.

Interactions Between Nearby HVDC Converters

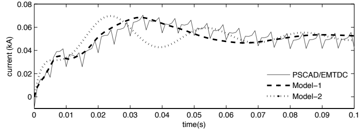
A simple Network for model Validation

- Two HVDC lines, ac filters, ac transmission line, and a generator.
- A pulse of magnitude of 5 % and duration 0.3s was applied to the rectifier current controller input.

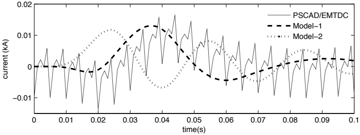


Comparison of EMT, SS-traditional, and SS-Dynamic Phasor Approach.

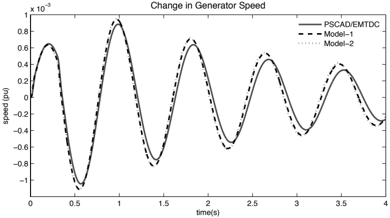
(a) Change in Idcr of HVDC1



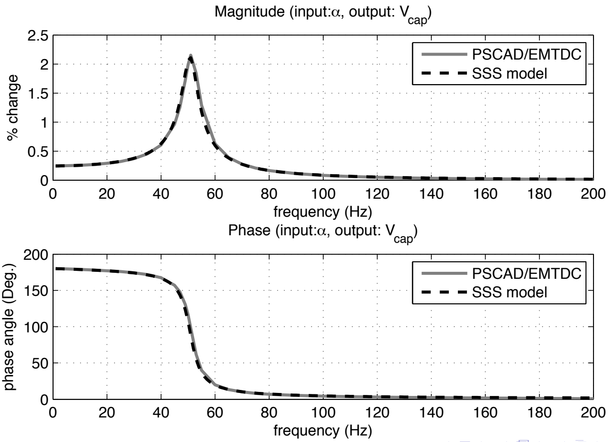
(b) Change in Idcr of HVDC2



Rotor Oscillations: SS-traditional and SS-Dynamic Phasors give same results

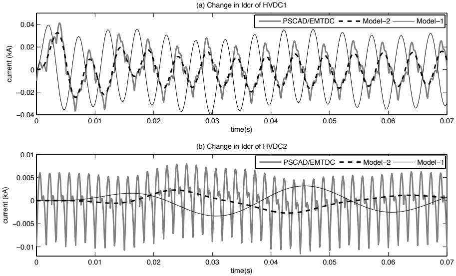


Frequency Response of the Model – EMT Vs SS-Dynamic Phasor

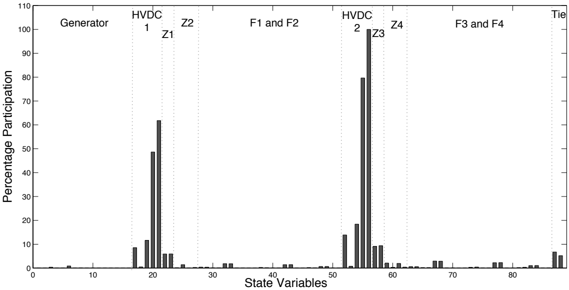


Response to a 200 Hz signal

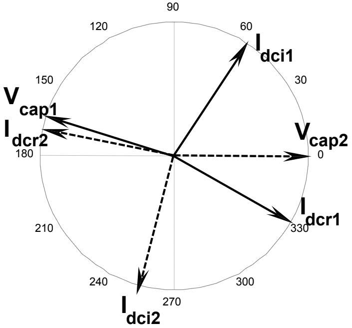
Changes in Rectifier side DC currents for a 5 %, 200Hz sinusoidal change of the HVDC1 rectifier side AC source voltage (VS1).



Participation Factors ⇒ presence of an interaction between the two HVDC converters

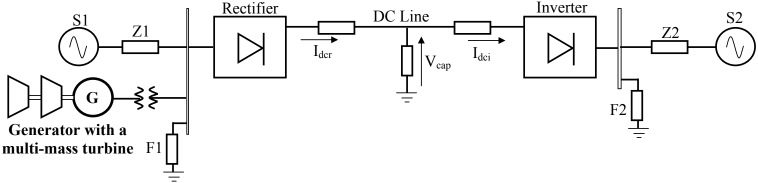


Mode Shape ⇒ state variables of the two converters oscillate against each other



Torsional Interactions

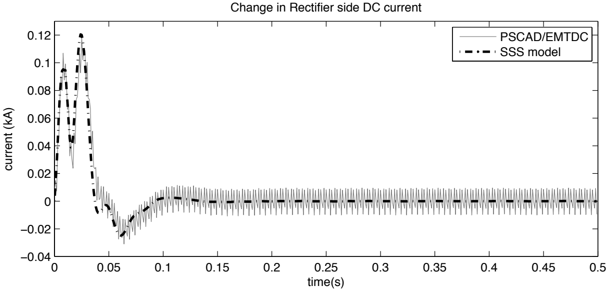
- The CIGRE benchmark HVDC test system with some modifications.
- A synchronous generator is connected at rectifier side AC bus to supply half of the P-Q requirement of rectifier.



Comparison of EMT and SS-Dynamic Phasor

SS-Dynamic-Phasor provides accurate results in the frequency range of interest

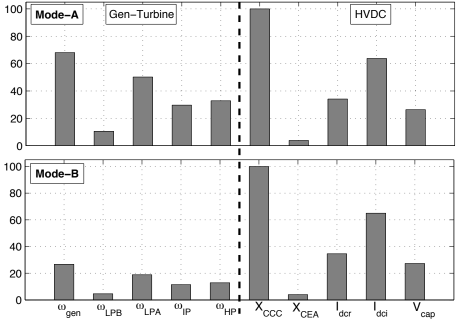
10 % change in rectifier current reference for 10 ms



Torsional Interaction Modes

Mode	**Freq. (Hz)**	**D (%)**	**Major Participants**
A	16.24	-0.03	HVDC-Generator-Turbine
B	16.36	1.05	HVDC-Generator-Turbine

Participating states are identified using Participation Factors



Publications

1. C. Karawita, U.D. Annakkage, *A Hybrid Network Model for Small Signal Stability Analysis of Power Systems*, IEEE Transactions on Power Systems, Vol 25, 2010, Page(s): 443–451.
2. C. Karawita, U.D. Annakkage, *Multi-Infeed HVDC Interaction Studies Using Small-Signal Stability Assessment*, IEEE Transactions on Power Delivery, Vol 24, 2009, Page(s): 910–918.
3. C. Karawita, U.D. Annakkage, *HVDC-Generator-Turbine Torsional Interaction Studies Using A Linearized Model With Dynamic Network Representation*, International Conference on Power System Transients (IPST), Kyoto, Japan, June 2009.

Current Research Work

- SSR between DFIG based Wind Power Plant and series compensated transmission lines (Hiranya).
- SSI between nearby LCC-HVDC and VSC-HVDC terminals (Kevin – MH).
- SSR mitigation using FACTS controllers (TGS).
- Transient Stability Simulation using Dynamic Phasors (Rae – MH).

Chandana has developed an SSR–Small Signal Analysis Program

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