Motors for Electric Cars
And Robotic Application

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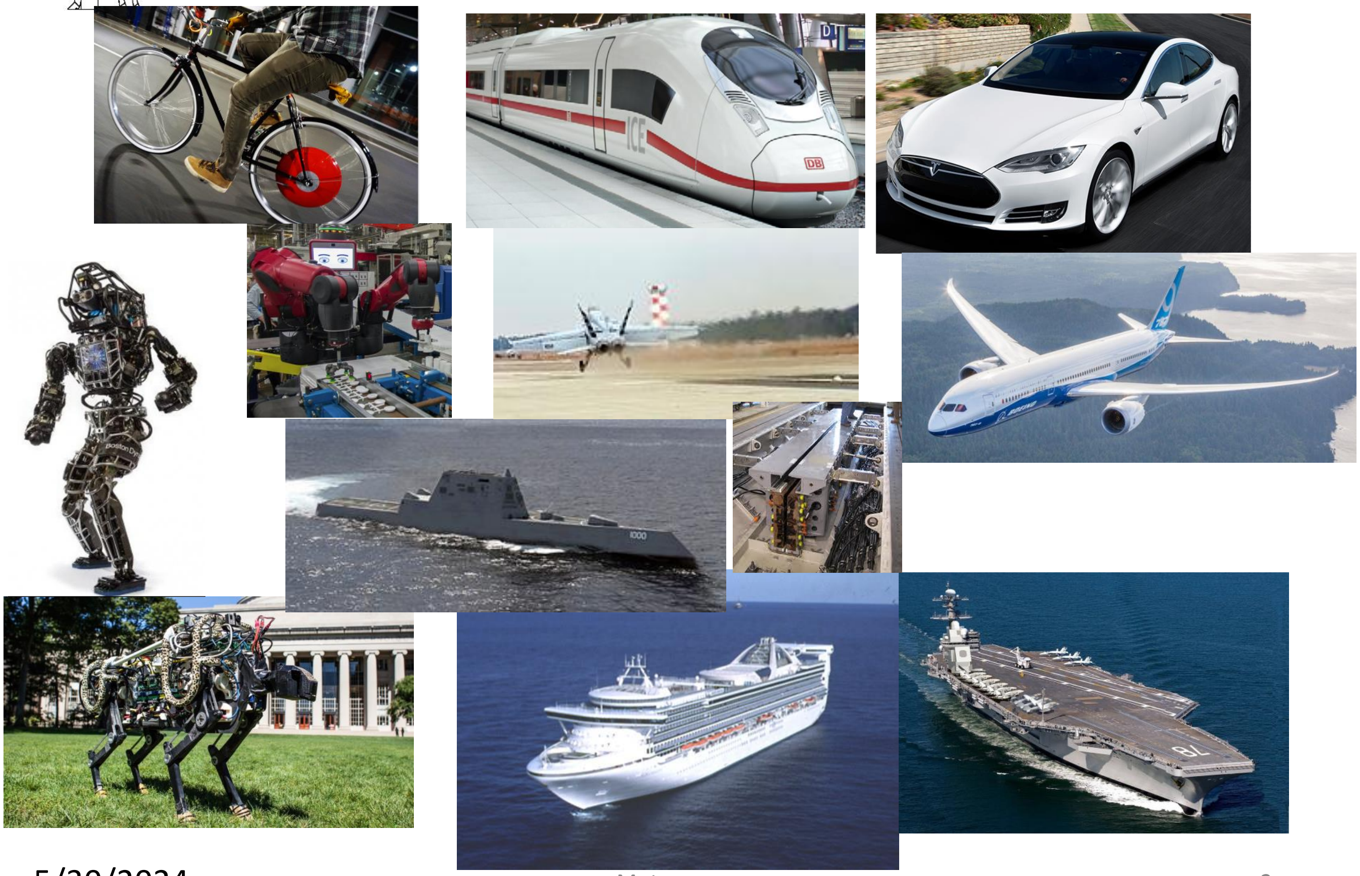
Motors have a wide range of modern applications



For now, we limit our attention to Cars and Robots
Electric Propulsion has certain Advantages:

In Cars

- Permits regeneration of braking energy for cars and bikes
- Use different sources of energy (including renewables)

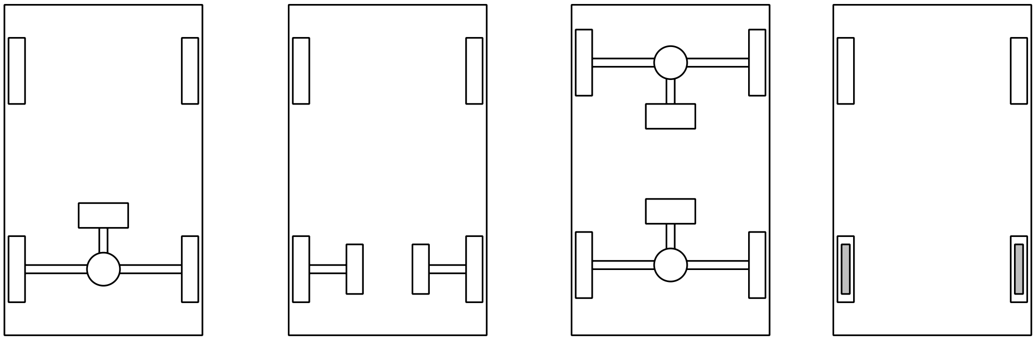
In Robotics

- Replace hydraulics and pneumatics for improved efficiency and reduction of leaks
- Better control characteristics: compliant control

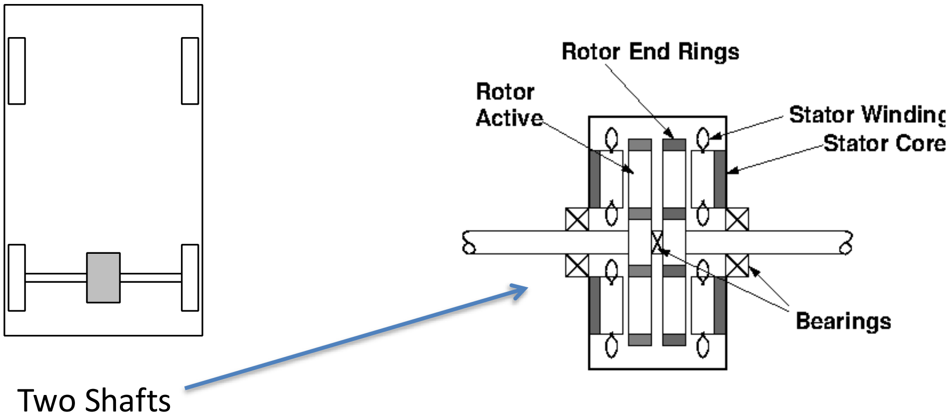
The two have common elements

- Wide Speed Range (including reversing)
- Wide Torque Range (Positive and Negative)

Start with Cars: There are many configurations:



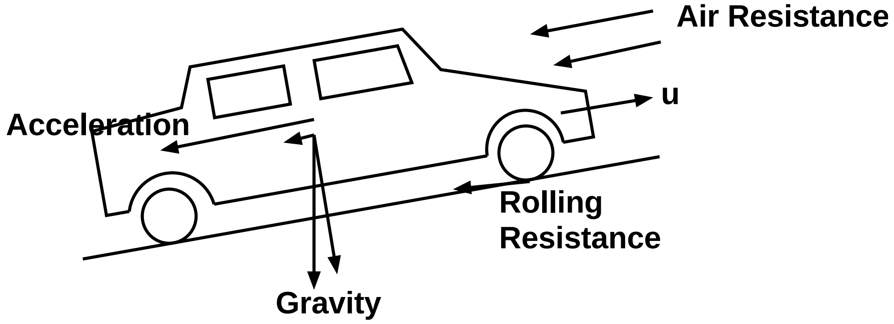
One Motor Two Motors Front and Back Wheel Motors



Two Shafts

For now, we will consider only a single motor driving the car. So what does the motor have to do?

Propulsion Force on a Car



$$F = M\frac{du}{dt} + \frac{1}{2}\rho_{air}C_dA_fu^2 + C_rMg + Mg\sin\theta_{hill}$$

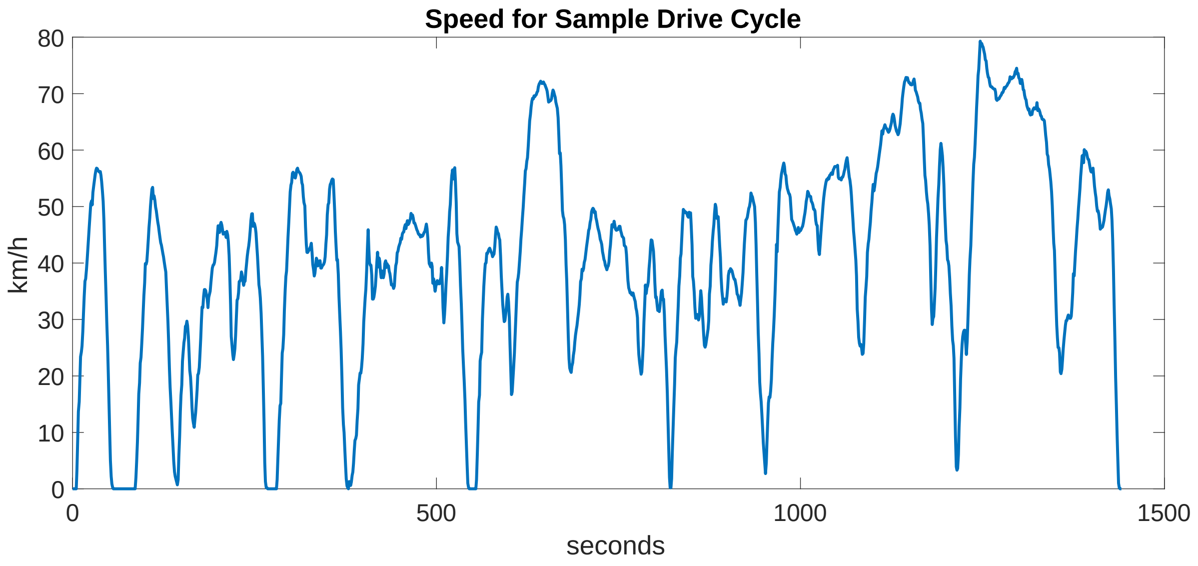
Acceleration

Air Drag

Rolling Resistance

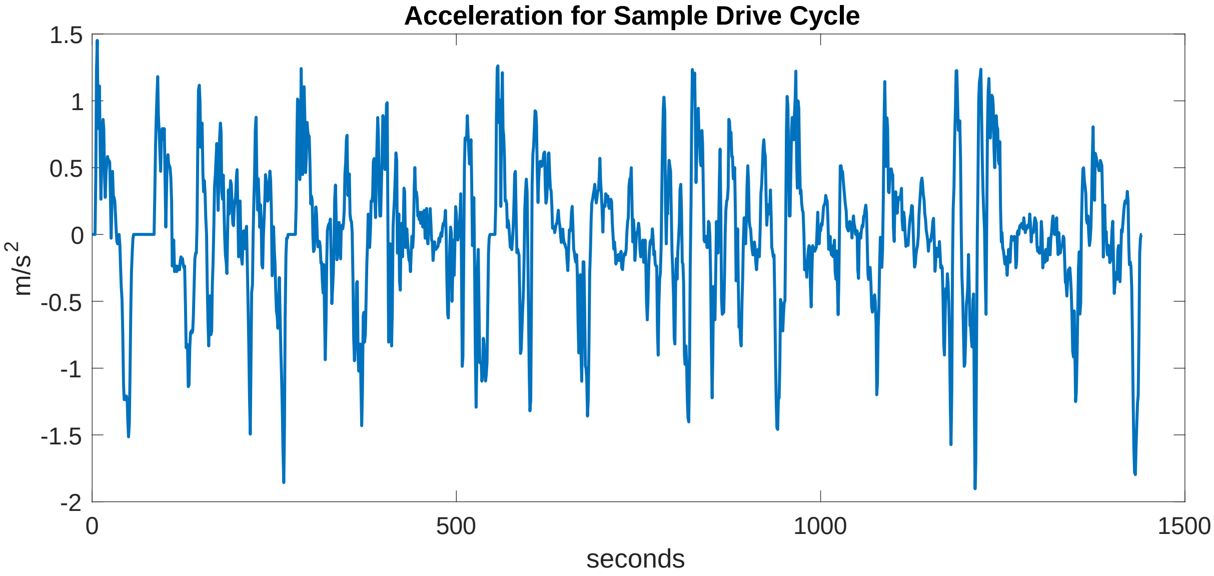
Gravity

Starting Point: How Fast Does The Car Go? This is often determined by recording what a driver does.



Note this is a modest drive cycle…

Speed can be differentiated to get acceleration



Now we can translate force and car speed to what the motor must do

$$W = \frac{R_g}{R_w} u$$

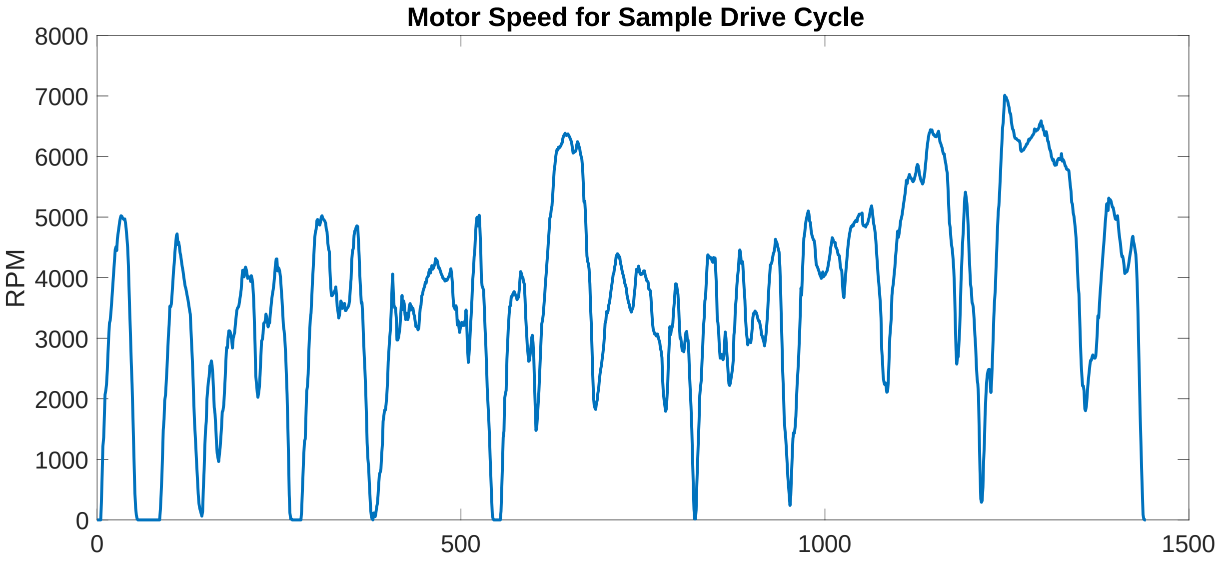
$$T = F \frac{R_w}{R_g}$$

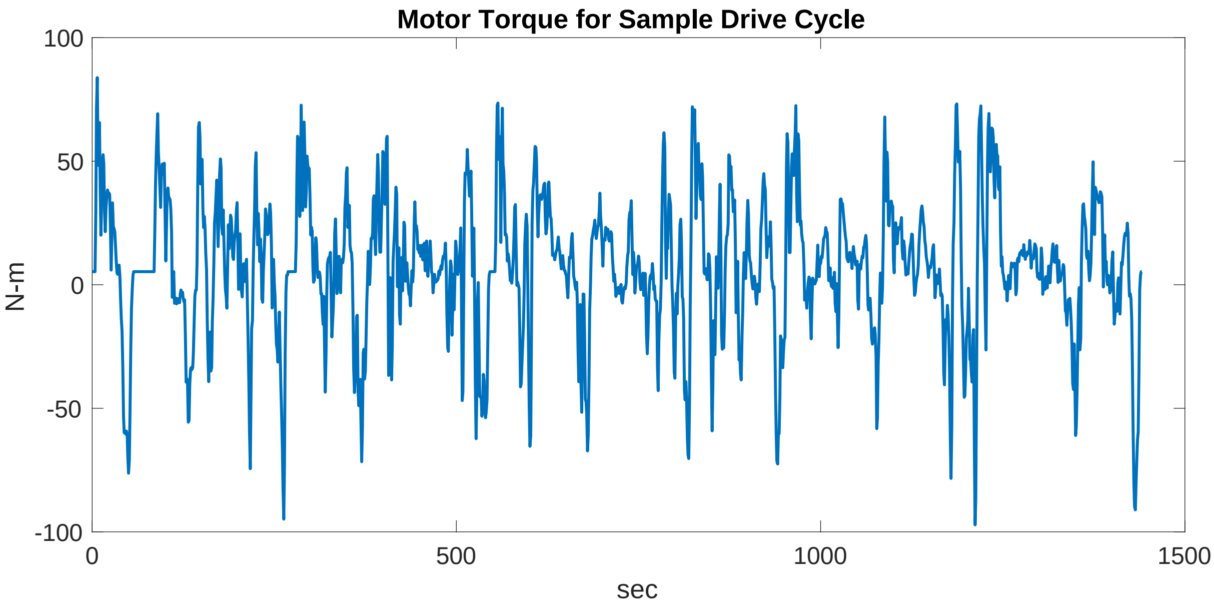
R_g is gear ratio

R_w is wheel radius

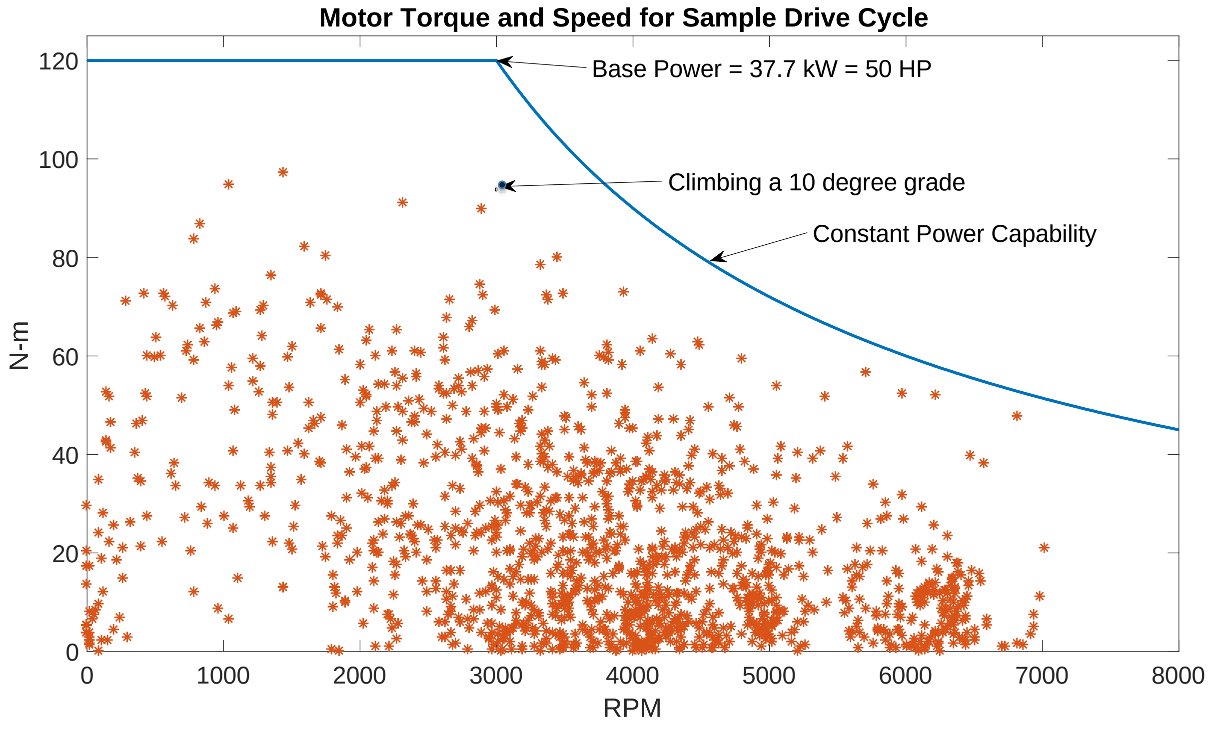
u is speed in m/s

$$N = \frac{60}{2p} W$$




Motor Torque for Sample Drive Cycle

Presented another way, we can superimpose on motor capability (We will have more to say about motor capability later)

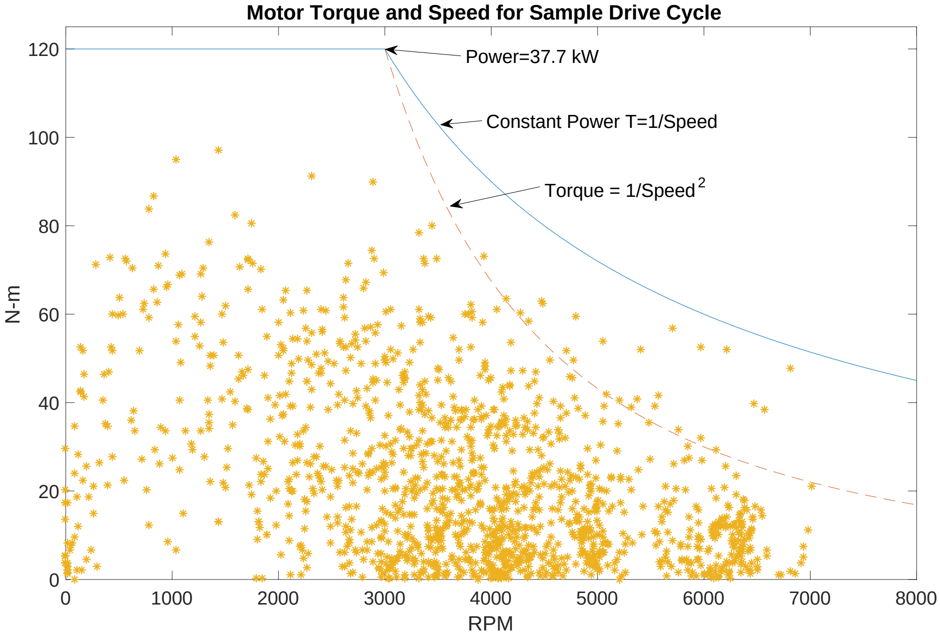

Motor Torque and Speed for Sample Drive Cycle

Base Power = 37.7 kW = 50 HP

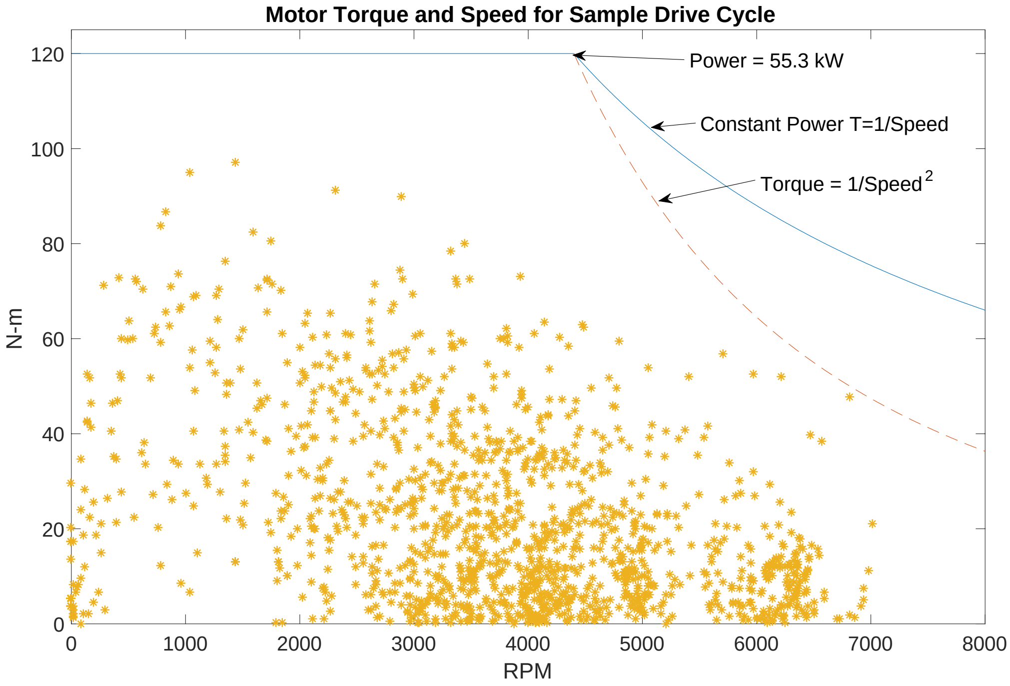
Climbing a 10 degree grade

Constant Power Capability

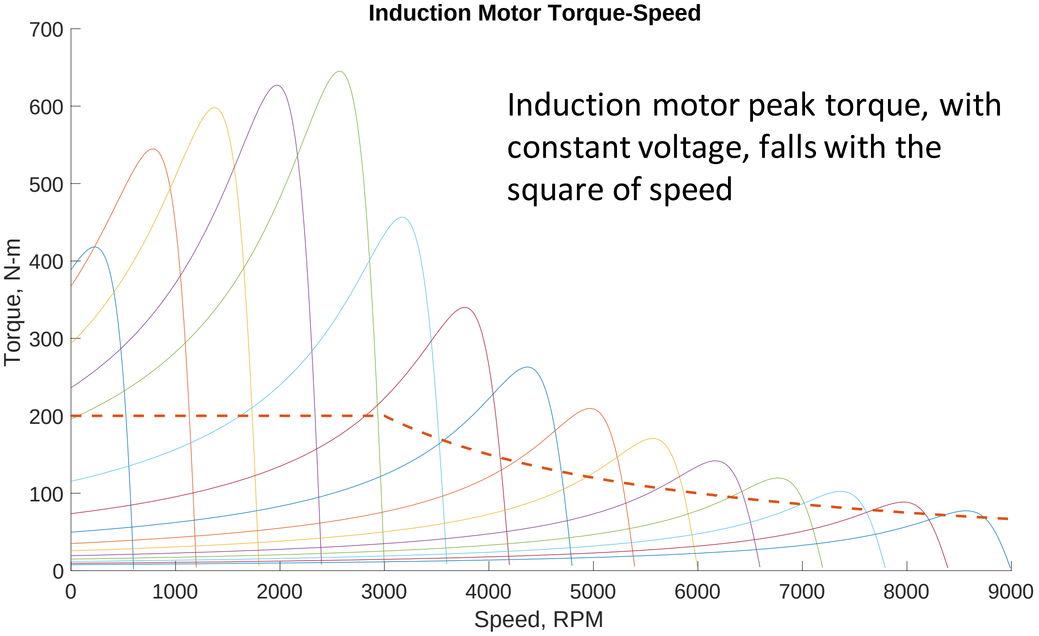
But what if the high speed motor operation is less than constant power?



Setting base speed higher increases motor rating



Here is why you might want to worry about that. An issue with all variable speed machines, if you expect a 'constant power' range of speed



Induction motor peak torque, with constant voltage, falls with the square of speed

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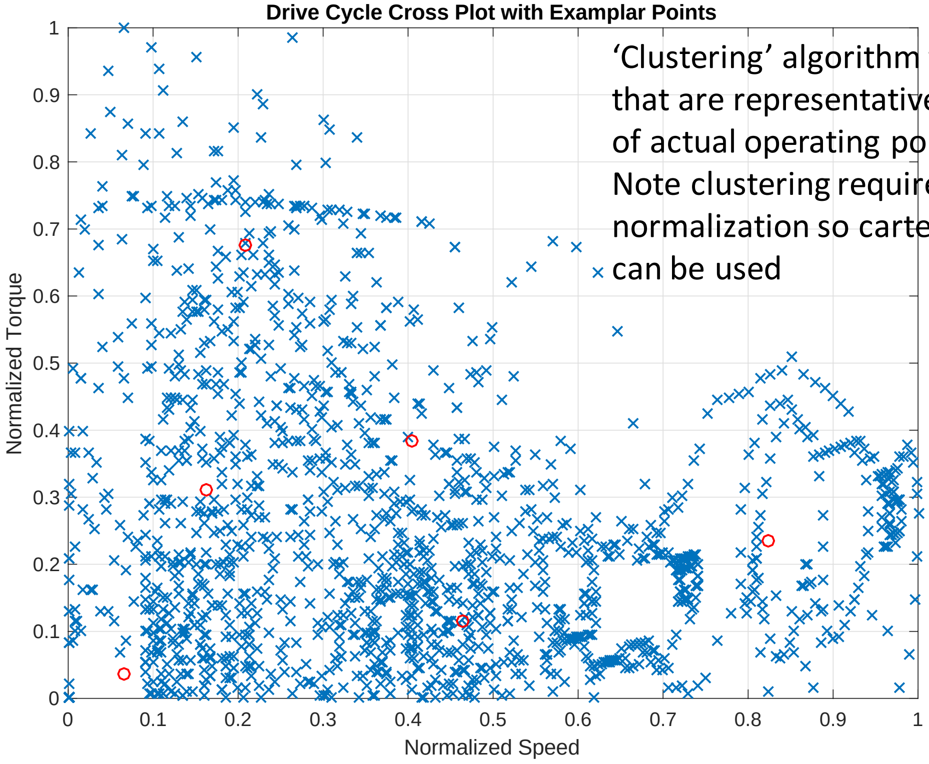
An Auto drive cycle has a large number of operating points



'Clustering' algorithm finds points that are representative of clusters of actual operating points. (Red)
Note clustering requires normalization so cartesian distance can be used

Clustering results

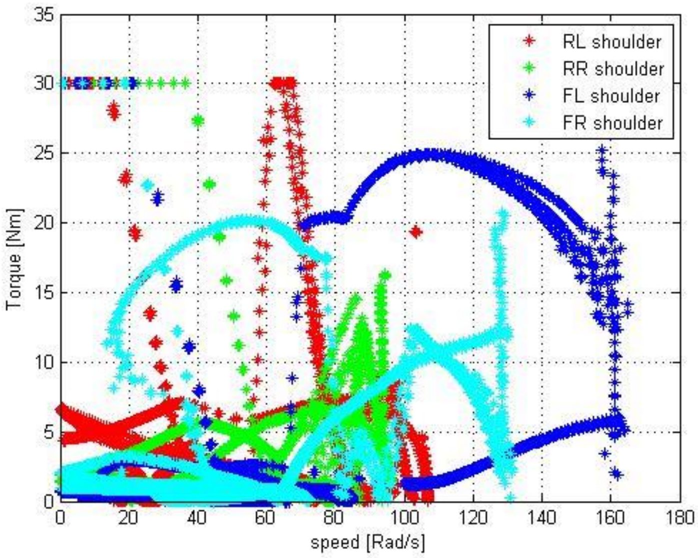
- Representative points have different number of associated points.
- Clustering algorithm repeatedly re-assigns points to clusters, moves cluster points to geometric mean and repeats until no points change clusters.
- ‘Error’ is a measure of the *size* of a cluster (not error)
- For efficiency calculation, weight cluster points by number of actual points in the cluster
- A comparison (with simple motor model) of these 6 points and the full drive cycle indicates agreement in efficiency to about ½%

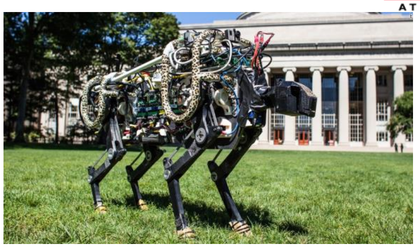
Reading Speed Data
New Sample Points

Speed,	Torque,	Number,	Weight	Error
0.0648	0.0378	429	0.238	0.096
0.4643	0.1149	442	0.245	0.122
0.8243	0.2339	299	0.166	0.155
0.2087	0.6754	202	0.112	0.144
0.1633	0.3110	237	0.132	0.122
0.4039	0.3834	192	0.107	0.129

Motors for Robots have similar requirements

Torque vs. Speed for Cheetah Motors





Cheetah Robot: Torque/Speed

Motor Design Features:

- Design for purpose
- Relatively large diameter, short stack
- High pole order (18)
- High number of slots per pole
- Single stage gear
- High phase order (8)

Here is one of the advanced motors for the Cheetah

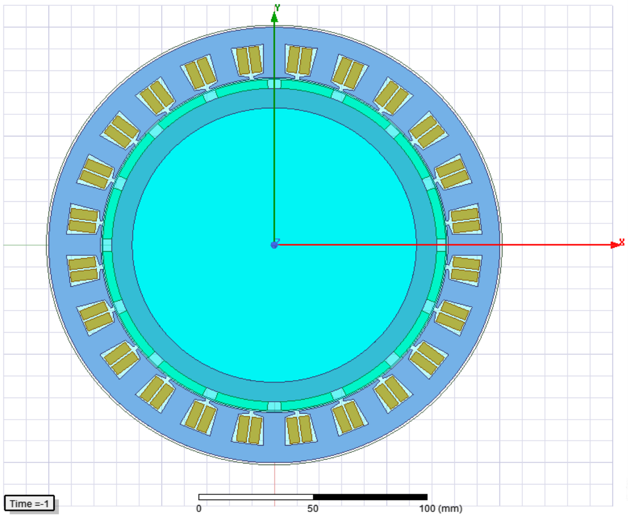
Motor Performance

Motor	Emoteq	MIT #1	MIT #3
Stator Excitation	3φ Sinusoidal	3φ Sinusoidal	8φ Fine-Grain Commutation
Mass [kg]	1.3	1.0	0.86
Max Torque [Nm]	21 ± 25% (Data) 10 (Experiment)	27	30
Motor Constant [Nm/$W^{0.5}$]	0.43	0.51	0.62
Shear Stress [kPa / psi]	110 / 16 52 / 7.5	110 / 16	170/ 24.8

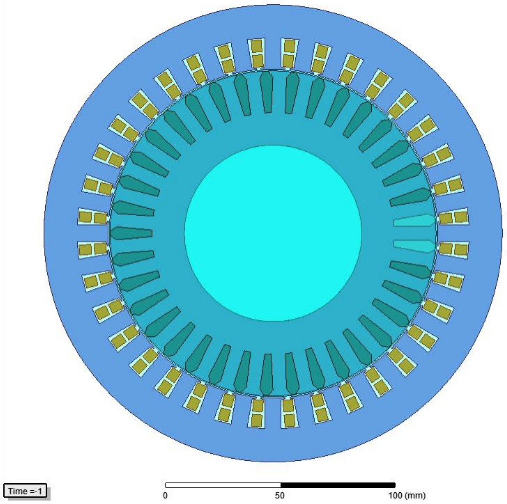
- Common 127-mm OD and 15-mm maximum axial stack length
- Emoteq: Allied Motion HT05001 - Commercial
- MIT #1: N. Farve, SM Thesis, EECS - Experimental
- MIT #2: A. Banerjee et al, ICEM, 2012 - Theoretical
- MIT #3: M. Angle, PhD Thesis, EECS - Built and Tested

Here are the kinds of machines we are looking at most intensively:

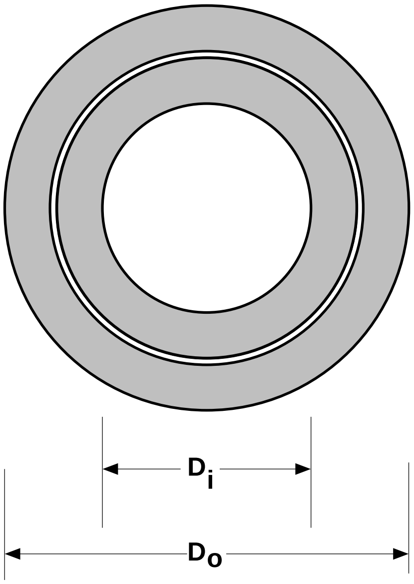
Surface Mount PM

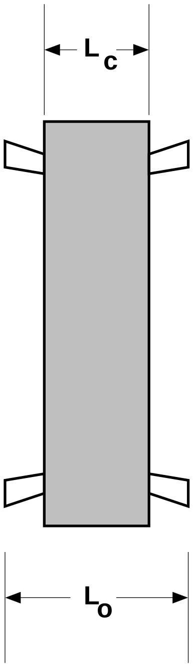


Induction Motor



Motors look sort of like this

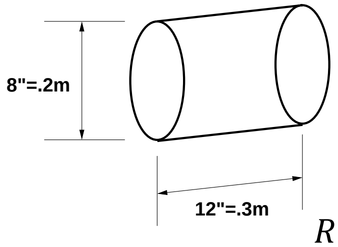


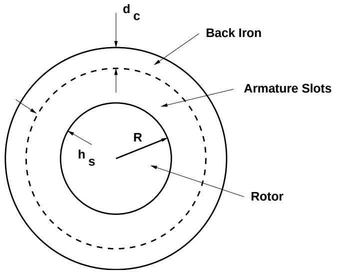


Back of the envelope:
Suppose we want 60 kW at 3,000 RPM:
Torque is:

$$\frac{60{,}000\,W}{300\,rad/\sec} \simeq 200Nm$$

Then suppose we are allowed this envelope:
And allocate the radial regions like this:





$$R = .05m, \ell = .25m$$

$$2\pi R^2 \langle \tau \rangle = T$$

$$\langle \tau \rangle = \frac{200}{2\pi \times .05^2 \times .25} \approx 51\text{ kPa}$$

Roughly what to expect

Motor Type	Average Shear
Standard large industrial induction motor	13 kPa
High performance industrial induction motor	35 kPa
Low Speed Mill Motor	45 kPa
IPS Motor (19 MW, 150 RPM)	76 kP
Advanced Induction Motor	100 kPa
Large Permanent Magnet Motor	120 kPa
Superconducting Synchronous Motor	340 kPa

Required surface current density:

$$<\tau>=\frac{1}{2}|B_1||K_z|$$

(If Peak B_1 = 1 T)

$$K_z=\frac{2<\tau>}{B_1}\simeq 102kA/m$$

How about back iron allocation?

$$B_c=B_1\frac{R}{pd_c}$$

$$if\ B_c=1.5\quad and\ p=5,$$

$$d_c=\frac{B_1}{B_c}\frac{R}{p}=\frac{1}{1.5}\frac{.05}{5}\simeq\frac{2}{3}cm$$

Suppose:

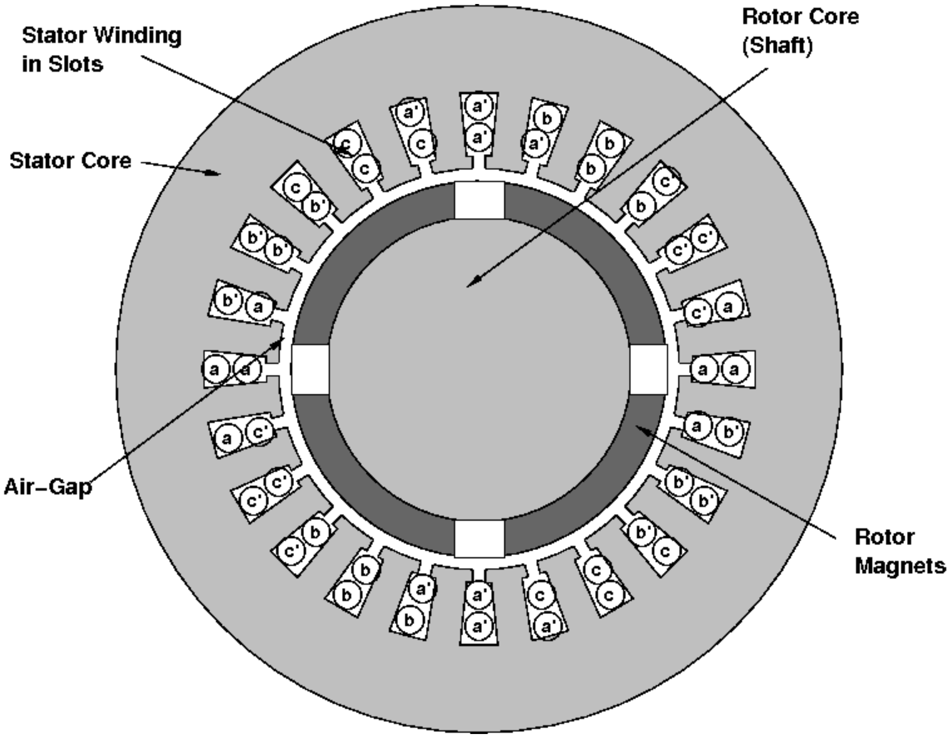
$$h_s\simeq 4cm$$

$$if\ \lambda_s=\frac{1}{2}$$

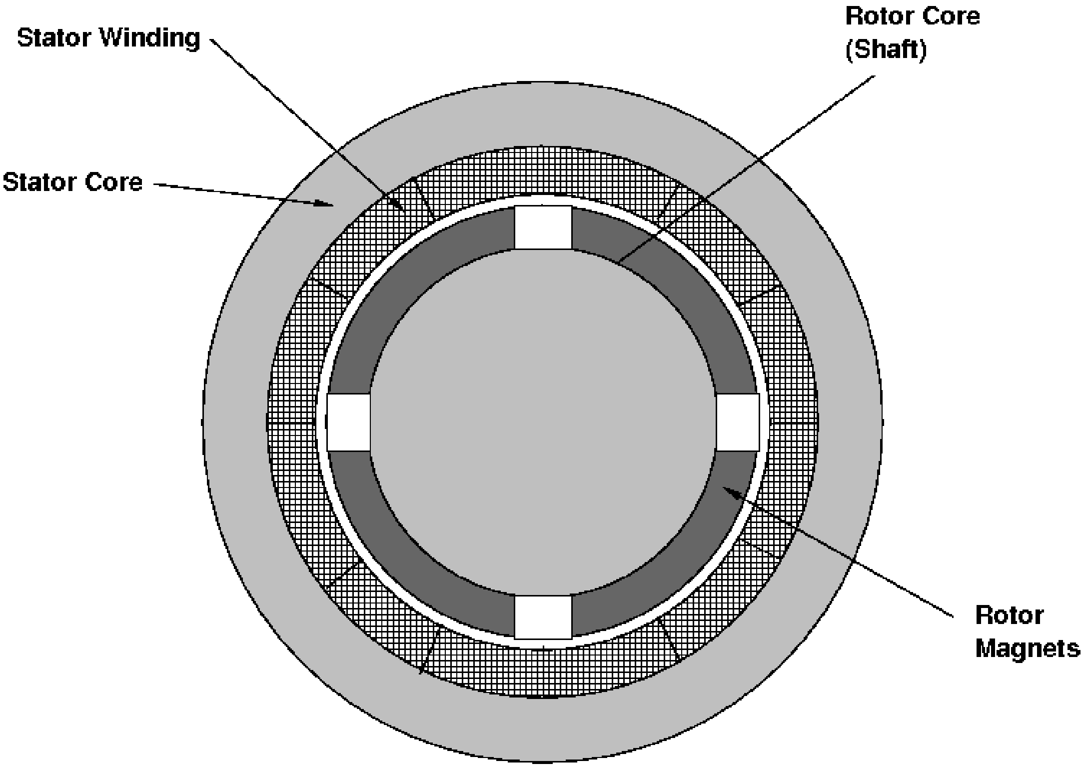
Then:

$$J_z=\frac{101{,}000\,A/m}{\frac{1}{2}\times .04}\simeq 5\times 10^6 A/m^2=5A/mm^2$$

Cross Section View: Surface Magnet Machine: Note windings



Alternate: Surface Mount (‘Iron Free’) Armature Winding



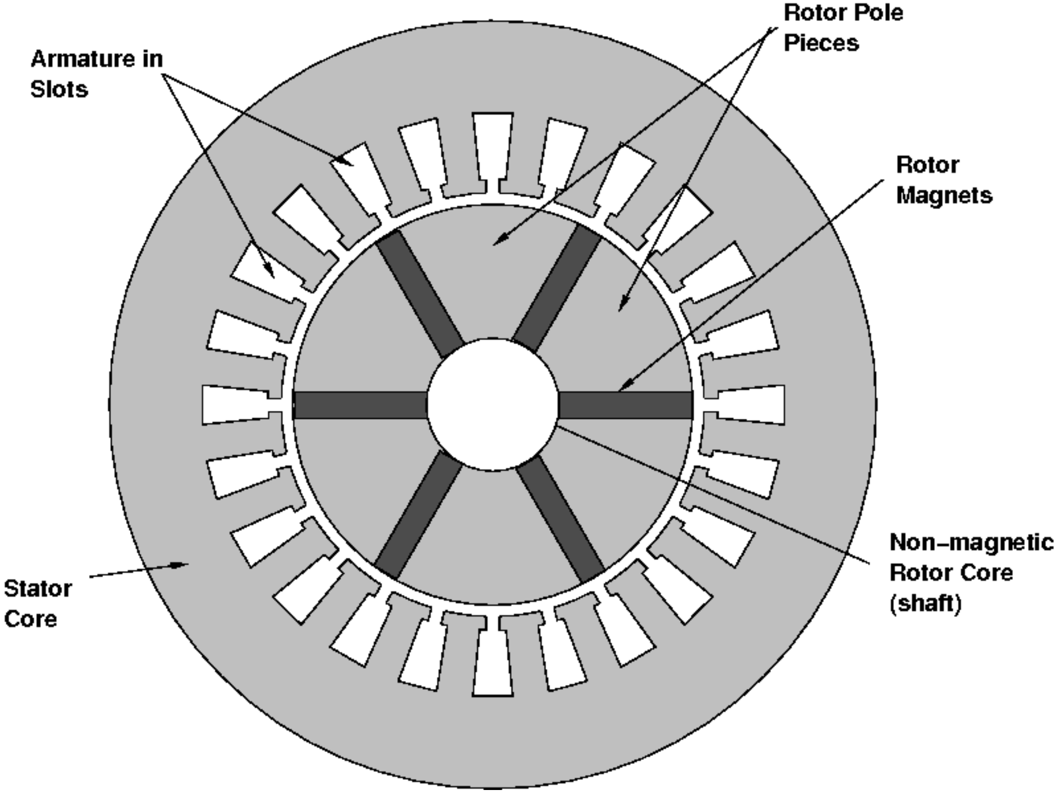
Stator Winding

Rotor Core (Shaft)

Stator Core

Rotor Magnets

Magnets Inside the Rotor



Rotor Pole Pieces

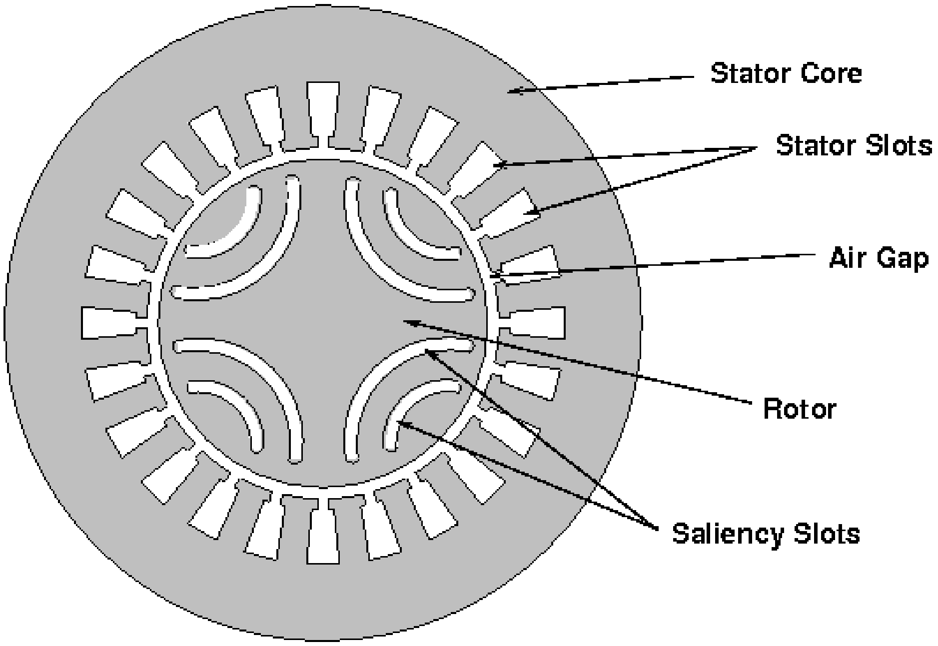
Armature in Slots

Rotor Magnets

Stator Core

Non–magnetic Rotor Core (shaft)

Machine Design for Very High (negative) Saliency



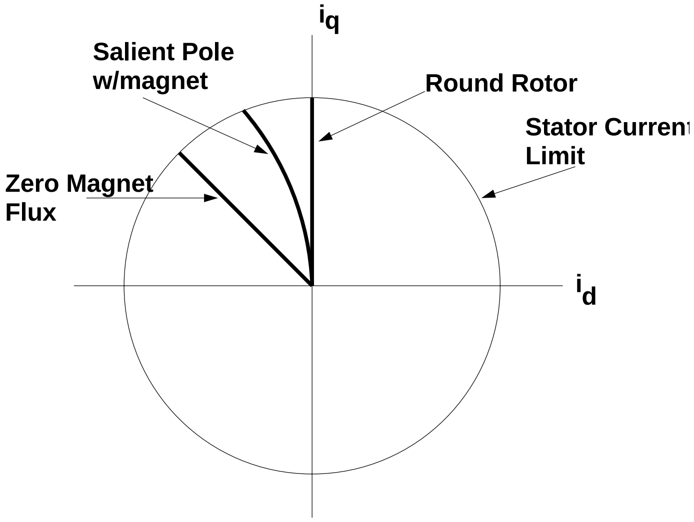
Torque in a 3-phase machine

$$T^e = \frac{3}{2}p\left(\lambda_d i_q - \lambda_q i_d\right)$$

In a machine with permanent magnet excitation this is:

$$T^e = \frac{3}{2}p\left(\left(\lambda_f + L_d i_d\right) i_q - L_q i_q i_d\right)$$

$$= \frac{3}{2}p\left(\lambda_f i_q - \left(L_q - L_d\right) i_d i_q\right)$$



Note that per-unit flux achievable for a given terminal voltage is:

$$y = \frac{V}{wl_B}$$

And this is related to current by:

$$y^2 = \left(1 + x_d i_d\right)^2 + \left(x_q i_q\right)^2$$

This is the equation that describes an ellipse in the i_d, i_q plane

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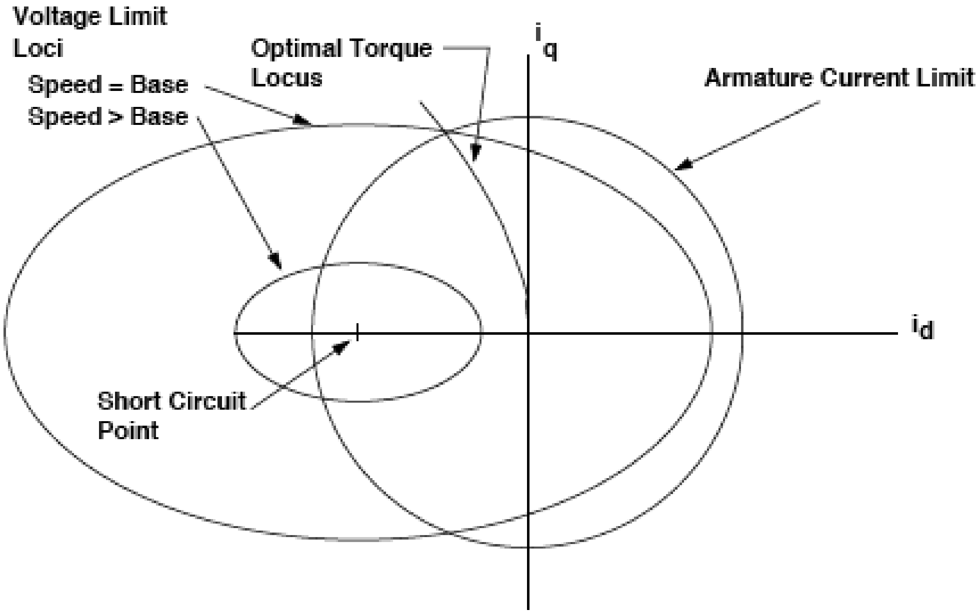
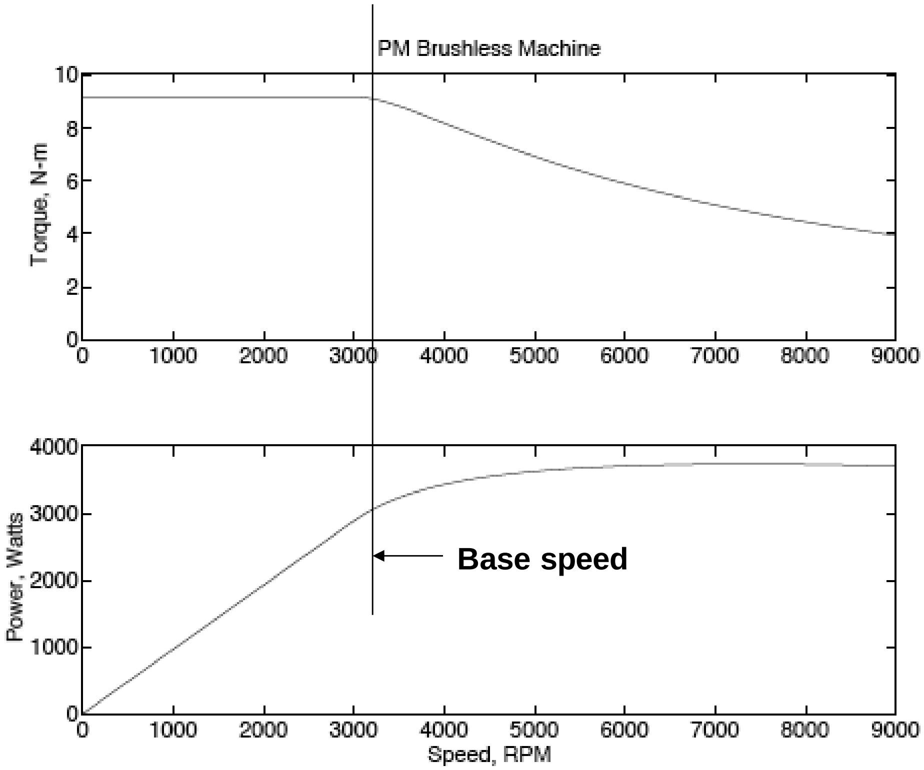
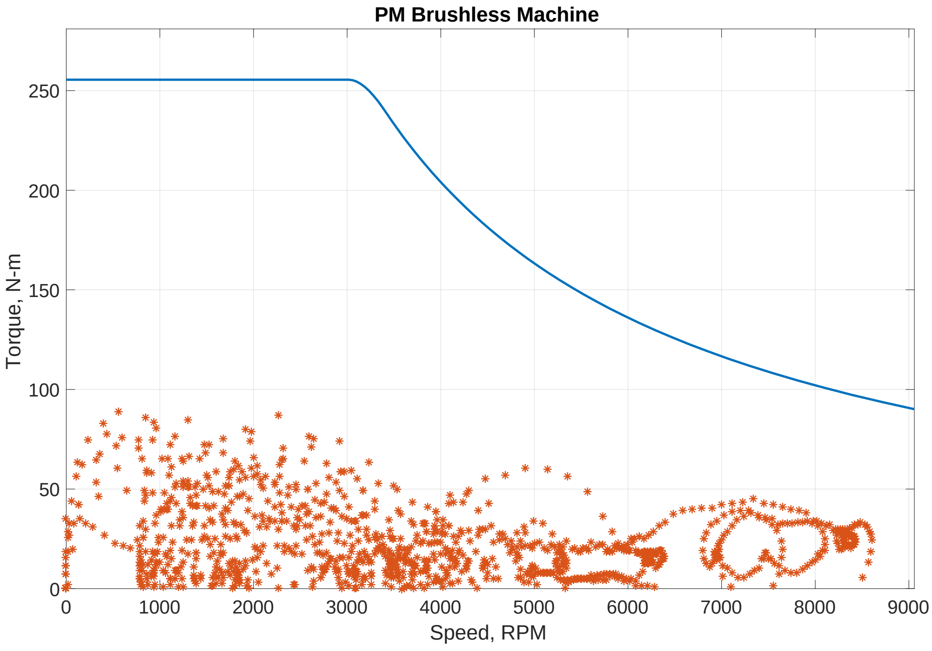


Here is torque-speed capability of a particular motor

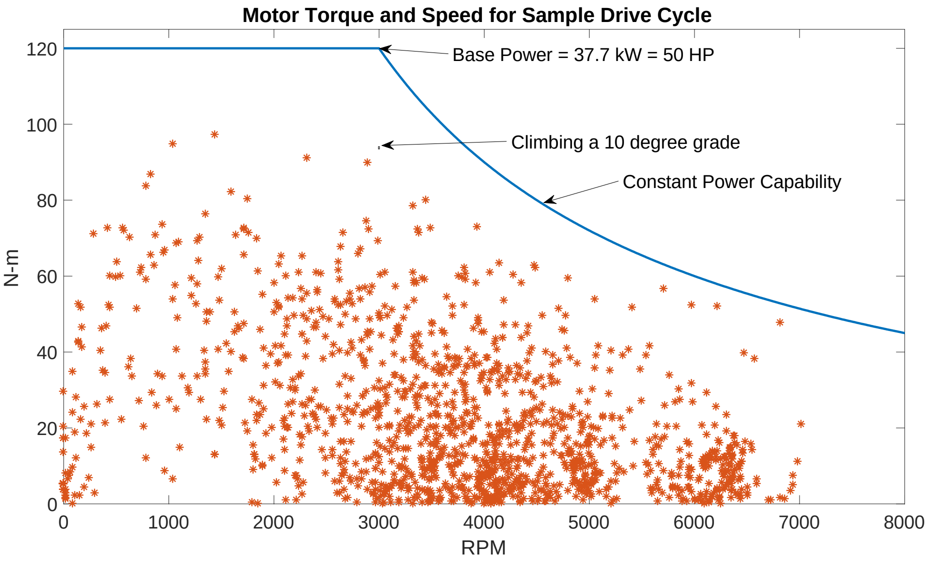


Base speed

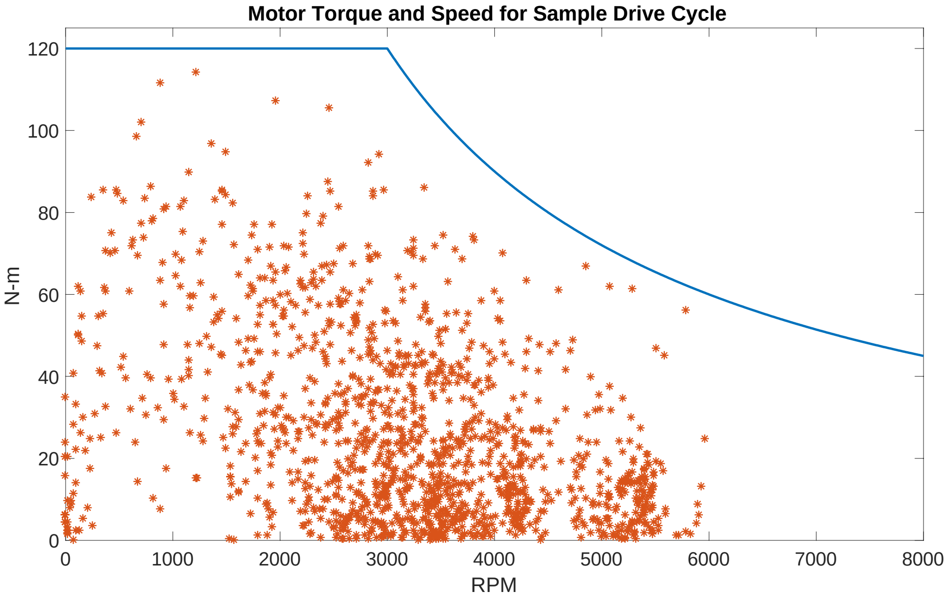
Capability of one of the PM Motors
A bit oversized for that small car…



We have seen this one before: Drive cycle superimposed on motor capability: gear ratio is 10



This is the same situation with a gear ratio of 8

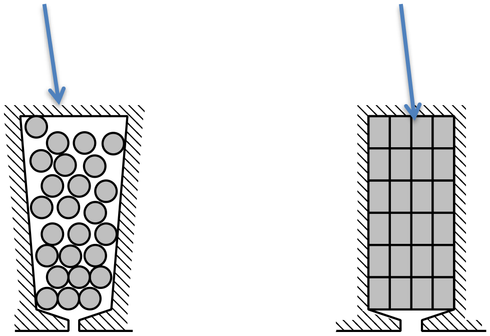


And there are a number of things to consider

- Heat Removal from Stator $\text{Thermal Flux} = P\frac{1-h}{h}$

  These machines are likely water jacket cooled
- Heat Removal from rotors
  - Fluid internal to rotor?
  - Forced Air?
  - Across the Air Gap?
- Magnet Heating: Drag Loss, Space Harmonics, Inverter Harmonics
- From Conductors in Slots:

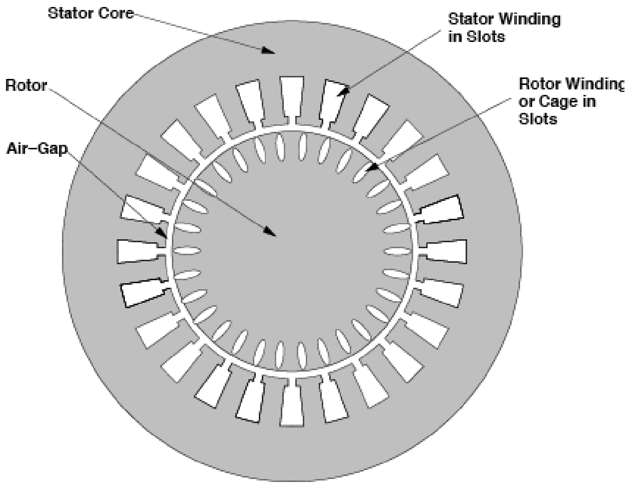
Round Conductors Rectangular Conductors

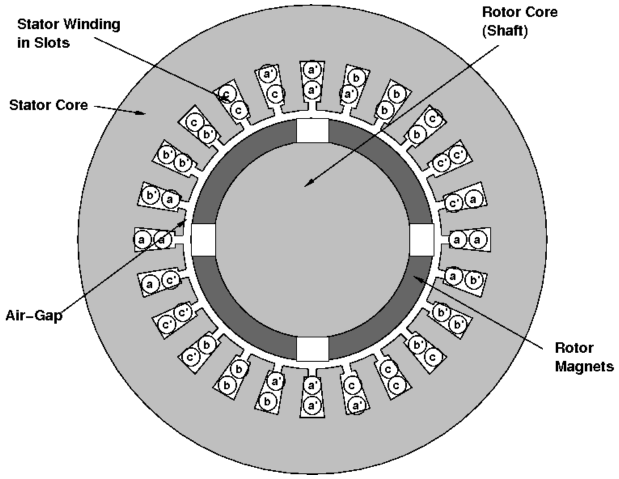


Thermally Conductive Encapsulant?

Close Packing: Better Thermal Conductance
Maybe good reason to use rectangular slots?
Concentrated Coils?
Hairpin Construction?

Look at the two machine types:

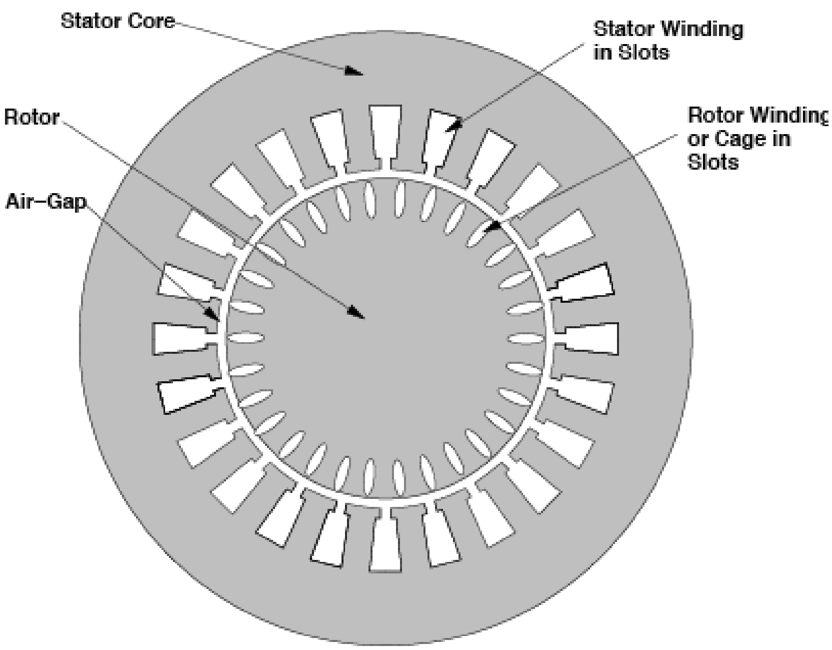




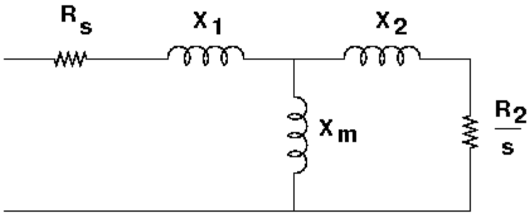
Note they are not a whole lot different:

- Flux Density about the same
- PM machine can support higher reaction current
  - IF the limit is heating
- I designed an Induction Motor
- Used data from a Prius drive motor for the PM

Some of the important parts of an induction motor



Here is the conventional equivalent circuit



$$X_m = \frac{3}{2}\frac{4}{\pi}\frac{\mu_0 R \square N_s^2 k_s^2}{p^2 g}$$

$$X_2 = \omega \frac{12 \square N_s^2 k_s^2}{N_R} L_{\text{slot}} + \omega \frac{6}{\pi}\frac{\mu_0 R \square N_s^2 k_s^2}{g}\left(\frac{1}{(n_+ p)^2} + \frac{1}{(n_- p)^2}\right)$$

$$R_2 = \frac{12 \square N_s^2 k_s^2}{N_R} R_{\text{slot}}$$

Stator elements R_s and X_1 are calculated using ordinary but detailed methods

Ordinary circuit techniques give us I_2 and then the first law of thermodynamics gives us output power and torque

$$P_{ag} = 3\left|I_2^2\right|\frac{R_2}{s}$$

$$P_D = 3\left|I_2^2\right|R_2$$

$$P_{mech} = P_{ag} - P_D = 3\left|I_2^2\right|\frac{R_2}{s}(1-s) = T\Omega = T\frac{\omega(1-s)}{p}$$

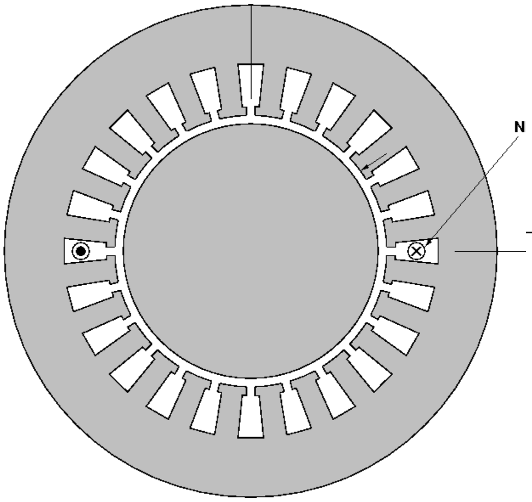
$$T = 3\frac{p}{\omega}\left|I_2^2\right|\frac{R_2}{s}$$

Of course this picture is far too simplified to be of any use. It is missing at least:

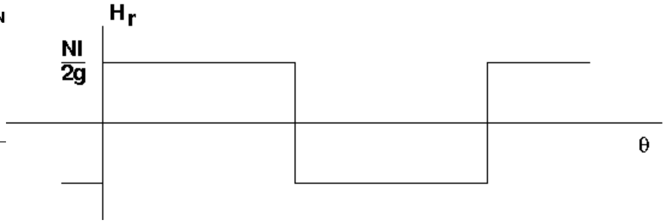
- Rotor End Rings
- Core Loss
- Stray Load and No-Load Loss
- Friction and Windage Loss

But all of that stuff can be added, and it still falls short.

One thing: windings are lumpy: Consider just one coil in the stator (one coil per pole per phase):



Air-Gap Magnetic Field looks like this:

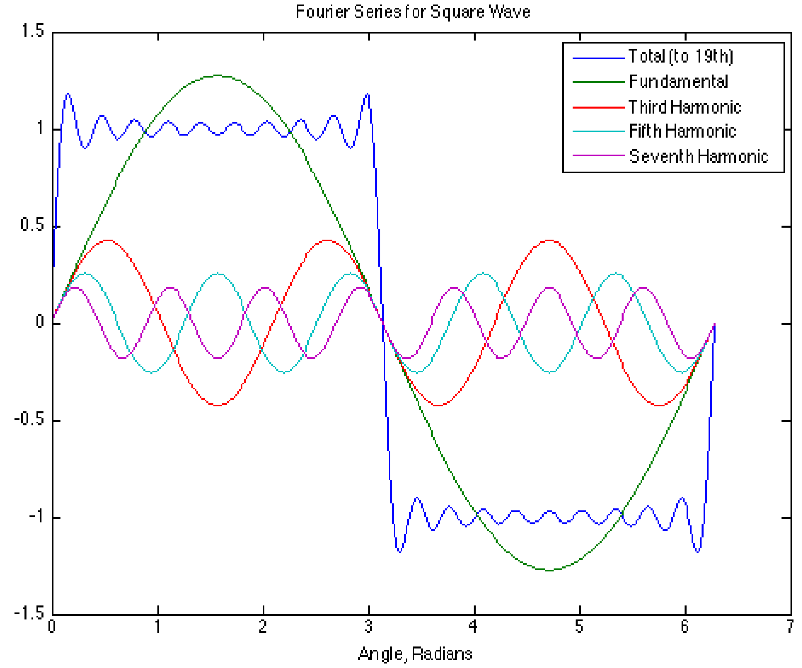


$$H_r = \sum_{\text{n odd}} \frac{4}{n\pi}\frac{NI}{2g}\sin(n\theta)$$

$$= \sum_{\text{n odd}} H_n \sin(n\theta)$$

$$H_n = \frac{4}{n\pi}\frac{NI}{2g}$$

The total MMF is the sum of an infinite number of space harmonics: here are the first few and the sum of orders up to 19th space harmonic



Now suppose we have a three-phase winding with balanced three phase currents: the total field for one harmonic will be like this:

$$H_{rn} = H_n \left\{ \begin{array}{l} \sin(npq)\cos(wt) \\ +\sin n\left(pq - \frac{2p}{3}\right)\cos\left(wt - \frac{2p}{3}\right) \\ +\sin n\left(pq + \frac{2p}{3}\right)\cos\left(wt + \frac{2p}{3}\right) \end{array} \right\}$$

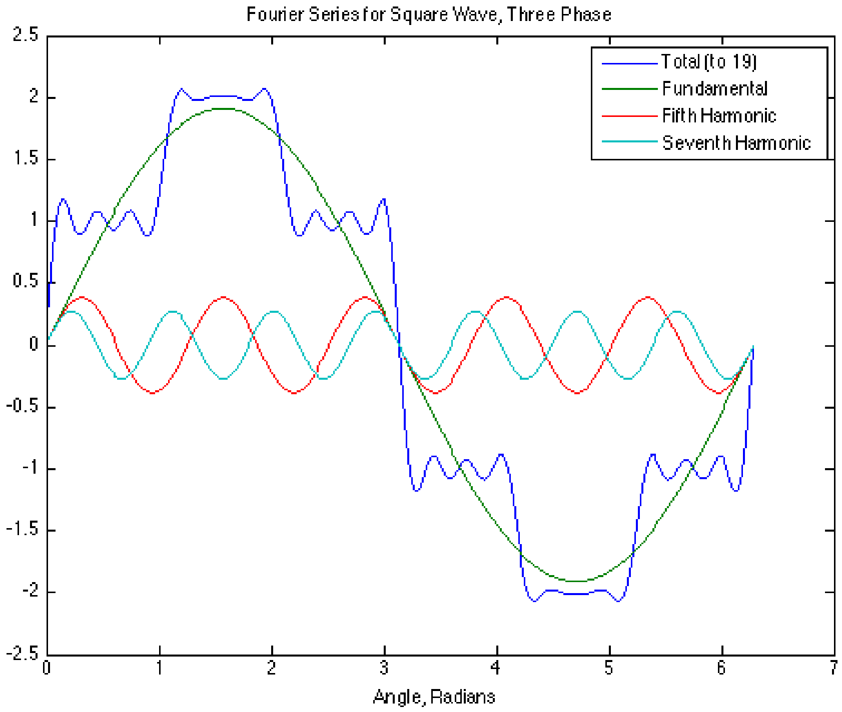
$$= \frac{H_n}{2} \left\{ \begin{array}{l} \sin(npq + wt) + \sin(npq - wt) \\ +\sin\left(npq + wt - (n+1)\frac{2p}{3}\right) + \sin\left(npq - wt - (n-1)\frac{2p}{3}\right) \\ +\sin\left(npq + wt + (n+1)\frac{2p}{3}\right) + \sin(npq - wt) + (n-1)\frac{2p}{3} \end{array} \right\}$$

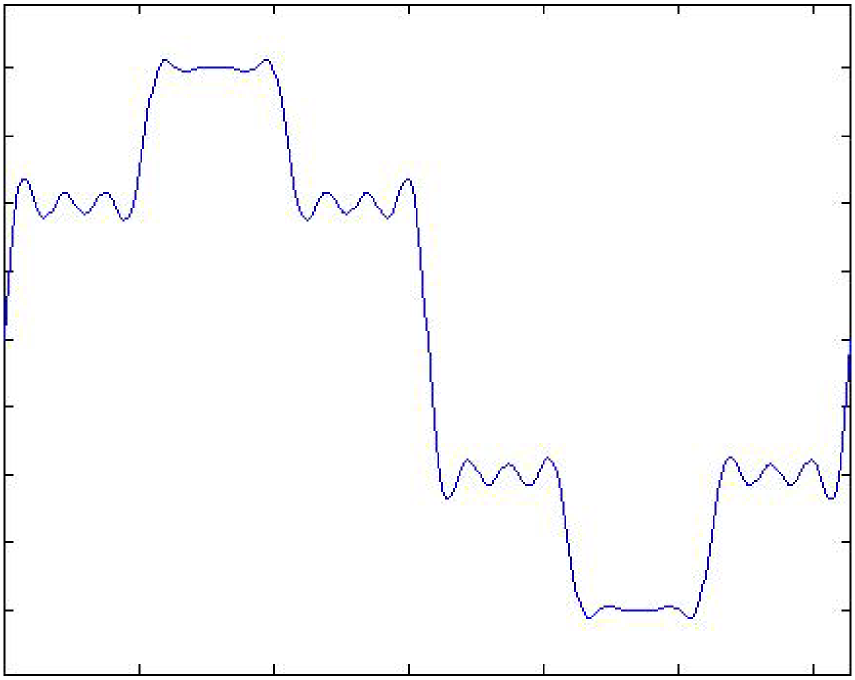
This give us two flavors of traveling wave in the air gap (forward and backward)
Note the triplens go away.

$$H_{rn} = \frac{3}{2} H_n \sin(npq - wt) \quad \text{for} \quad \text{n=1,7,13,...}$$

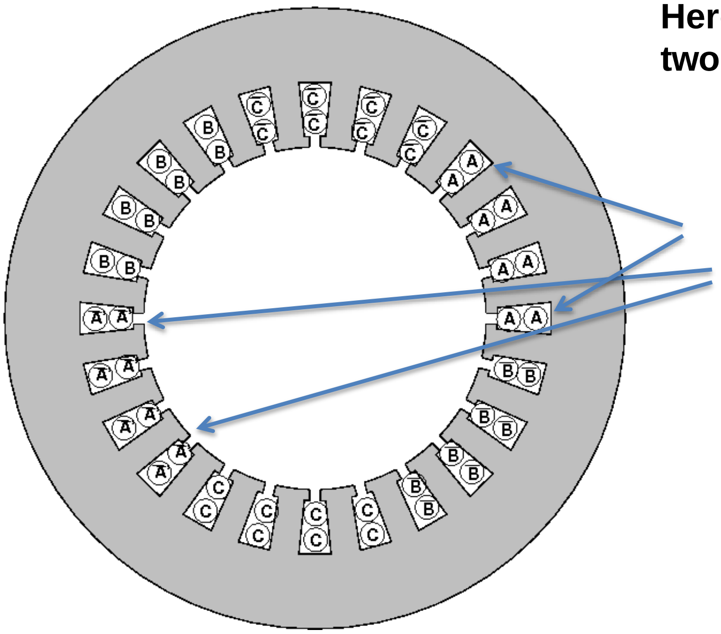
$$H_{rn} = \frac{3}{2} H_n \sin(npq + wt) \quad \text{for} \quad \text{n=5,11,...}$$

This is the Fourier Series at t=0, Same Orders: You can see where the active currents are





Winding Format is Important Here is a 24 slot stator with a two-layer, full pitch winding



Phase A winding between these slots And these slots

End windings connect upper layer on one side with lower layer on the other side

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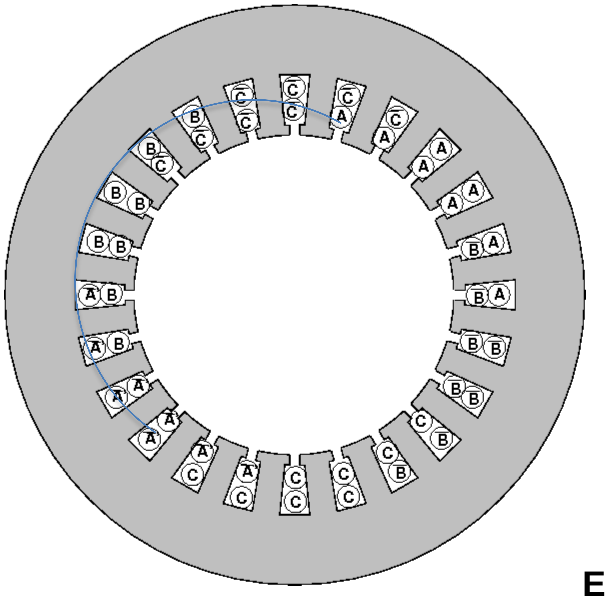
To use all of the slots, we have a distributed winding



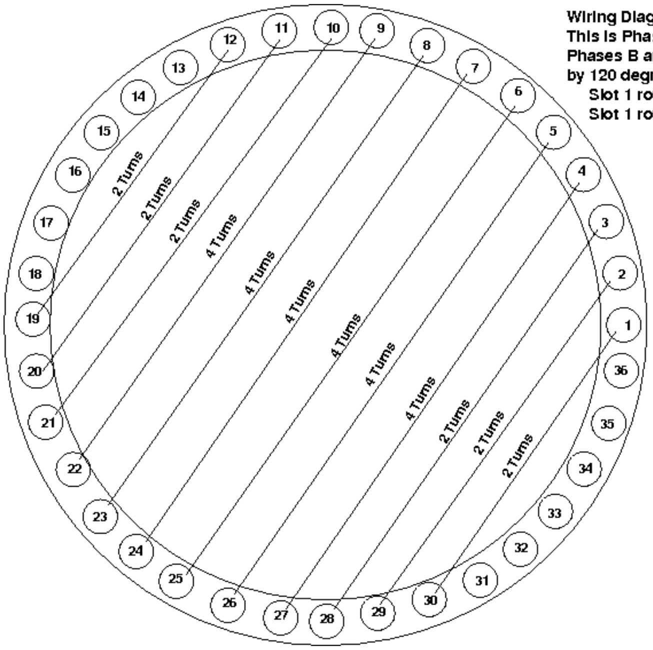
To improve performance, we often 'short pitch' a winding See how the inside layers of the winding have been slid 2 slots in the counter-clockwise direction. This one is called 5/6 pitch (really 10/12:

- coil throw is 10 slots,
- full pitch throw would be 12 slots.

End turns are shorter and this will have an impact on space harmonics

A ‘concentric’ winding pattern might be like this: with

- Turns count of 2-2-2-4-4-4
- Coil Throw of 7-9-11-13-15-17 slots



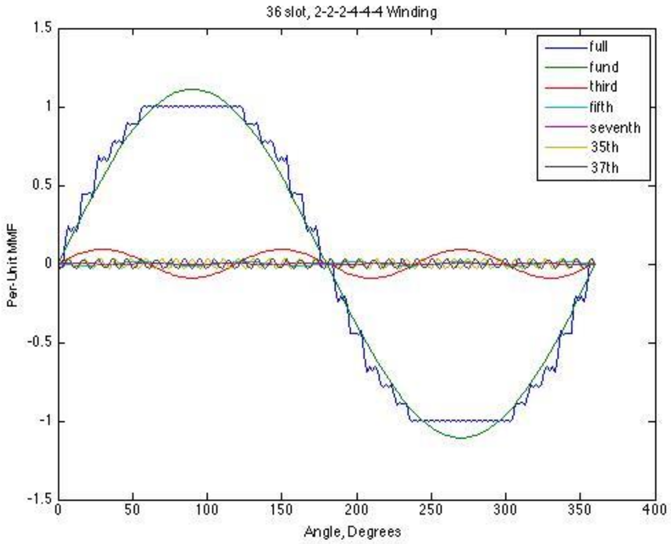
Wiring Diagram
This is Phase A
Phases B and C are this rotated
by 120 degrees:
Slot 1 rotates to Slot 13 for B
Slot 1 rotates to Slot 25 for C

Note that three sets of turns like this fits, with an overlap of 3 slots at the ends of each of the three phase belts

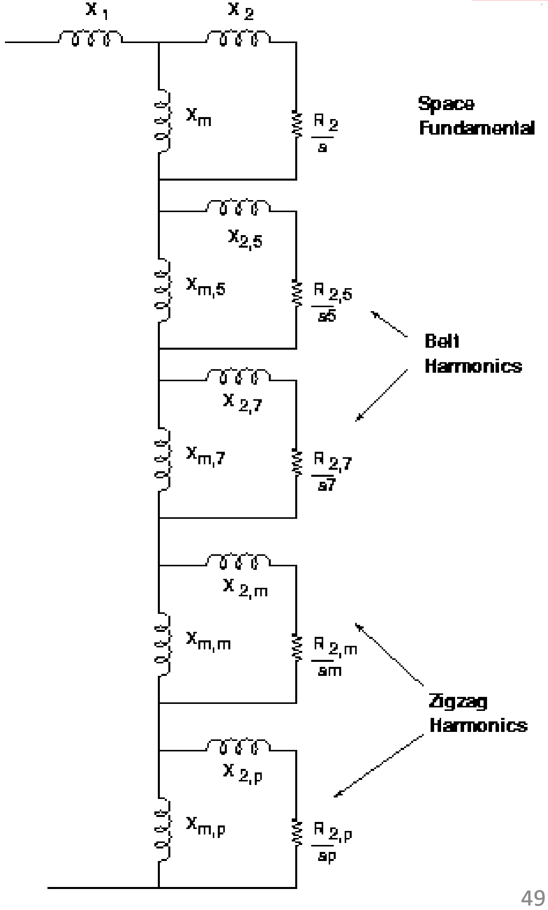
---

Here is a pretty good approximation to the waveform for one phase. It has space harmonics:

- For a three-phase winding, third harmonics cancel
- Fifth and Seventh are both quite small for this case
- Slot order +/- 1 are bigger (but still not really big)



We can build up an equivalent circuit for the induction motor:



Space harmonics are driven by the same stator current and their fluxes add.
So the equivalent circuit should look like this
Major harmonics are 5, 7 and $\frac{N_s}{p} \mp 1$

Parameters of the space harmonic circuits can be estimated just like the fundamental, but note the frequencies and slips are all different

---

Frequency 'seen by' the rotor conductors:

$$H_{rn} = \frac{3}{2} H_n \sin(np\theta \mp \omega t)$$

$$\theta = \theta' + (1-s)\frac{\omega}{p} t$$

$$H_{rn} = \frac{3}{2} H_n \sin\left(np\theta' + \left((n \mp 1) - ns\right)\omega t\right)$$

$$\omega' = \left((n \mp 1) - ns\right)\omega$$

Space fundamental running (n=1), s small, (small)
Space fundamental starting (n=1), s=1,
Harmonics running (n=5,7,…) s small
Note some harmonics (5, 11,…) frequency is negative (these are 'backward rotating')

$$w_1' = sw$$

$$w_1 = w$$

$$|\omega'_n| \approx (n \mp 1)\omega$$

So if rotor impedances are frequency dependent, we need to be cognizant of rotor frequency and its effects.

Induction Motor Power Evaluation:

- Circuit (last slide) can be evaluated for currents
- Power across the air gap for each harmonic is:

$$P_{agn} = 3\left|I_{2n}\right|^2 \frac{R_{2n}}{S_n}$$

- Dissipation for each harmonic is:

$$P_{dn} = 3\left|I_{2n}\right|^2 R_{2n}$$

- And torque for each harmonic is:

$$T_n = p\frac{P_{agn}}{w_n}$$

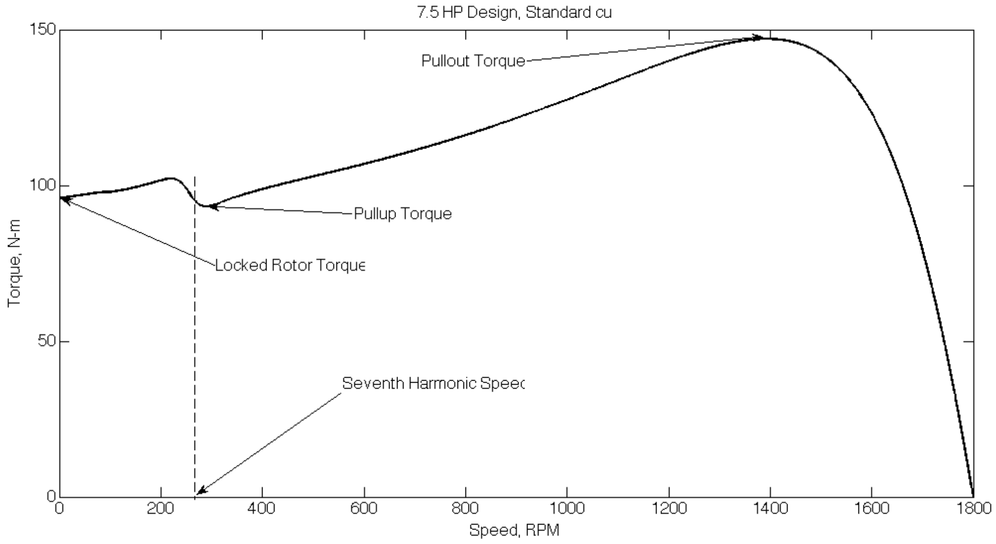
- Then power input, power dissipated and torque converted can be simply added up
- (and had better match!)

A few design vignettes to entertain and alarm

- Improper Pitch in an Induction Motor
- Optimal Slip for Automotive Drive
- Air Gap Trade Study and a Warning
- Garbage In, Garbage Out
- Another trade study: duty cycle

An example of space harmonic effect: the lowest order space harmonic (aside from fundamental) that affects torque profile is seventh.

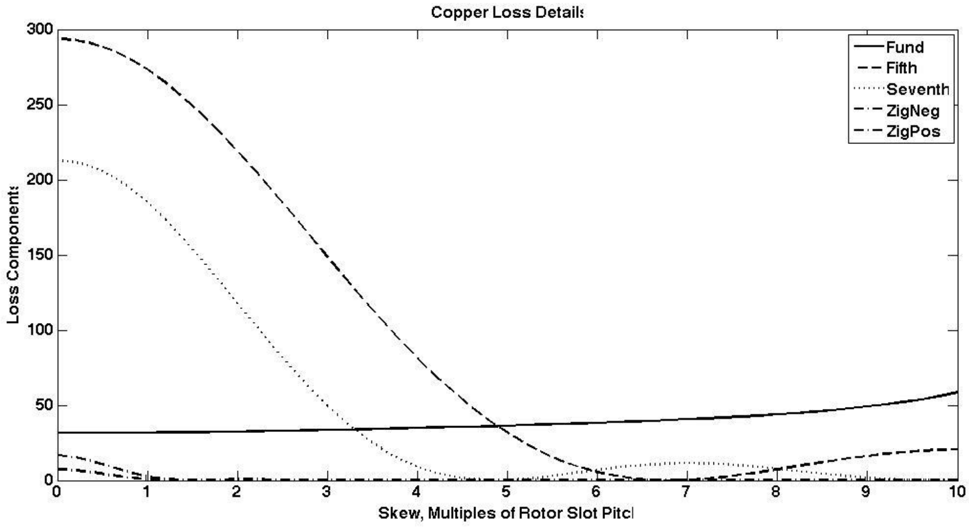
This is important for across the line start, but not so much for traction motors. But harmonics still cause losses



This is about a 48 slot, 4 pole motor (think Tesla Car)

Plotted is harmonic loss (rotor), harmonic by harmonic, plotted against rotor skew

- This is for the 2/3 pitch winding
- ‘Skew’ is rotor bar rotation from one end to the other
- This is an example of a badly designed stator:
- Loss estimated for each harmonic circuit



Here are four different ways of laying out that winding

Full and 5/6 pitch correspond with earlier slides

	24 Slot, 2 Pole Winding																							
Slot #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

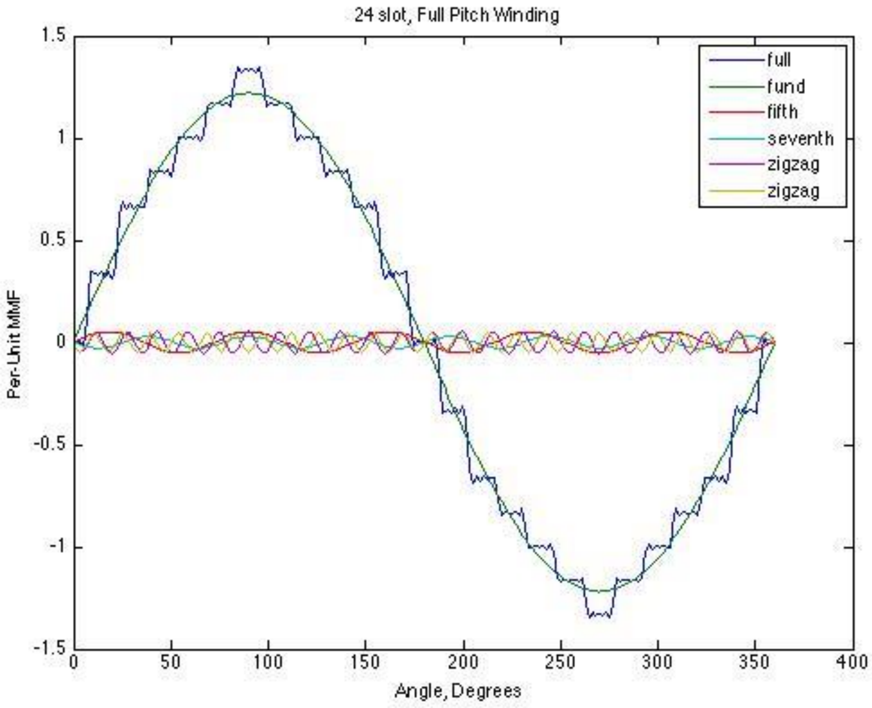
	Full Pitch, Two Layer																							
Top	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C	B'	B'	B'	B'
Bottom	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C	B'	B'	B'	B'

	2/3 Pitch, Two Layer																							
Top	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C	B'	B'	B'	B'
Bottom	B'	B'	B'	B'	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C

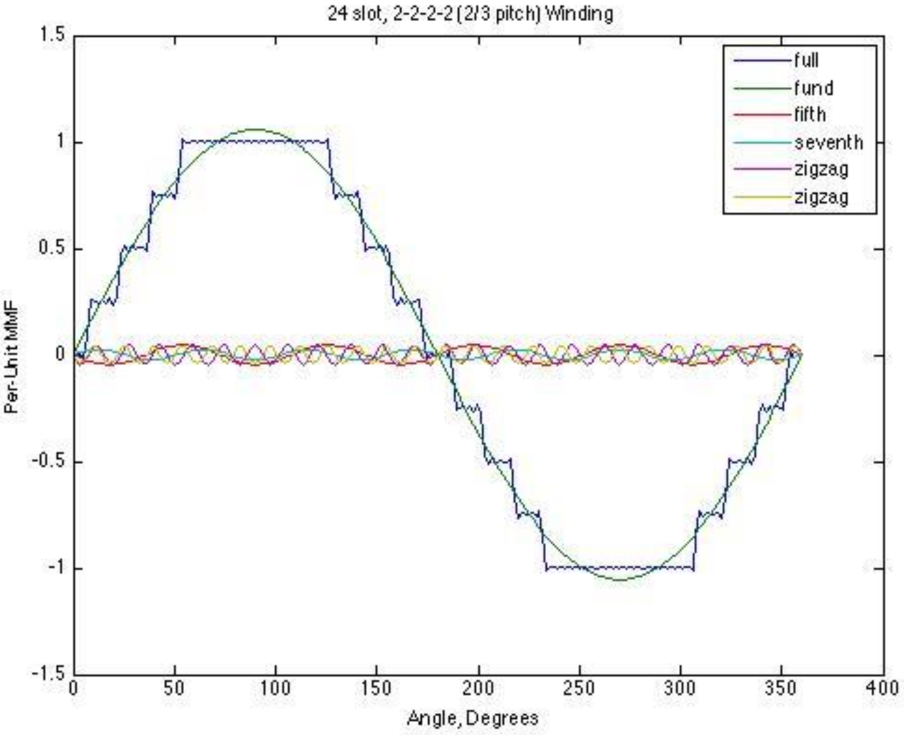
	5/6 Pitch, Two Layer																							
Top	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C	B'	B'	B'	B'
Bottom	B'	B'	A	A	A	A	C'	C'	C'	C'	B	B	B	B	A'	A'	A'	A'	C	C	C	C	B'	B'

	Concentric, Lapped Winding Ratio of Turns																							
PhaseA	1	2	3	3	2	1							-1	-2	-3	-3	-2	-1						
PhaseB	-2	-1							1	2	3	3	2	1							-1	-2	-3	-3
PhaseC					-1	-2	-3	-3	-2	-1							1	2	3	3	2	1		

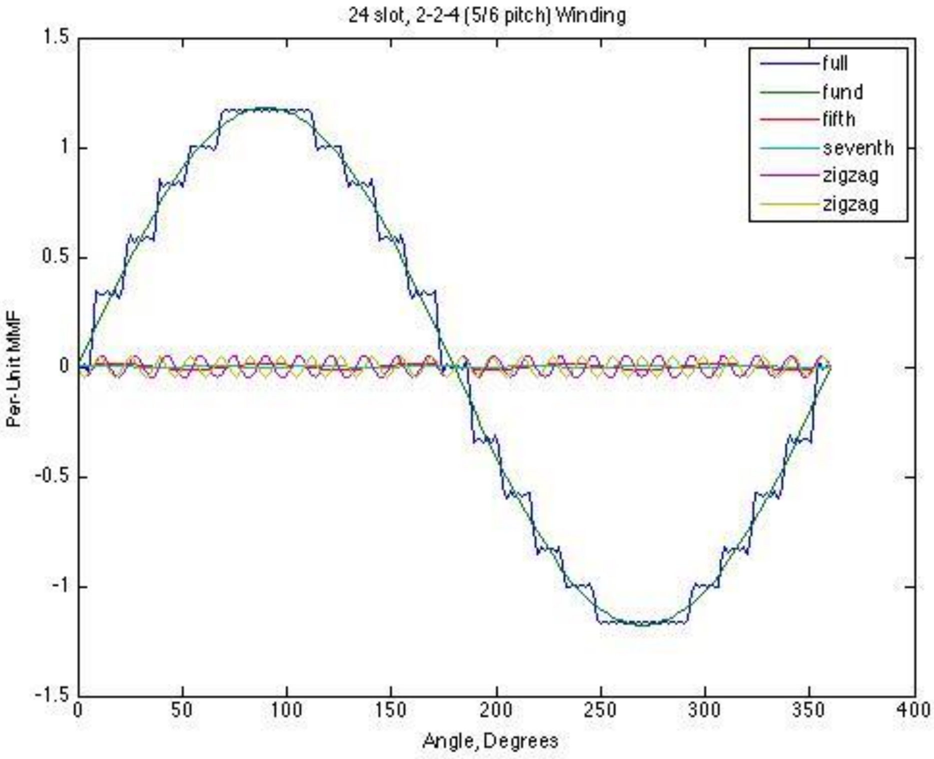
Fourier Analysis of the Full Pitch, Three Phase Winding



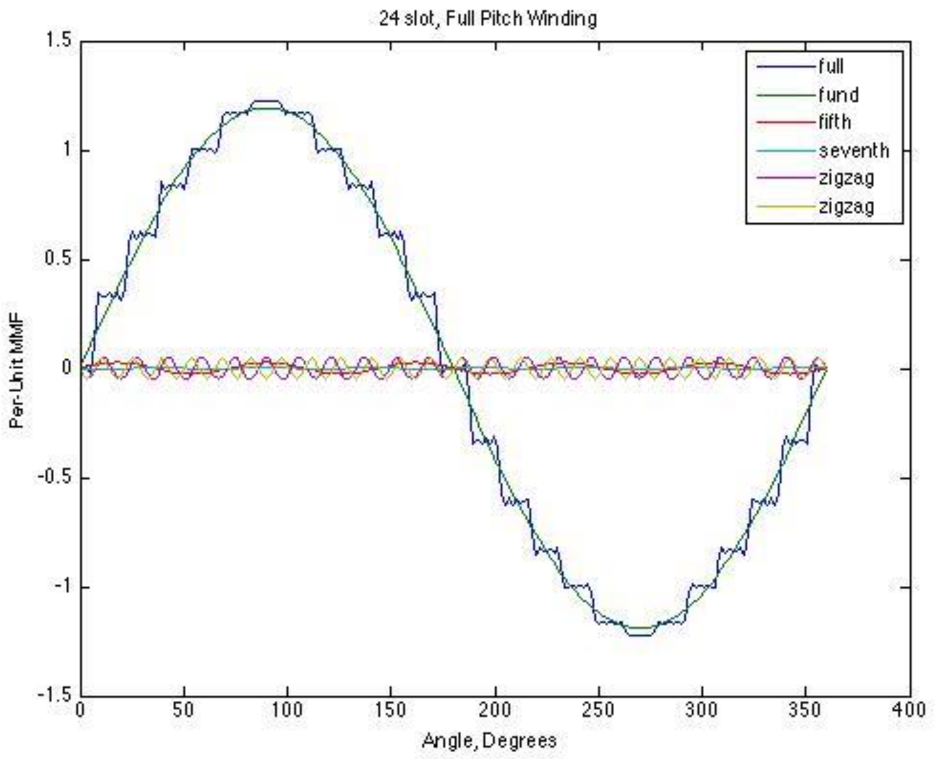
Same analysis for a 2/3 pitch winding



Note the 5/6 pitch winding has smaller 5th and



The concentric winding is also pretty good



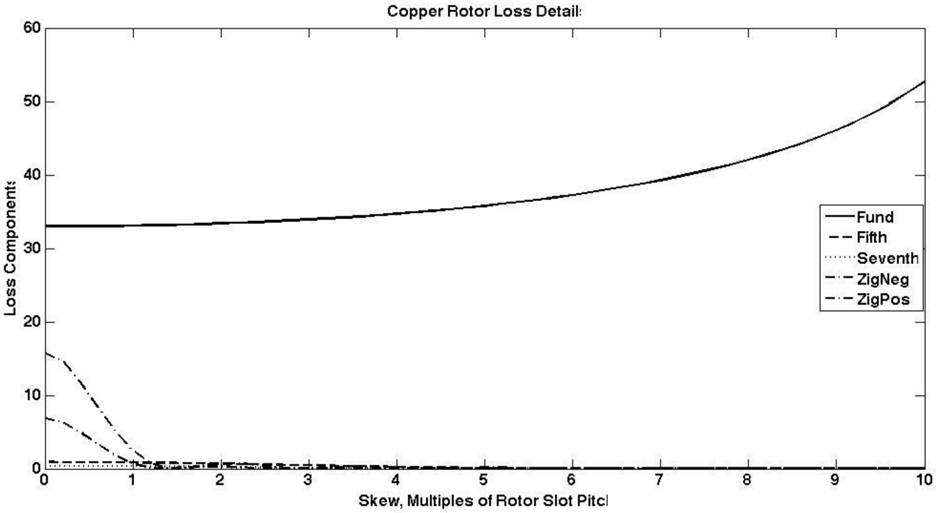
Some boring details about those windings

- Full pitch has highest fundamental winding factor
- 2/3 pitch has lowest, and still high harmonic factors
- 5/6 pitch has middling fundamental, but very low harmonic factors
- ‘Conc’ (concentric) has good fundamental factor and relatively low harmonic factors (very low 7th)

Harmonic Winding Factors
48 Slot, 4-Pole Winding

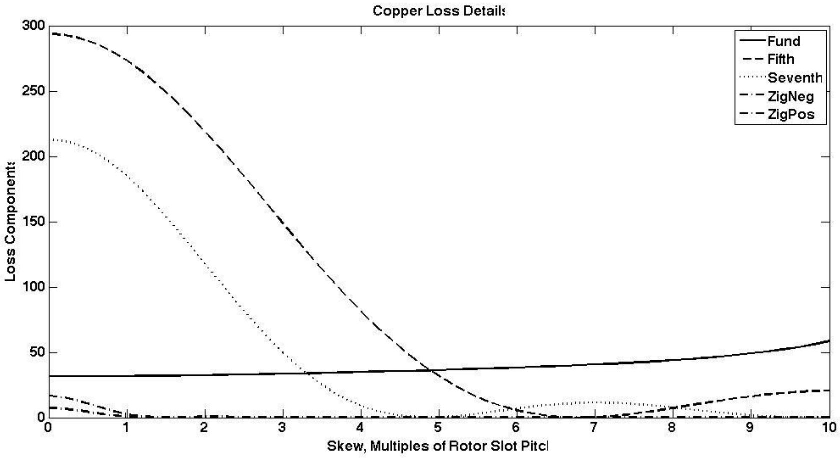
	K(1)	|K(5)|	|K(7)|	|K(23)|	|K(25)|
Full Pitch	0.958	0.204	0.158	0.958	0.958
2/3 Pitch	0.829	0.178	0.137	0.829	0.829
5/6 Pitch	0.925	0.053	0.041	0.925	0.925
Conc	0.936	0.104	0.025	0.936	0.936

I said that was a badly designed winding, and here is why:
This is the same plot for 5/6 pitch stator winding
Note 5th and 7th losses are much smaller with the 5/6 pitch winding.
Note the scale on the left is much smaller
The difference represents a loss in efficiency of about 2%

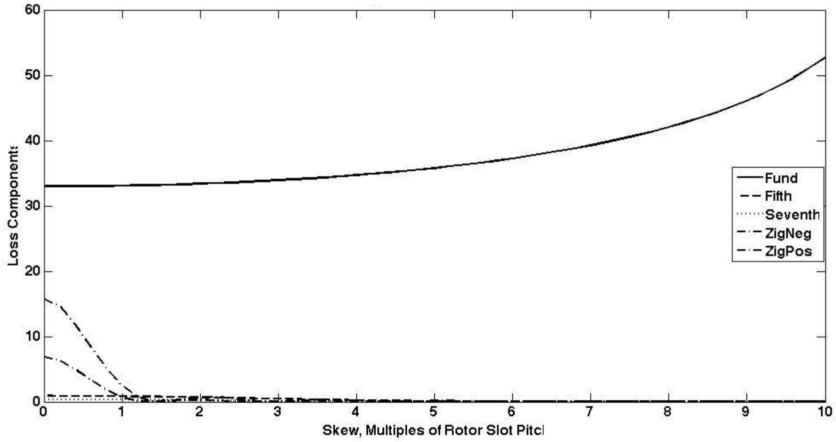


Two charts repeated in next slide

2/3 Pitch

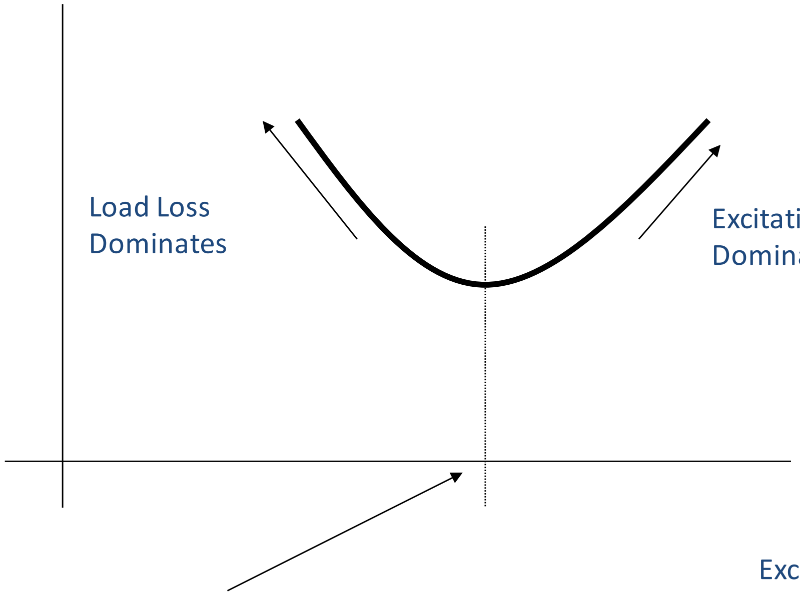


5/6 Pitch



But now consider excitation

Hypothesize that there is an optimal excitation level, sort of like this:



Total Loss

Load Loss
Dominates

Excitation Loss
Dominates

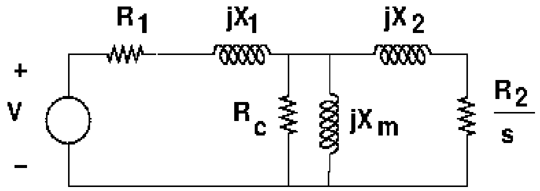
Excitation

Optimum moves left with increased speed

Excitation is flux density in the air gap

- This is adjustable in an induction motor (by voltage)
- It is fixed in a permanent magnet motor
- It is well known that as voltage is changed in an induction motor, efficiency changes with it, in about the shape of the previous slide
- The question is, can we take advantage of this? And what does it mean?
- We approach this with an approximate model
- And we find that it can help, even with a simplified control strategy.

Induction Motor: Basic Model
Assume core loss is proportional to voltage squared



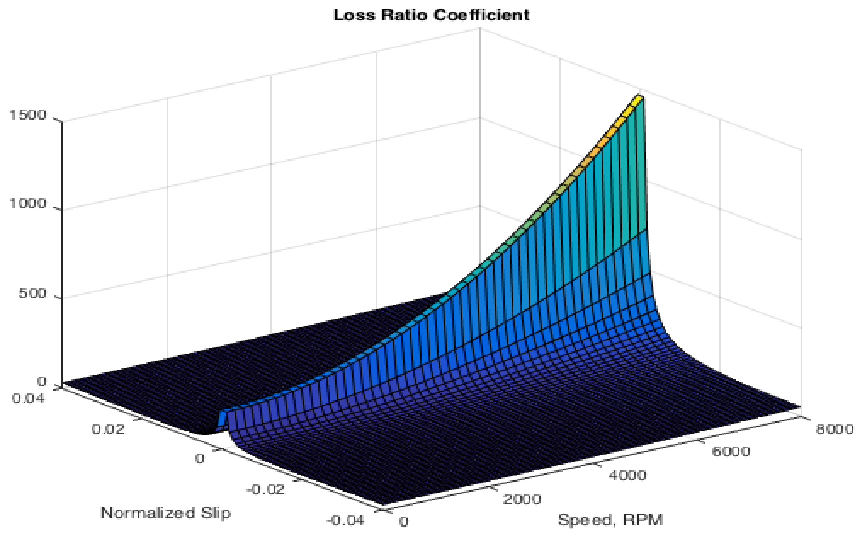
$$T = 3\frac{p}{w}\left|I_2\right|^2\frac{R_2}{s} \qquad s = s_B\frac{W_B}{W} \qquad T = 3\frac{p}{W_B}\left|I_2\right|^2\frac{R_2}{s_B}$$

$$P_d = 3R_1\left|I_1\right|^2 + 3R_2\left|I_2\right|^2 + 3\frac{\left|V_c\right|^2}{R_c}$$

The trick is to minimize dissipation for a given torque

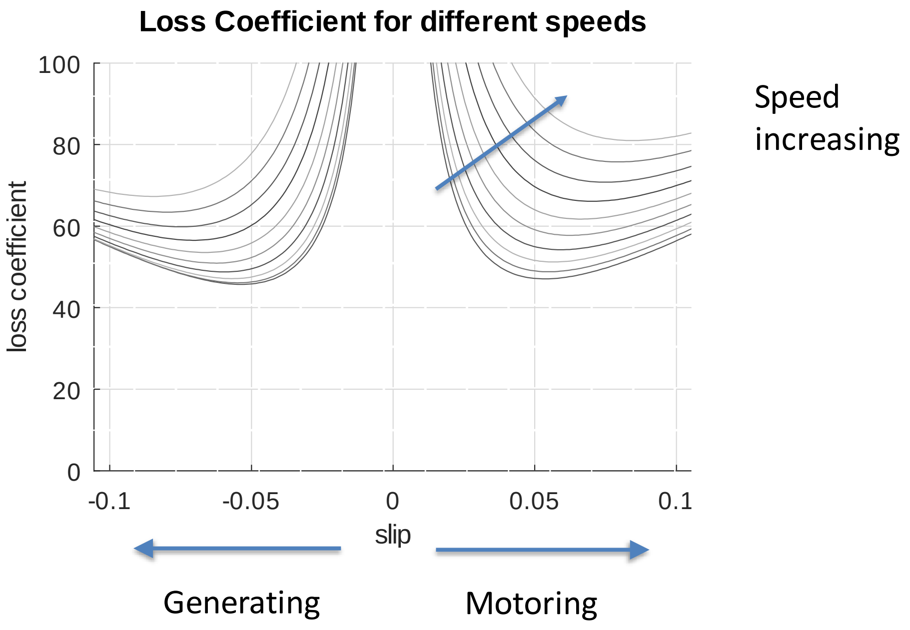
$$\left.\begin{aligned} T &= V^2 C(N,s_B) \\ P_d &= V^2 C_\ell(N,s_B) \end{aligned}\right\} \longrightarrow C_d(N,s_B) = \frac{C_\ell(N,s_B)}{\left|C(N,s_B)\right|}$$

That loss ratio as a function of speed and slip



It does have a minimum!

Here are cross-sections of that figure at different speeds

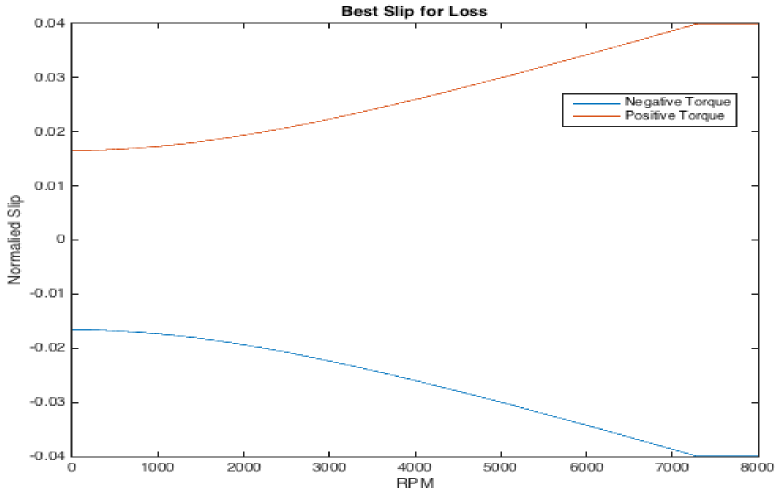


Speed increasing

Generating

Motoring

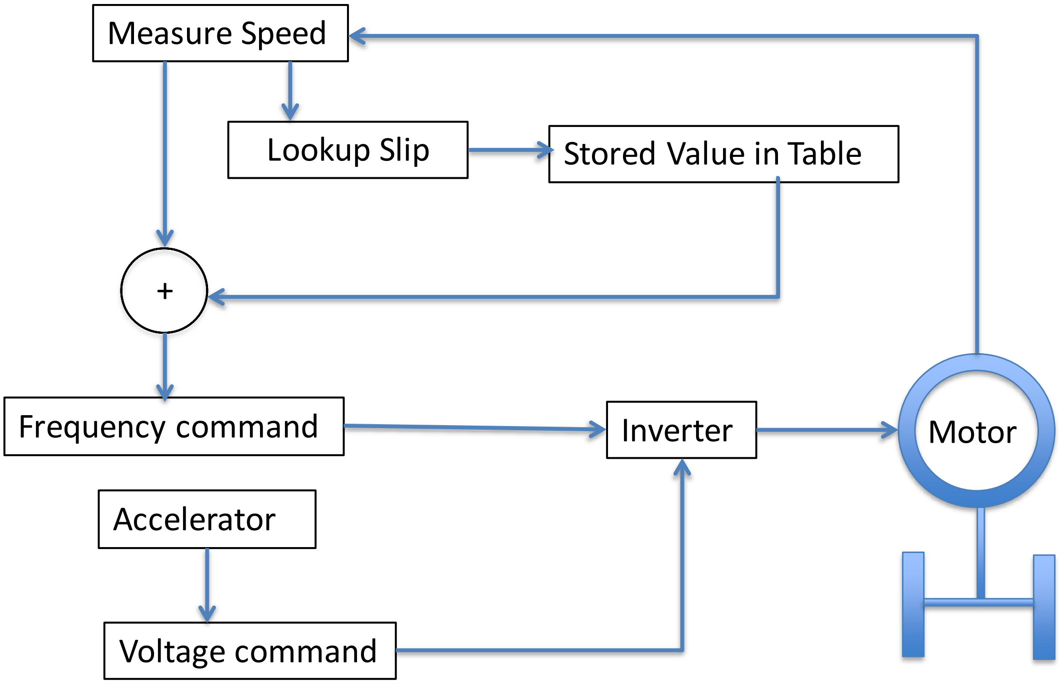
So for each speed, there is a slip that minimizes loss



Rough Estimate for Induction Motor Drive, Effective (Drive Cycle) Efficiency

Volts/Hz Control	84%
Voltage Optimized	91%

A control system that takes advantage of this should be straightforward to implement



Measure Speed

Lookup Slip

Stored Value in Table

+

Frequency command

Inverter

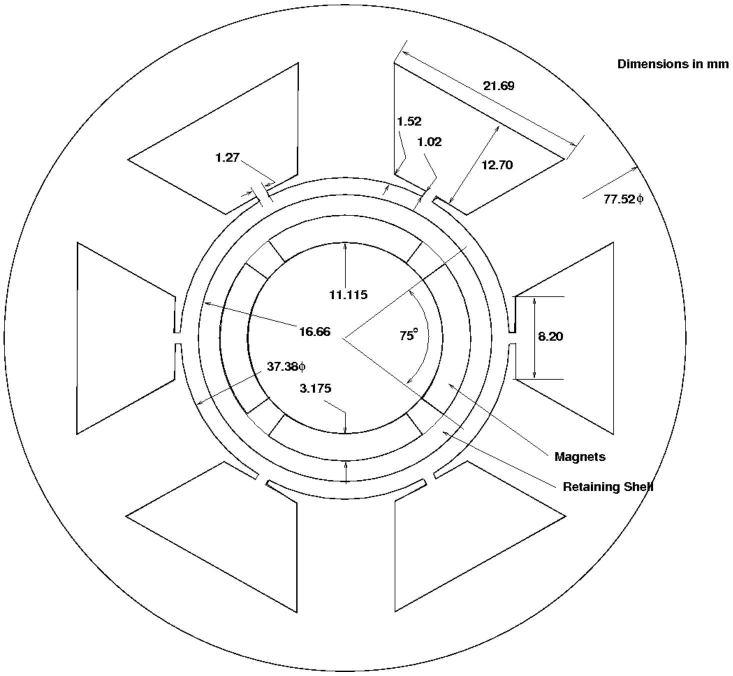
Motor

Accelerator

Voltage command

A small scale trade study

- Surface Mount Permanent Magnet Motor
- Modestly high speed machine
- Focus here is on losses in the machine



Question:

- What is the best airgap to use?
- What are the things that determine best air gap?

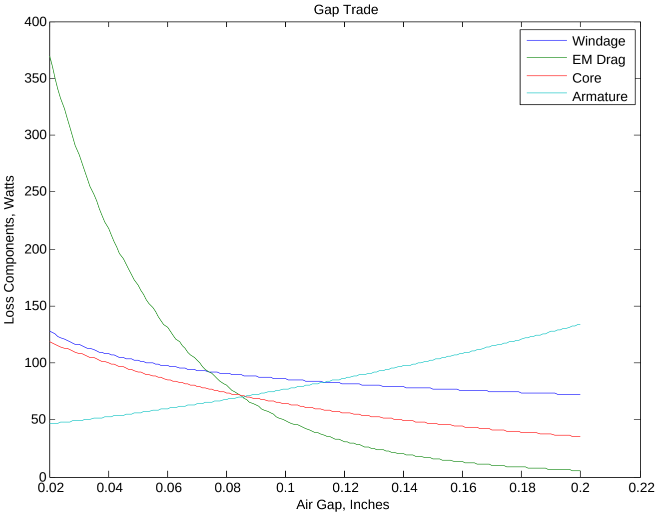
Here are a couple of things that might happen if gap is too small:

- Rotor overheating leading to loss of magnetization
- Higher rotor loss than anticipated

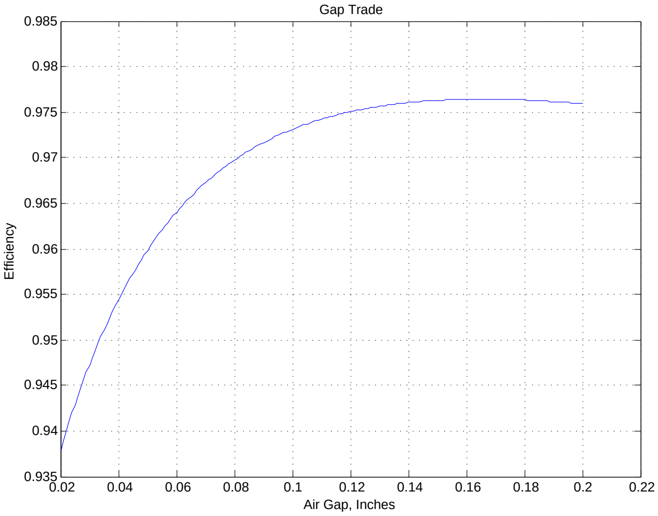
On the other hand, if the gap is too large

- Interaction magnetic field is smaller than it would be
- Leading to higher armature losses

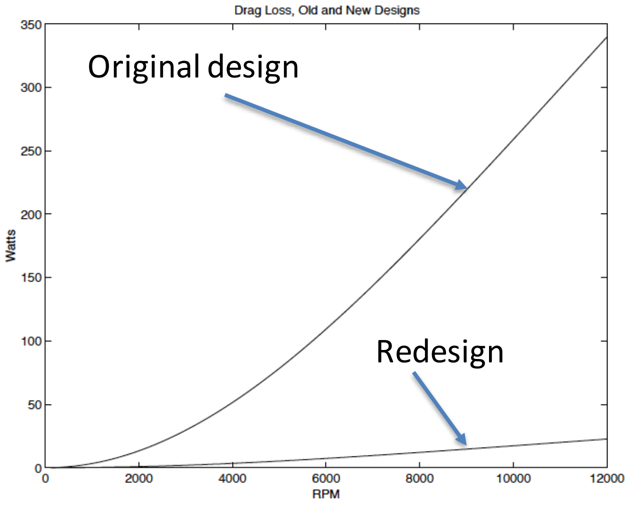
This trade study was to determine the proper air-gap dimension for this motor. (Not the same one as before)



Unlike some popular conception, the mechanically minimum air-gap is not the best



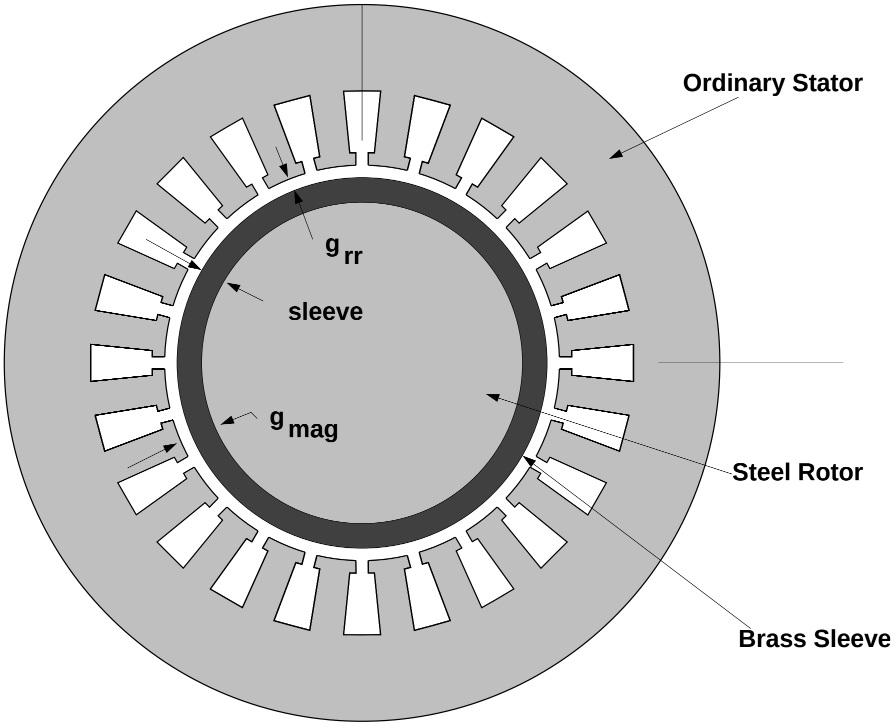
An example of the first bad thing on this list:
A combination of stator slots that are too big and air-gap that is too small caused a rotor to overheat and demagnetize. We had to re-design the motor.



Original design

Redesign

Design Error in a High Speed Induction Motor



Design (as a verb) is the process of determining what you are going to make.

Design (as a noun) is a collection of things:

- Materials to use:
  - Copper or Aluminum? Or something else
  - Grade and thickness of steel
- Dimensions
  - Air Gap Radius and clearance
  - Length
  - Slot dimensions
  - Etc

Attributes of a design are things like

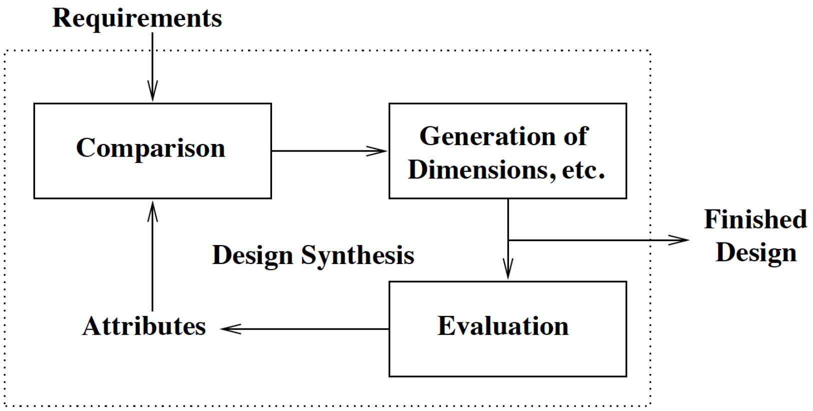
- Cost
- Efficiency
- Weight

Optimization is Finding the Best Motor for the job

- And that usually means maximizing or minimizing some combination of the attributes

‘Design’ (verb) is often represented as a loop

- One Iterates until a suitable, or perhaps optimal soluition is found.
- Methods differ for
  - Evaluation
  - Generation of dimensions (synthesis)
  - Breaking out of the loop (‘done’)



Requirements

Comparison

Generation of Dimensions, etc.

Finished Design

Design Synthesis

Attributes

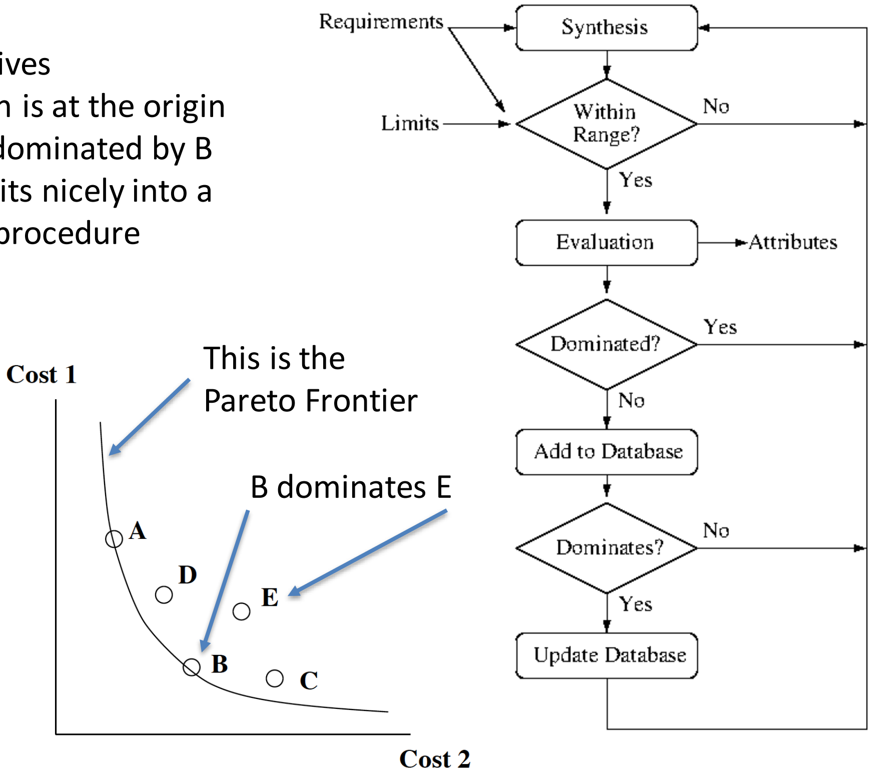
Evaluation

Multi-Attribute:

Finding alternatives

- ‘Best’ Solution is at the origin
- Solution E is dominated by B
- This process fits nicely into a Monte Carlo procedure

Vilfredo Federico Damaso Pareto (born *Wilfried Fritz Pareto*; Italian: (15 July 1848 – 19 August1923)



Requirements

Synthesis

Limits

Within Range?

No

Yes

Evaluation

Attributes

Dominated?

Yes

No

Add to Database

Dominates?

No

Yes

Update Database

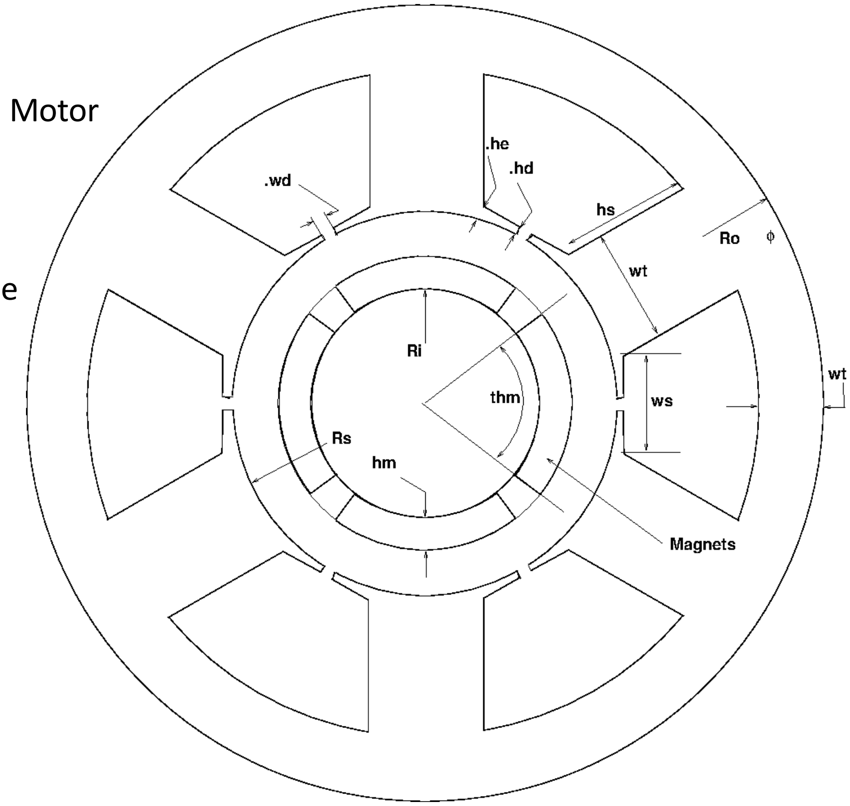
This is the Pareto Frontier

B dominates E

Example:
Permanent Magnet Motor
Rating: 1000 watts
Speed: 1000 RPM

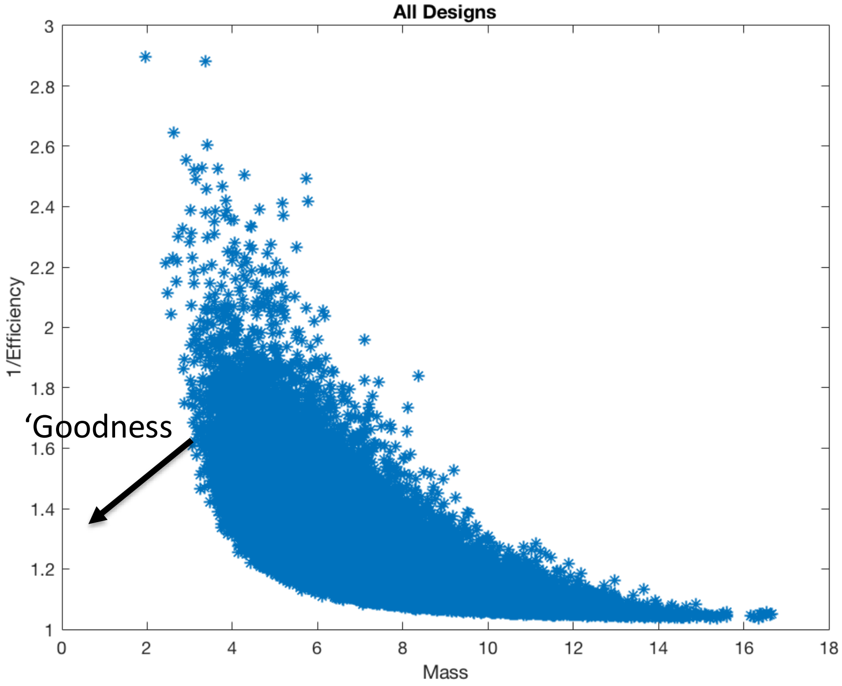
For this problem, we allow 7 dimensions to vary: R, g, hm, l (not shown) wt, hs, dw (wire dia not shown)

We will have more to say about the process of evaluation later



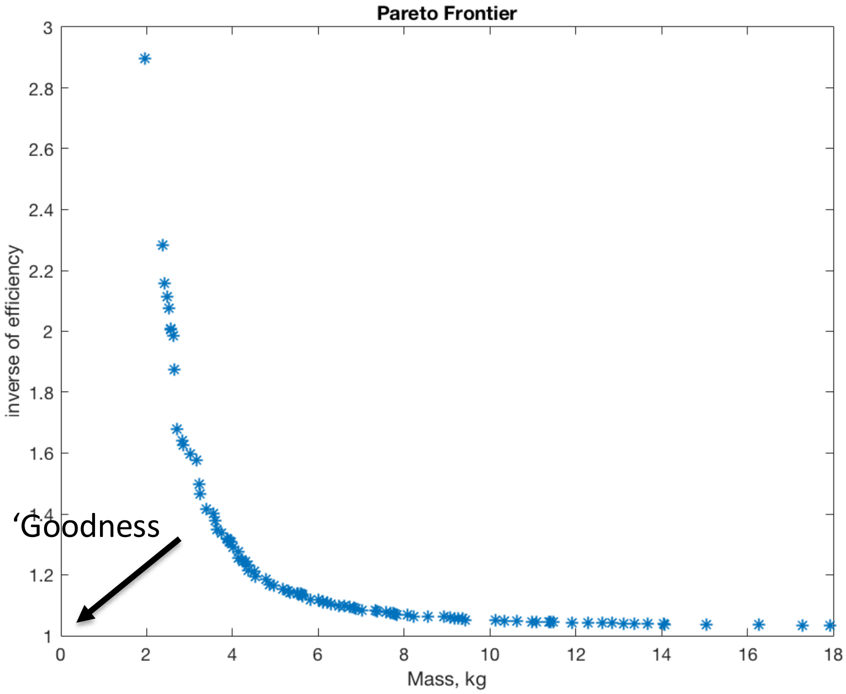
Let the Monte Carlo Process run

1. Random Designs that are viable
   (Viable: dimensions within preset limits, other limits observed)
2. About 1.1 million dots here



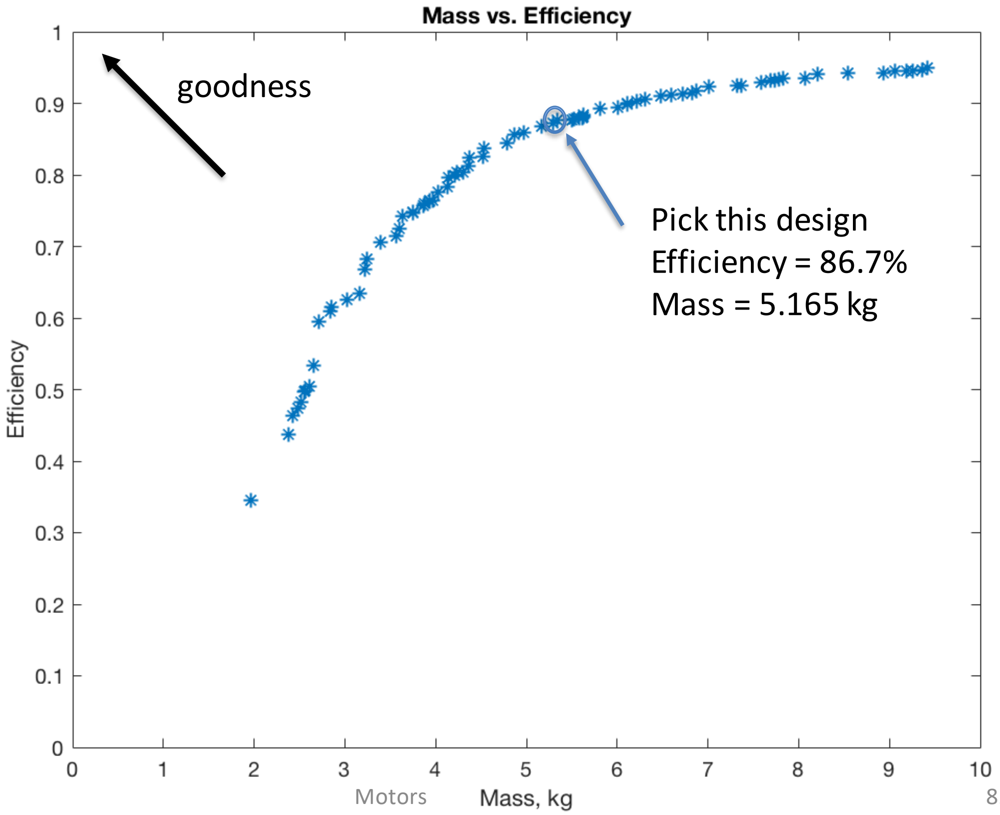
'Goodness

If we remove the designs that are ‘dominated’ (inferior), we get a ‘pareto’ frontier



‘Goodness

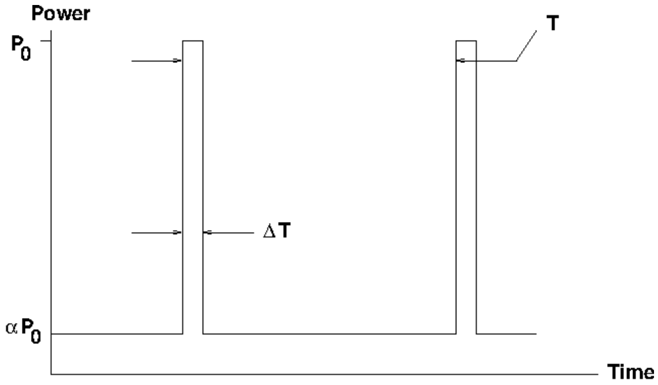
Or, maybe make this easier to read



goodness

Pick this design
Efficiency = 86.7%
Mass = 5.165 kg

But suppose that motor is to operate against a pulse type load like this:

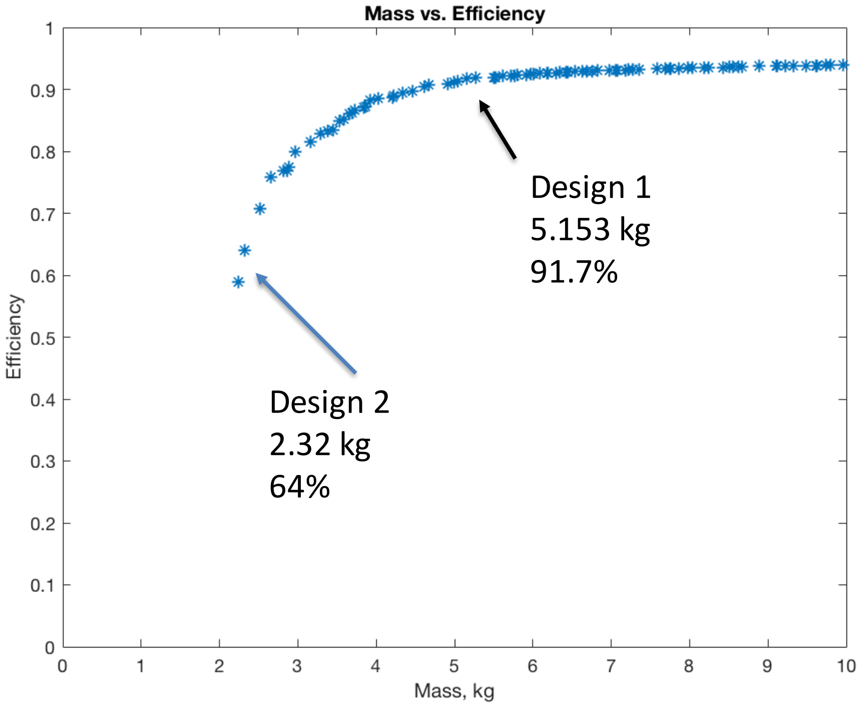


$\alpha = .1$
$\Delta = .05$

$$\eta_{eff} = \frac{\text{Energy Out}}{\text{Energy In}} = \frac{\alpha(1-\Delta)+\Delta}{\frac{\alpha(1-\Delta)}{\eta_1}+\frac{\Delta}{\eta_2}}$$

For that 5.165 kg machine, $\eta_{eff} \approx 62.2\%$

So repeat the procedure with the intended duty cycle



Design 1
5.153 kg
91.7%

Design 2
2.32 kg
64%

So one takeaway here is that the <u>optimal</u> motor depends on what it is to be driving, and relatively how important are efficiency and mass.