Frequency-dependent modeling of components in power electronics systems

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Outline of presentation

- Parameters for characterization of component behavior
- Extraction of rational function-based models
- Inclusion of models in circuit simulators
- Applications
Model classification

- Linear, time-invariant
- Frequency dependent behavior
- Black-box model (terminal equivalent)
  - No information about internal voltages/currents
Characterizing terminal behavior

- Admittance parameters \( i(\omega) = Y(\omega)v(\omega) \)
- Scattering parameters \( b(\omega) = S(\omega)a(\omega) \)
- Voltage transfer functions \( v_{\text{int}}(\omega) = H(\omega)v_{\text{ext}}(\omega) \)

Many other options...

Interacting with system

Non-interacting
Terminal characterization by admittance parameters

\[ i = Yv \]

\[
\begin{bmatrix}
I_1(\omega) \\
I_2(\omega) \\
\vdots \\
I_n(\omega)
\end{bmatrix}
=\begin{bmatrix}
Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\
Y_{21}(\omega) & Y_{22}(\omega) & \cdots & Y_{2n}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1}(\omega) & Y_{n2}(\omega) & \cdots & Y_{nn}(\omega)
\end{bmatrix}
\begin{bmatrix}
V_1(\omega) \\
V_2(\omega) \\
\vdots \\
V_n(\omega)
\end{bmatrix}
\]
Obtaining the admittance parameters

• Calculations
  • Using information about internal structure and material parameters
    Example: Cables
  • Reduced-order modeling of system of (known) linear components
    Example: network equivalents

• Measurements
  • Vector network Analyzer
  • Gain-phase, S-parameters
  • Impedance measurement
  • Other
Modeling using rational functions

\[ \sum_{m=1}^{N} \frac{R_m}{j\omega - a_m} + D = C(j\omega I - A)^{-1}B + D \]
Why rational model?

- **Analytical transformation from frequency domain to time domain (accurate!)**

  \[ y(s) = \sum_{m} \frac{r_m}{s-a_m} \quad \Rightarrow \quad y(t) = \sum_{m} r_m e^{at} \]

- **Recursive convolution in time domain (fast!)**

  \[ i(t) = y(t) \ast v(t) \quad \Rightarrow \quad \begin{cases} \mathbf{x}_n = \mathbf{a} \mathbf{x}_{n-1} + \mathbf{v}_{n-1} \\ i_n = \mathbf{c}^T \mathbf{x}_n + \gamma v_n \end{cases} \]
Rational fitting via polynomials

\[
f(s) \approx \frac{a_0 + a_1 s + a_2 s^2 + \ldots}{1 + b_1 s + b_2 s^2 + \ldots} \quad s = j\omega
\]

\[f(s)(1 + b_1 s + b_2 s^2 + \ldots) \approx a_0 + a_1 s + a_2 s^2 + \ldots\]

\[
A \mathbf{x} = \mathbf{b} \quad s_1, s_2, \ldots, s_K
\]

\[b_1, b_2, \ldots a_1, a_2, \ldots\]

\[
f(s) = \sum_m \frac{r_m}{s - a_m} + d
\]

- Ill-conditioning
- Biased
- Unstable poles
Vector Fitting (Pole Relocation)*

- Method for fitting a pole-residue model to frequency data

\[ f(s) \approx \sum_{m=1}^{N} \frac{r_m}{s-a_m} + d \quad s = j\omega \]

- Robust, efficient, accurate
- Guaranteed stable poles

- Freely available since 1999
- Widely applied for rational modeling of power systems, high-speed electronics, microwave structures, ...

Vector fitting: Pole identification

\[(1 + \sum \frac{\tilde{c}}{s - \tilde{a}}) f(s) = \sum \frac{c}{s - a} \]

\(\sigma(s)\)

\(\vec{a} \text{: specified (initial) poles}\)

\(Ax = b\)

\(\tilde{c}, c\)

\(f(s) = \sum \frac{c}{s - a} = \prod \frac{(s - z)}{(s - \tilde{a})} = \prod \frac{(s - z)}{(s - \tilde{z})}\)

\(\sigma(s)\)

The poles of \(f\) are equal to the zeros of \(\sigma\)!

The new poles are used as initial poles \(\rightarrow\) iterative procedure
Vector fitting: Zeros calculation

\[
\sigma(s) = \sum_{m=1}^{N} \frac{\tilde{c}_m}{s - \overline{a}_m} + 1
\]

\[
\{z_m\} = eig(\tilde{A} - b \cdot \tilde{c}^T)
\]

Unstable poles are "flipped" into the left half-plane \(\rightarrow\) guaranteed stable poles
Example SAW filter

20th order approximation
Example SAW filter

80th order approximation
Stability of time domain simulation

\[ Y(\omega) \approx \sum_{m=1}^{N} \frac{R_m}{j\omega - a_m} + D \]

Requirements:
1. Poles in left half plane: \( \{a_m\} < 0 \)
2. \( \text{eig(Re}\{Y(\omega)\}) > 0 \)
3. \( \text{eig}(D) > 0 \)

Passivity condition (symmetrical model)
Passivity enforcement by perturbation

- Perturb the model's parameters so that the model becomes passive*
  - Perturb only elements in \( \{R_m\}, D \)
  - Make change to \( Y(s) \) minimal

\[
\Delta Y = \sum_{m=1}^{N} \frac{\Delta R_m}{s-a_m} + \Delta D \cong 0
\]

Linearization (eigenvalue perturbation)

\[
eig(\Re\{Y + \sum_{m=1}^{N} \frac{\Delta R_m}{s-a_m}\}) > 0
\]

\[
eig(D + \Delta D) > 0
\]

LS problem

Constraint

\[
A_{sys} \Delta x \cong 0
\]

Constrained LS problem

(e.g. QP)

\[
B_{sys} \Delta x < c
\]

\( \Delta x \): elements of \( \{\Delta R_m\}, \Delta D \)

Example: transformer modeling

Pass. enf.

10 kV 400 V

1 2 3 4 5 6

Fitting band

10^5 - 10^3

Frequency [Hz]

10 kV 400 V

Measured Simulated

10x V_4

V_2, V_3

0 0.1 0.2 0.3 0.4 0.5 0.6

Time [ms]

0 200 400 600 800 1000 1200 1400

Voltage [Volt]

0 50 100 150 200 250 300 350 400

Voltage [Volt]

V_1

0 0.1 0.2 0.3 0.4 0.5 0.6

Time [ms]

Original Passivated

10^3 - 10^5

10^6
Including model in EMTP-type circuit solvers

\[
\begin{bmatrix}
I_1(\omega) \\
I_2(\omega) \\
\vdots \\
I_n(\omega)
\end{bmatrix}
= 
\begin{bmatrix}
Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\
Y_{21}(\omega) & Y_{22}(\omega) & \cdots & Y_{2n}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1}(\omega) & Y_{n2}(\omega) & \cdots & Y_{nn}(\omega)
\end{bmatrix}
\begin{bmatrix}
V_1(\omega) \\
V_2(\omega) \\
\vdots \\
V_n(\omega)
\end{bmatrix}
\]

\[
Y(\omega) \approx \sum_{m=1}^{N} \frac{R_m}{j\omega - a_m} + D
\]
Lumped circuit equivalent

\[ Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \]

\[ Y_{ij} = \sum_{m=1}^{N} \frac{R_{ij,m}}{j\omega - a_m} + D_{ij} \]
Norton equivalent w/ controlled current source

\[ y(\omega) = \sum_m \frac{r_m}{j\omega - a_m} \quad \Rightarrow \quad y(t) = \sum_m r_m e^{at} \quad \text{Impulse response} \]

\[ i(t) = y(t) \ast v(t) \quad \Rightarrow \quad \begin{cases} 
  x_n = \alpha x_{n-1} + v_{n-1} \\
  i_n = c^T x_n + \gamma v_n
\end{cases} \quad -c^T x_n \]
Example 1: High-frequency transformer modeling

45 MVA (three-phase),
137 kV / 8.5 kV
YNd
Admittance measurements

Gain-phase
Current sensor
Admittance measurements

Calibration for
- Frequency variation in current sensor sensitivity (V/A)
- Insertion impedance effects
- Measurement cables
Model extraction (fitting + passivity enf.)

Eigenvalues of $Y$

Eigenvalues of $G = \text{Re}\{Y\}$
Time domain validation:
Voltage transfer \( \text{high} \rightarrow \text{low} \)

![Diagram showing voltage transfer between 137 kV and 8.5 kV](image)

- ~60 kHz
- ~2 MHz

![Graphs showing voltage transfer](image)
Example 2: High-frequency transformer modeling

92 MVA (three-phase, three-winding)
Admittance modeling w.r.t. H1, H2, H3, X1, X2, X3

Manufacturer's detailed model

Voltage transfer from H1 to X1

Measurement-based black-box modeling offers superior accuracy
Final remarks

• Error magnification
  • Some admittance matrices have a combination of large and small eigenvalues (typically at low frequencies). Examples: cables, transformers, coils
  • The small eigenvalues tend to be lost in the measurement/modeling. Can lead to catastrophic error magnifications. Requires special handling.

• Passivity enforcement
  • The passivity enforcement may in some instances corrupt model behavior. May need a few trials before reaching a good model.

• Time delay effects
  • Systems with embedded delays may require excessive model orders
  • Delay-based rational modeling (e.g. traveling wave models)