On the Complexity of Dynamic Directed Evidential Networks with Conditional Belief Functions Construction and Belief Propagation

Wafa Laâmari[1] Boutheina Ben Yaghlane[2]
Christophe Simon[3]

[1]LARODEC, ISG de Tunis, Tunisia
[2]LARODEC, IHEC de Carthage, Tunisia
[3]CRAN, ESSTIN, France

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Context (1)

Problems

Available knowledge in real-world applications has been characterized by the uncertainty.

Proposed Solutions

Several network-based approaches have been developed for modeling uncertain knowledge such as probabilistic, possibilistic and evidential graphical models.

Context (2)

Problems

Existing uncertain knowledge change also from time to time.

Proposed Solutions

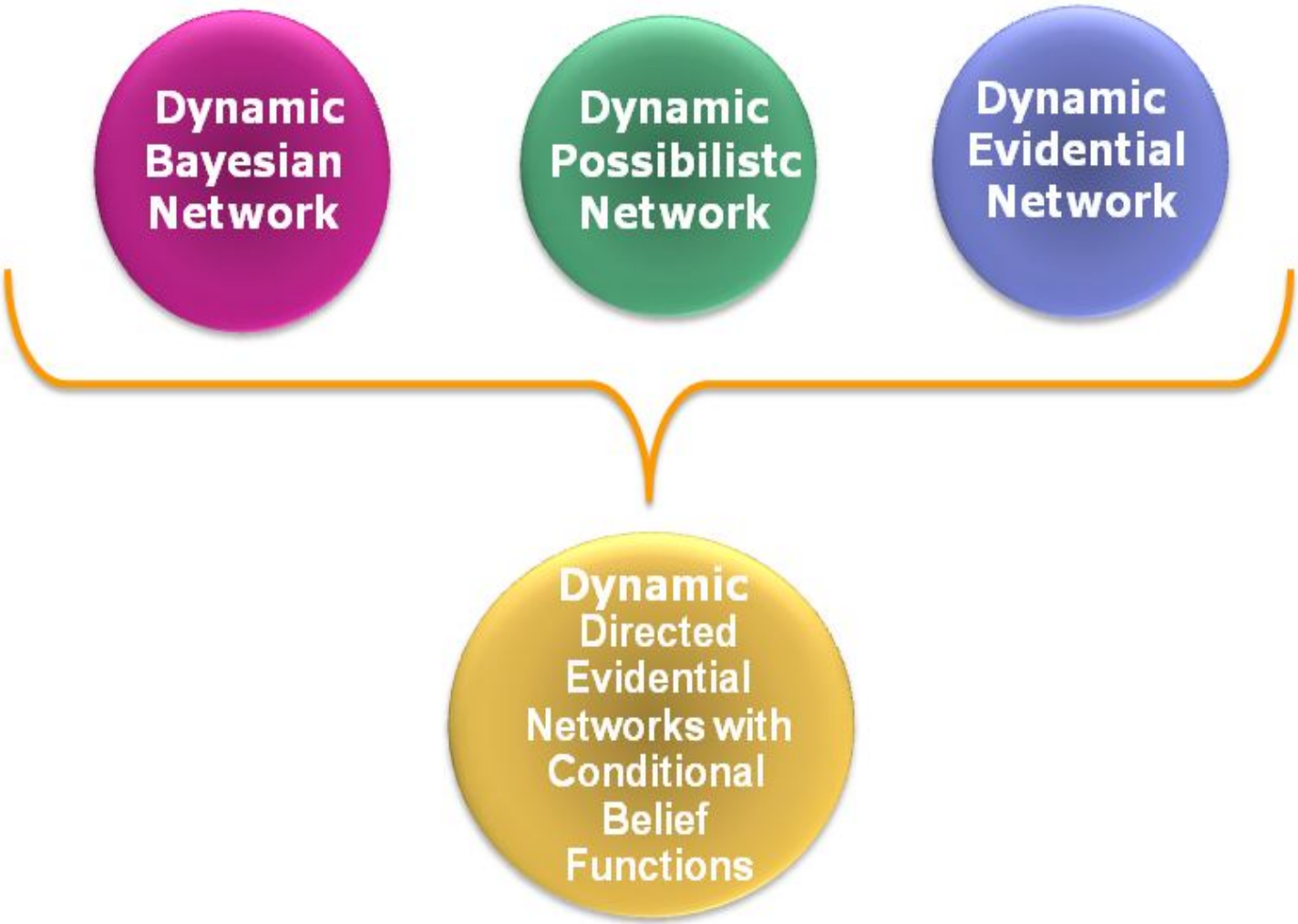
Various methods have been developed to take into account changes over time when reasoning under uncertainty, including those relying on network-based approaches.

Context (3)

- Represents the temporal dimension when modeling uncertainty using the probabilistic formalism.
- This formalism handles efficiently the aleatory uncertainty, but the epistemic uncertainty is modeled by an uniform distribution which is not always efficient to represent the total ignorance.

- Models uncertain sequential data with possibilistic theory.
- Does not express the imprecision in knowledge.

- Models the temporal evolution of uncertain knowledge
- Based on an extension of the Bayes theorem to the representation of the theory of evidence, DEN does not fully exploit the abilities of the evidential formalism.


Dynamic Bayesian Network
Dynamic Possibilistc Network
Dynamic Evidential Network
Dynamic Directed Evidential Networks with Conditional Belief Functions

Outline

1 **DEVN**
- The Original Structure
- Modified Binary Join Tree (MBJT)

2 **The Background on Dynamic Directed Networks**

3 **Dynamic Directed Evidential Networks with Conditional Belief Functions (DDEVNs)**
- Graphical Representation of DDEVNs
- Propagation in DDEVNs

4 **Complexity analysis**
- Complexity of the MBJTs Construction Process
- The computational complexity of the inference process

5 **Conclusion and Future Work**

Formalism

A DEVN is a directed graphical model expressing the uncertainty by the means of the evidence theory.

- For each root node X, having a frame of discernment Ω_X constituted by q hypotheses, an a priori belief function $M(X)$ has to be defined by the following equation:

$$M(X) = [m(\emptyset) \quad m(A_1^X)....m(A_i^X)....m(A_{2q-1}^X)] \tag{1}$$

with $$m(A_i^X) \geq 0 \quad and \quad \sum_{A_i^X, A_i^X \in 2^{\Omega_X}} m(A_i^X) = 1 \tag{2}$$

$m(A_i^X)$ is the belief that X verifies the hypotheses of A_i^X.

- For other nodes, a conditional belief function $M[Pa(X)](X)$ is specified for each possible hypothesis A_i^X knowing the focal sets of the parents of X.

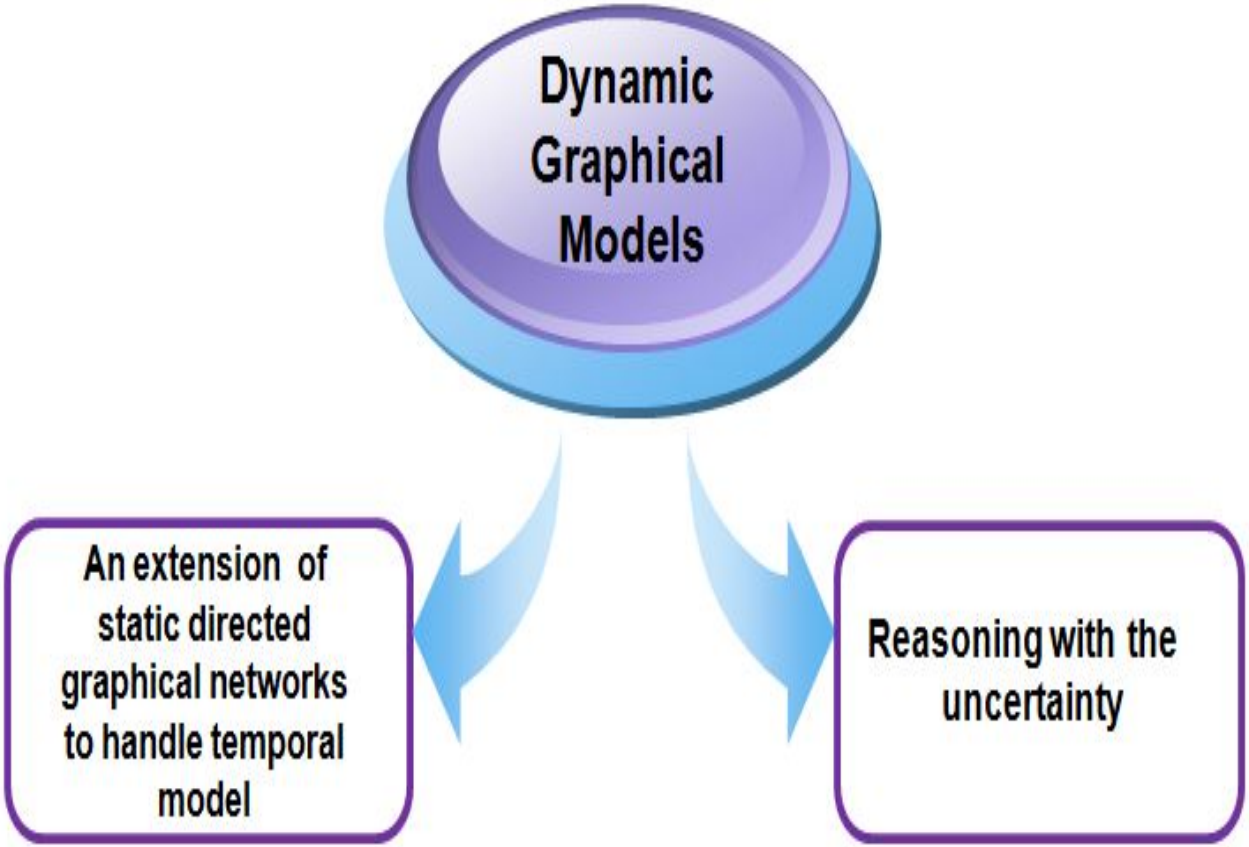
- The inputs *I* are a DEVN, an elimination Sequence.
- The output *O* is a static MBJT.

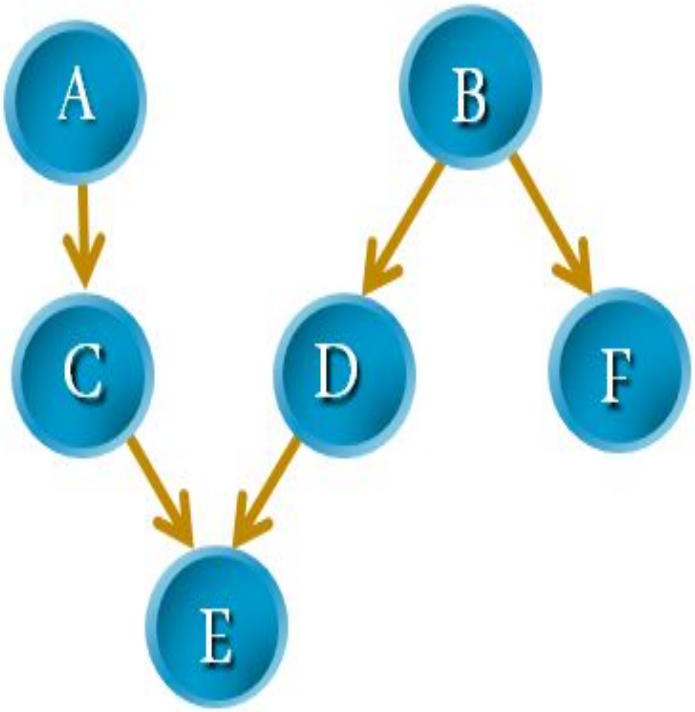
Algorithm

1. Find out the subsets of the hypergraph from the directed evidential network
2. Arrange the subsets of the hypergraph in a binary join tree.
3. Attach singleton subsets to the binary join tree
4. Make the tree binary again if the tree becomes non-binary when attaching a singleton subsets to it.
5. Draw rectangles containing the conditional relations between variables instead of circles containing just the list of these variables.

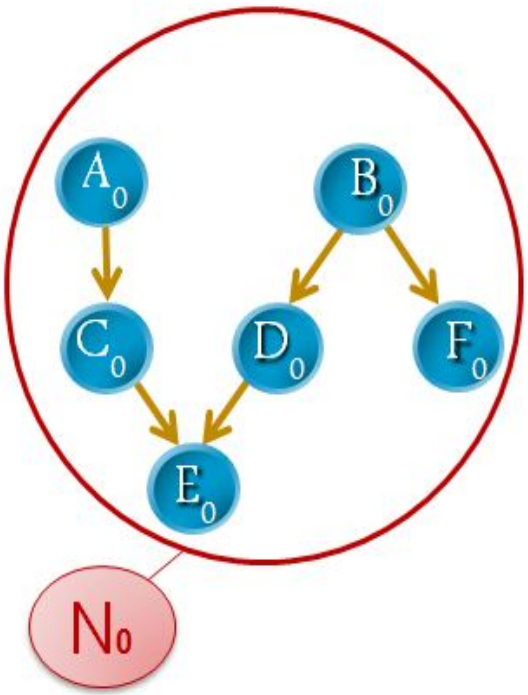
Propagation in DEVN

- Extension of Shenoy's algorithm for conditional belief functions
- Computation based on the GBT and the DRC rules of Smets.


Dynamic Graphical Models
An extension of static directed graphical networks to handle temporal model
Reasoning with the uncertainty

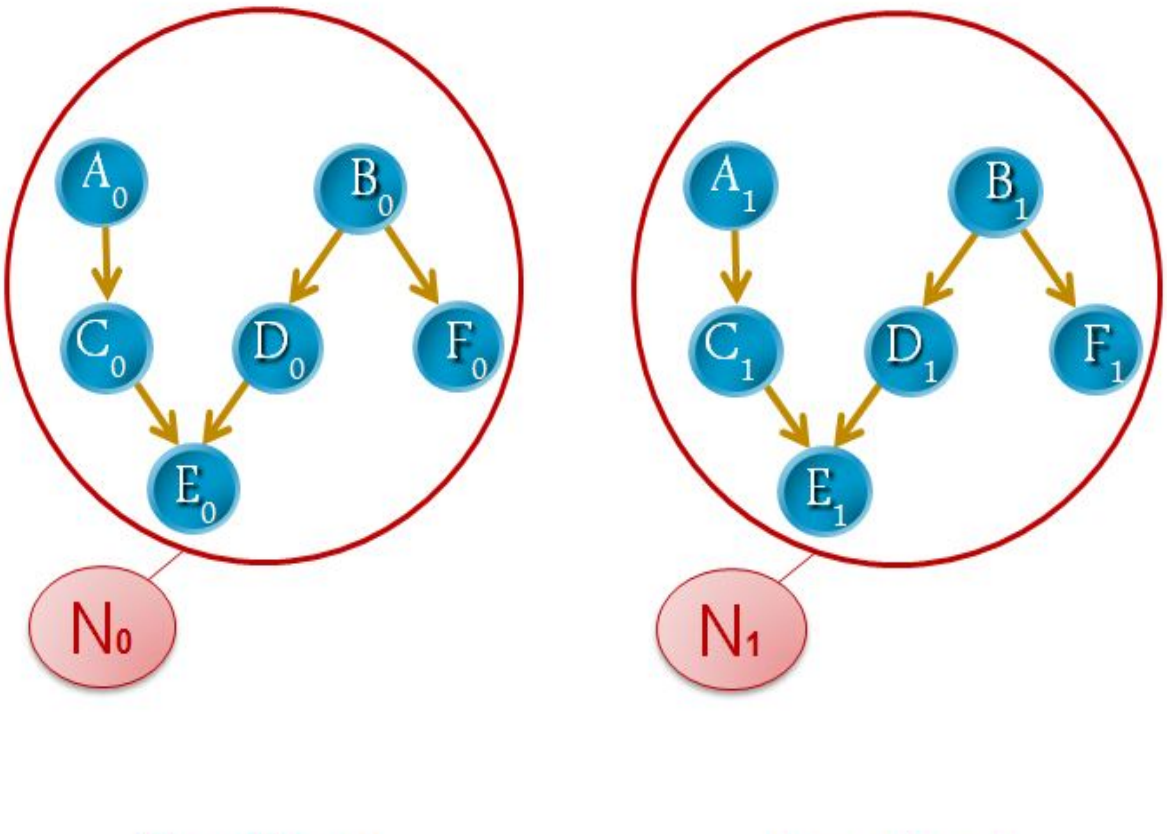

A
B
C
D
F
E

Representation

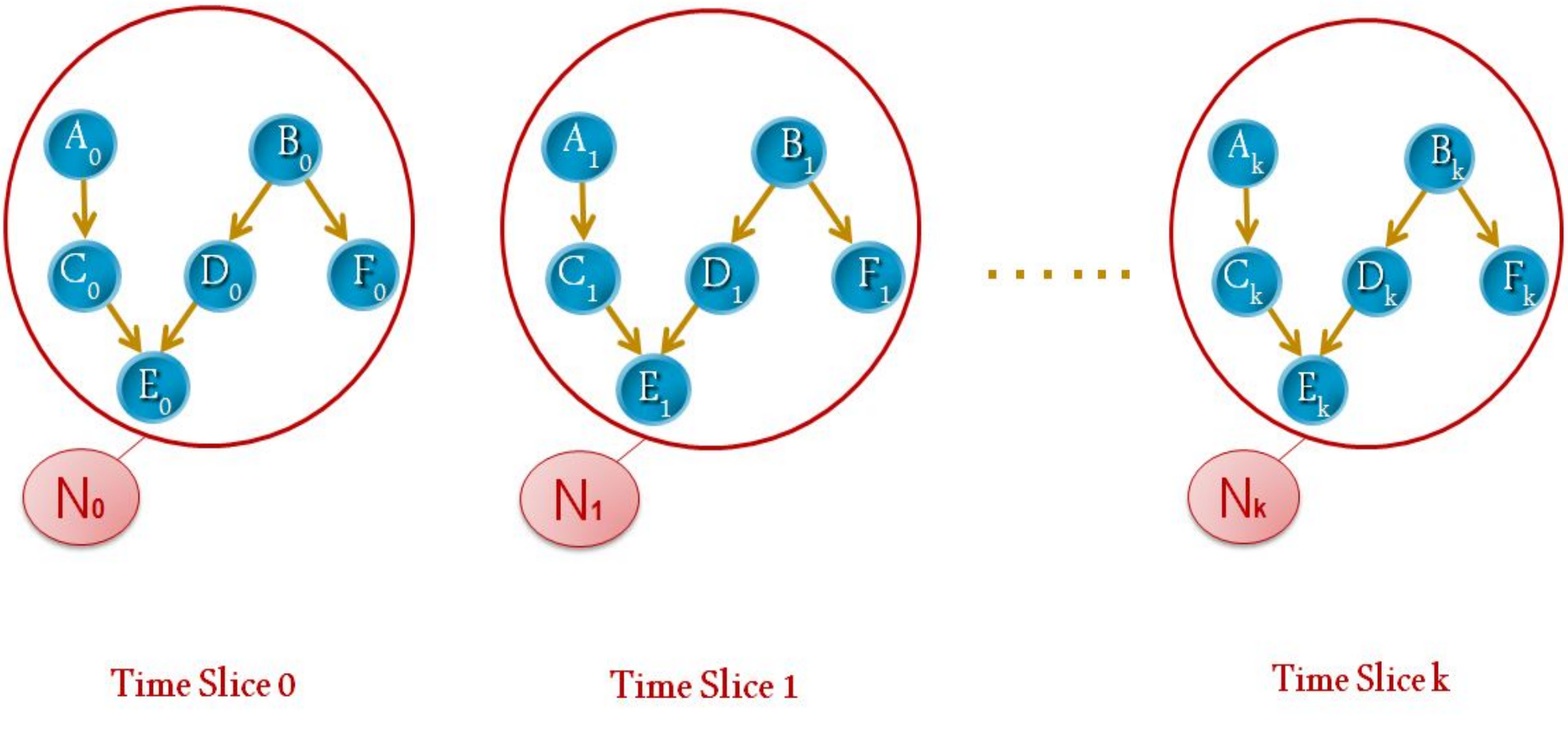


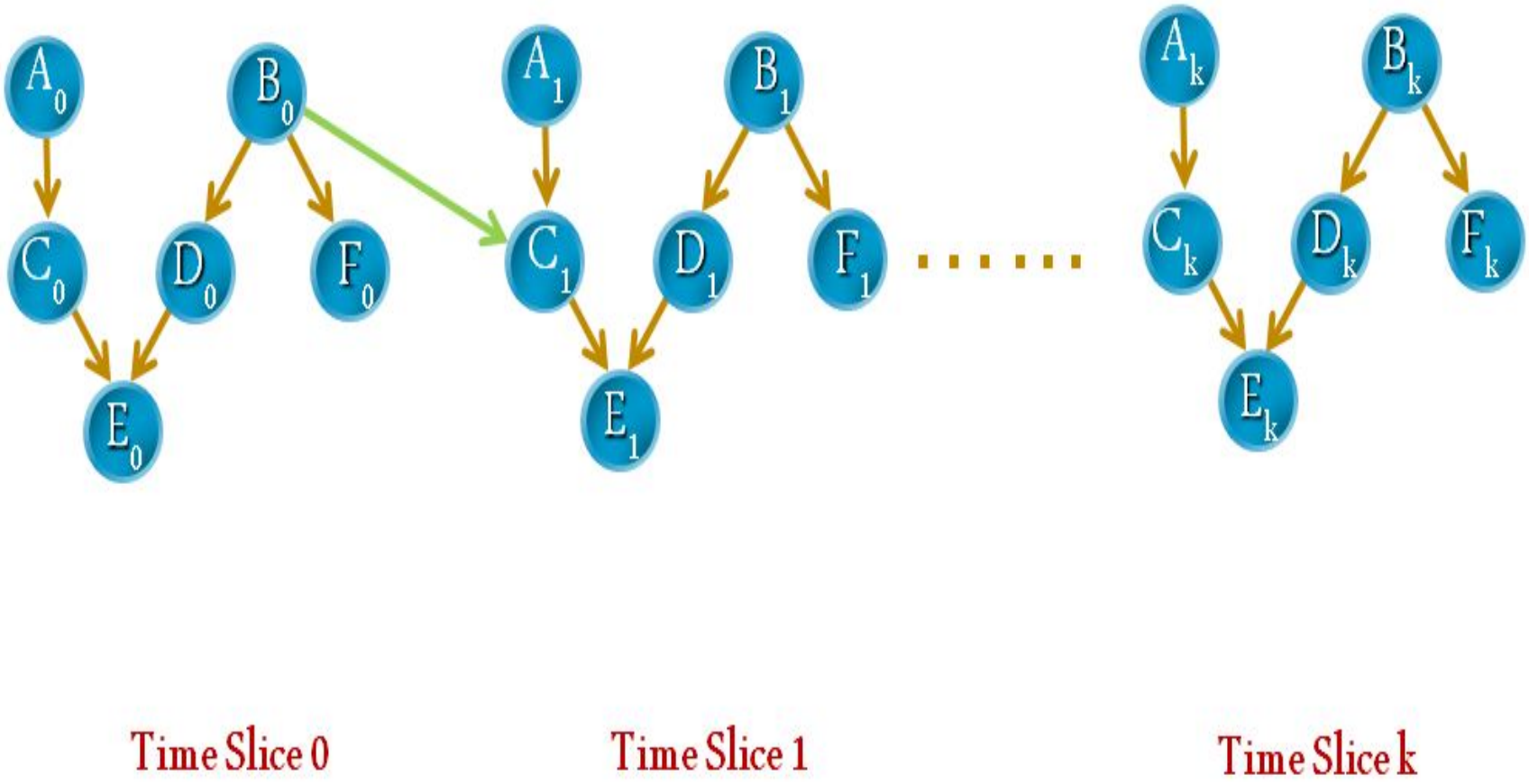
Time Slice 0

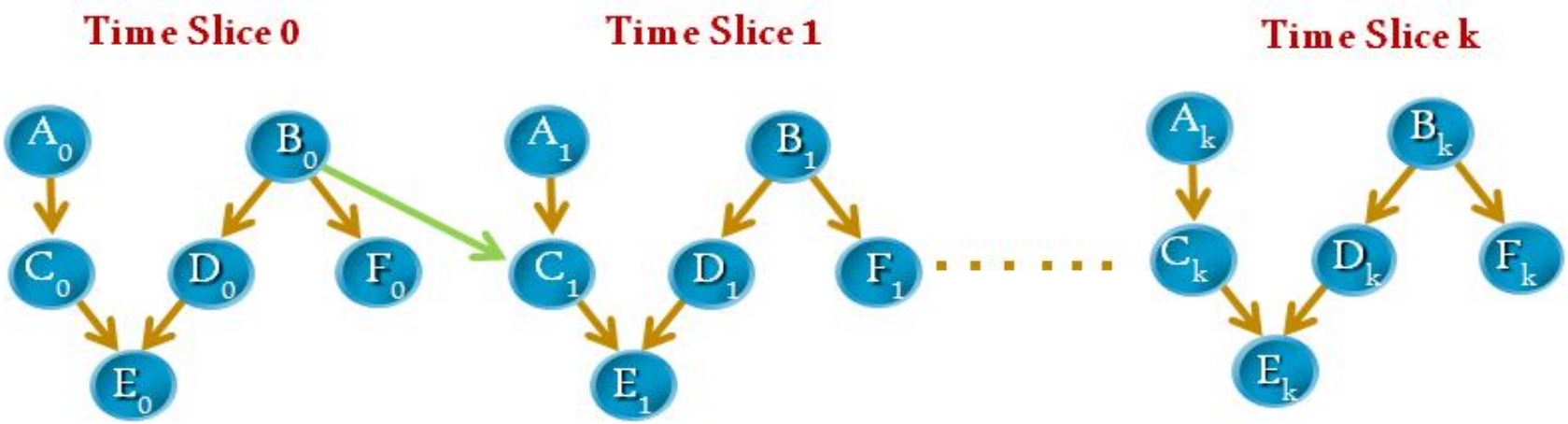
Representation



Representation




A0
B0
C0
D0
F0
E0
A1
B1
C1
D1
F1
E1
Ak
Bk
Ck
Dk
Fk
Ek
Time Slice 0
Time Slice 1
Time Slice k



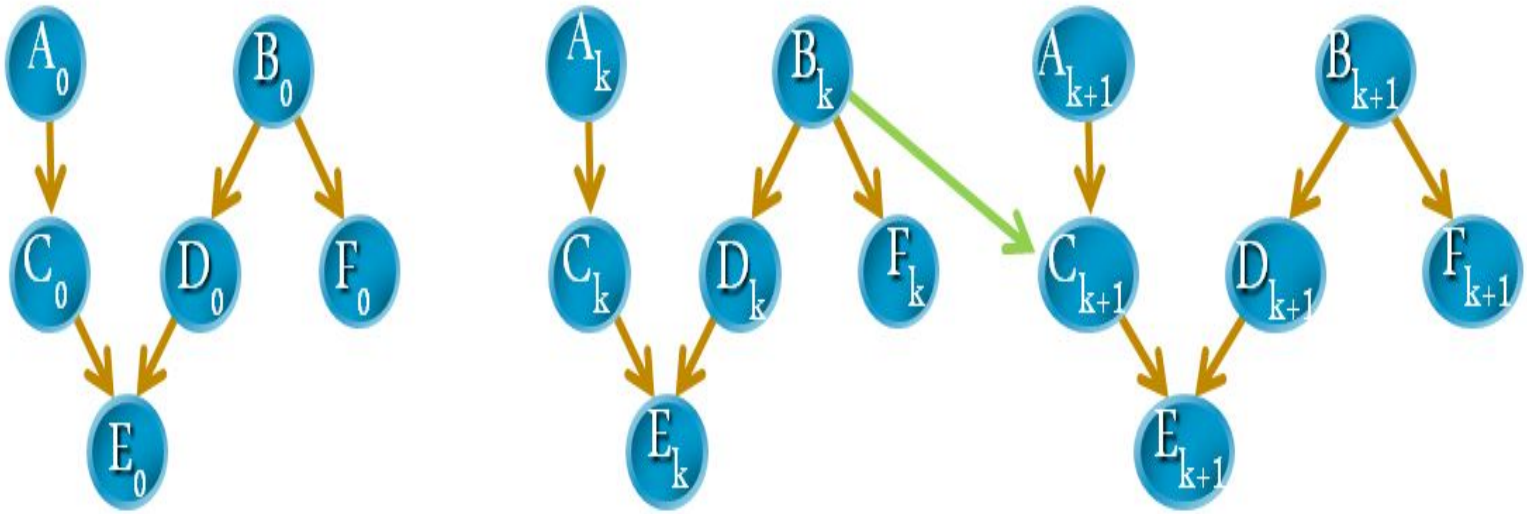
With a large number of time slices, the graphical structure becomes huge, and the inference process runs out of memory and takes too much time

With the new representation, a DDGM is defined as a couple (G_0, G_k), where:

- G_0 denotes the DGM corresponding to the initial time slice $k = 0$
- G_k denotes a 2-time slices DGM (2-TDGM) in which only two nodes are introduced to represent a same variable at successive time slices

G_0: time slice 0

G_k: time slice k > 0



The transition function defined between the current time slice k and the future time slice k+1 do not depend on k.

The distribution F(X_{k+1}) of the variable X at the time step k + 1 depends only on the distributions of its parent nodes in this time slice and also on the distributions of its parent nodes in the immediately preceding time step k.

Definition

The proposed formalism, named Dynamic Directed Evidential Network with Conditional Belief Functions (DDEVN), is the evidential counterpart of dynamic Bayesian network and dynamic possibilistic network

Formalism

A DDEVN is defined as a couple (D_0, D_k), where:

- D_0 denotes the DEVN corresponding to the time slice k = 0.
- D_k denotes a 2-time slices DEVN (2-TDEVN) in which only two nodes are introduced to represent a same variable at successive time slices.

Propagation algorithm in the DDEVN

$MBJTM_0$

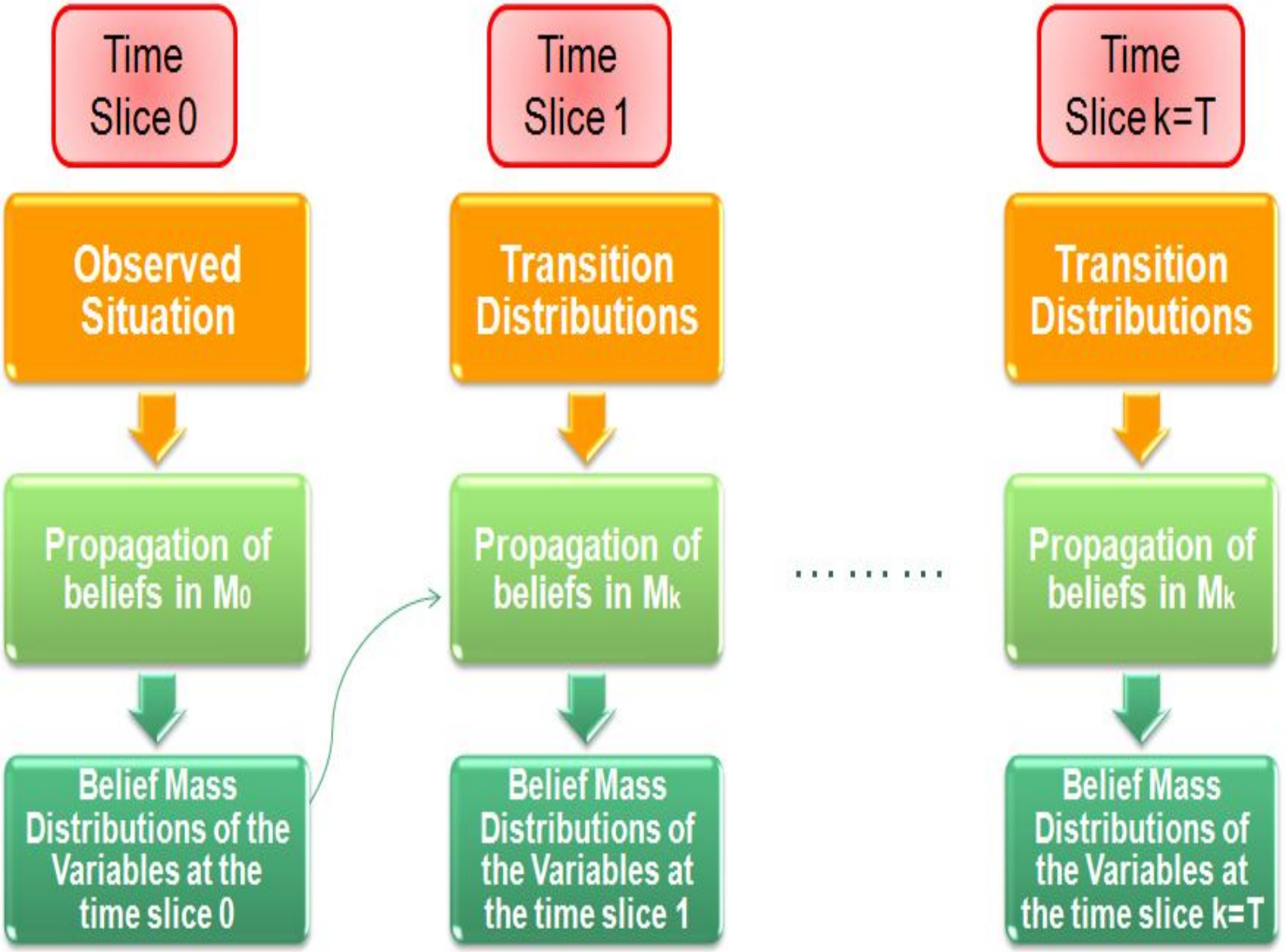
- Represents the initial time slice $k = 0$
- Is built from only the first time slice of the 2-TDEVN

Propagation of beliefs when k=0

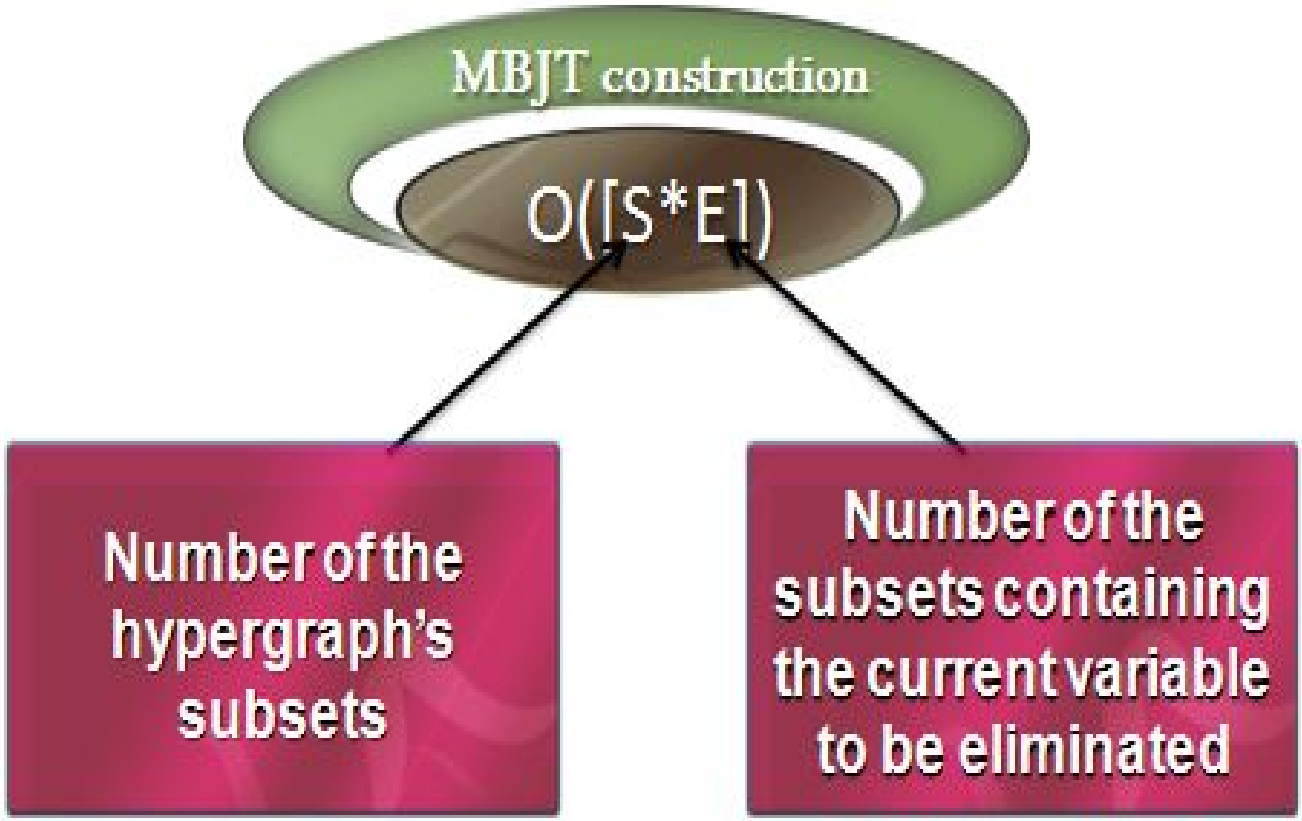
$MBJTM_k$

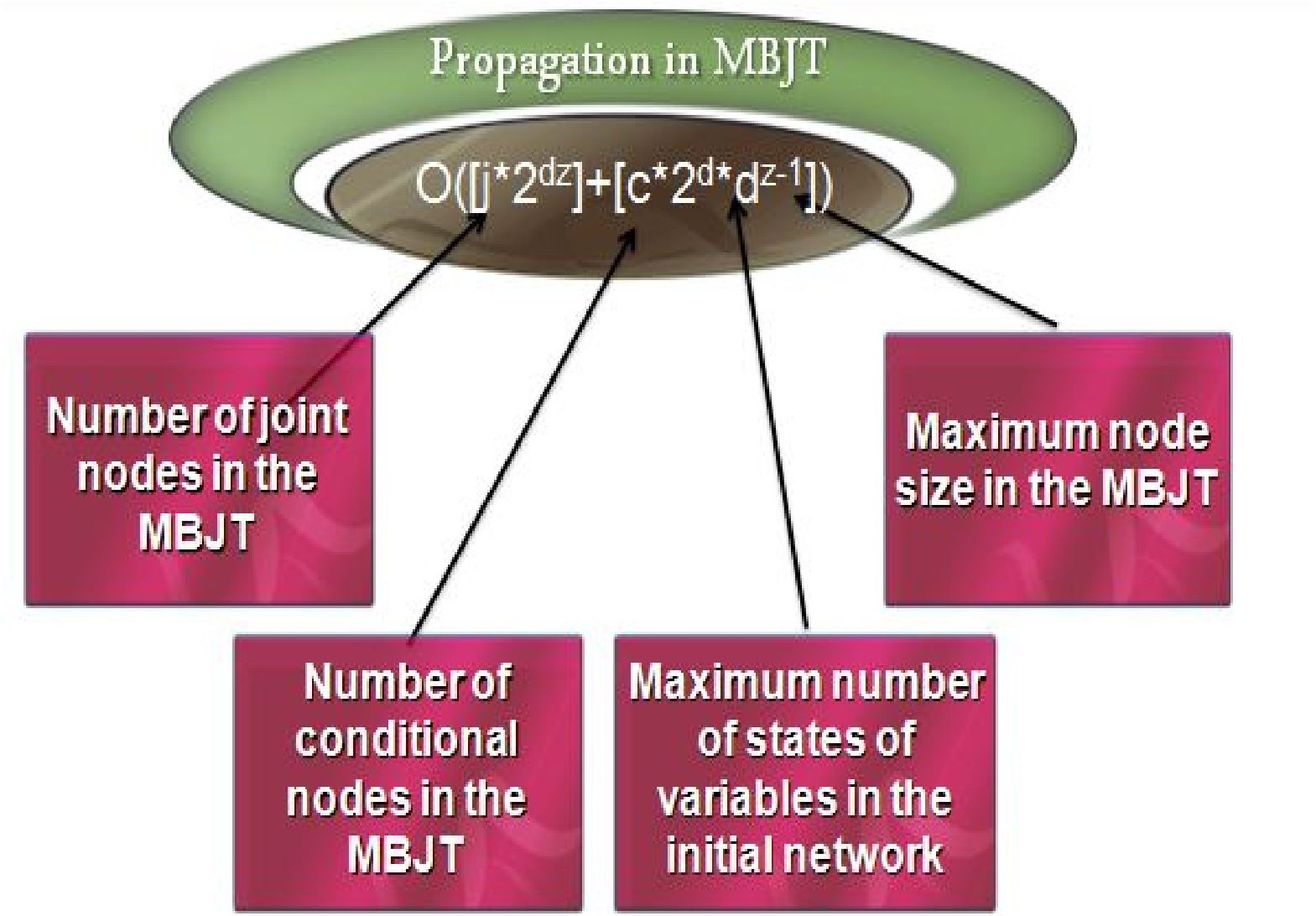
- Represents each time slice $k > 0$
- Is built from the 1.5 DDEVN

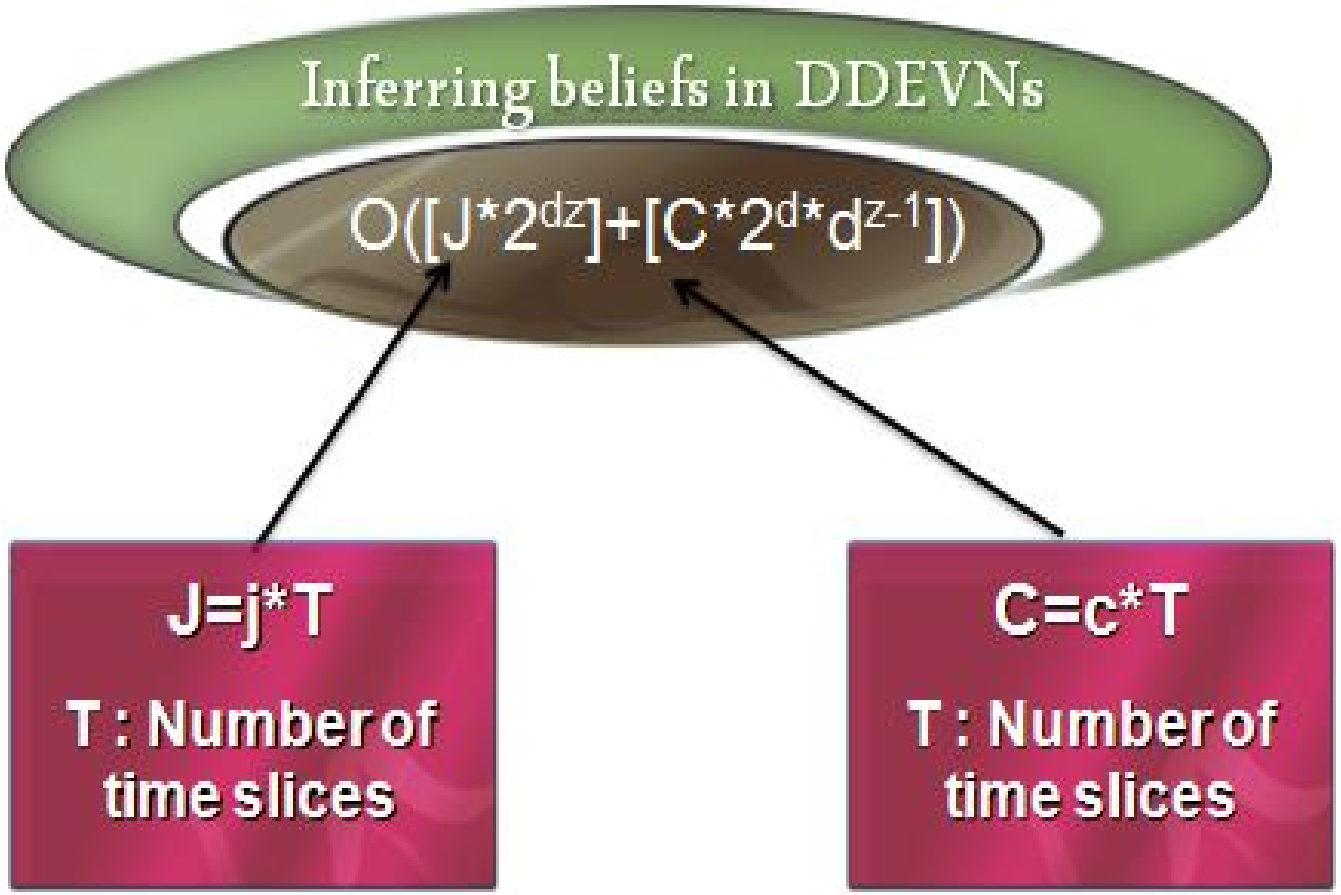
Propagation of beliefs when time slice k>0


Time Slice 0
Time Slice 1
Time Slice k=T
Observed Situation
Transition Distributions
Transition Distributions
Propagation of beliefs in M0
Propagation of beliefs in Mk
Propagation of beliefs in Mk
. . . . . .
Belief Mass Distributions of the Variables at the time slice 0
Belief Mass Distributions of the Variables at the time slice 1
Belief Mass Distributions of the Variables at the time slice k=T

We are interested in the computational complexity of the MBJTs Construction Process:




Propagation in MBJT
O([j*2^dz]+[c*2^d*d^z-1])
Number of joint nodes in the MBJT
Number of conditional nodes in the MBJT
Maximum number of states of variables in the initial network
Maximum node size in the MBJT


Inferring beliefs in DDEVNs
O([J*2^dz]+[C*2^d*d^z-1])
J=j*T
T : Number of time slices
C=c*T
T : Number of time slices

Extension of the static DEVN into the dynamic DEVN that we called DDEVN

Developing new algorithms to perform the propagation process in the dynamic directed evidential network with conditional belief functions will be of a great interest

Thank You for your attention