Possible Improvement to NSGA-III on Discrete Multi-Objective Optimizations

Peerasak Wangsom and Kittichai Lavangnananda
School of Information Technology
King Mongkut's University of Technology Thonburi
Bangkok 10140, Thailand
Email: peerasak.w@mail.kmutt.ac.th, and Kitt@sit.kmutt.ac.th

***Abstract*—The state-of-the-art multi-objective evolutionary algorithm (MOEA), *Nondominated Sorting Genetic Algorithm-III* (NSGA-III), has been recently proposed for the multi-objective optimizations and was shown superior to several MOEAs in continuous mathematical problems. Nevertheless, some works reported that NSGA-III yielded lower performance in discrete multi-objective optimizations where objective values are discrete. In such problems, NSGA-III faced the difficulty to generate nondominated solutions linked to all reference points. This work studies the possibilities to improve quality of solutions from NSGA-III. Multi-objective knapsack problem is used to evaluate the performance of NSGA-III comparing with NSGA-II. The results reveals that NSGA-III was unable to link all reference points, particularly in the best reference points of each objective. Observation of the solutions suggests that adding extra solutions pointing to a possible reference point of each objective may improve the performance of NSGA-III.**

Keywords—Discrete multi-objective optimization, Multi-objective evolutionary algorithm (MOEA), Nondominated Sorting Genetic Algorithm-III (NSGA-III), NSGA-II

I. Introduction

The key challenge of the multi-objective optimization problem lies in generating solutions having good quality in term of convergence and diversity. The convergence is to minimize the distance of solutions to the optimal front, while the diversity refers to maximizing the distribution of solutions over the optimal front. Obtaining a good balance between the convergence and the diversity has become a challenging task where Multi-Objective Evolutionary Algorithms (MOEA) have been applied. *Nondominated Sorting Genetic Algorithm-III* (NSGA-III) [1], one of the MOEAs, has been recently proposed in order to enhance the quality of solutions. NSGA-III is an improved version of NSGA-II [2], which is the most frequently-used MOEAs [3]. Instead of using NSGA-II function, called Crowding Distance, for maintaining the diversity among solutions, NSGA-III applies well-distributed Reference Points to preserve the diversity of nondominated solutions, also called Pareto solutions. NSGA-III was proven successful than some MOEAs in mathematic problems, which usually are continuous values, having more than two objectives [1], [3]. For this reason, it attracts some attention and has been applied in real applications [4].

In the discrete multi-objective optimization where values in each objective are discrete, however, the performance between NSGA-III and NSGA-II is still debatable. In the multi-objective knapsack problem, for instance, NSGA-III did not always superior to NSGA-II [3]. The work in [3] also concluded that NSGA-II yielded a higher ability to maintain the diversity among solutions, while NSGA-III performed better in generating convergence of solutions. In the open shop scheduling, NSGA-III also showed a limitation of distribution of Pareto solutions [5]. Even though the results reported that NSGA-III showed higher Hypervolume than NSGA-II in most cases, the number of Pareto solutions was considerably small (18 of 92 solutions).

According to results of previous works, using reference points in NSGA-III has difficulty in maintaining the diversity of the solutions in the discrete multi-objective optimization problems. Therefore, possibilities of improvement to NSGA-III in such problems exist. Therefore, this work aims to evaluate the performance and study the behavior of NSGA-III. In order to reaffirm the result of the previous work [3], experiments are carried out to compare the performance of NSGA-III and NSGA-II in the multi-objective knapsack problem with three objectives. However, this study uses different performance metric where Hypervolume [6] is adopted in the performance evaluation and report the number of Pareto solutions. The detail of the study is elaborated in the following section.

II. Performance Evaluation on Multi-objective Knapsack Problem

A. Multi-Objective Knapsack Problem

The knapsack problem is one of the most studied NP-hard problems in combinatorial constrained optimization [7]. In the multi-objective optimization perspective, the problem becomes the determination of available items that maximize all profit values while not exceeding the weight capacity.

In formal, a multi-objective knapsack problem K can be formulated as a set of (W, I, P) where W is a weight capacity, I is a set of items, and P is a set of profits. An item i is member of set I and it has a weight w_i and positive integer profits v_{ip} where p is member of set P. Decision variable x_i denotes whether item i is selected for the knapsack or not. The capacity of the knapsack was set to half weight and the knapsack problem can be formulated as follows :

$$max \sum_{i=1}^{n} v_{ip} x_i , \quad i \in I, p \in P, x_i \in \{0,1\} \quad (1)$$

subject to $$\sum_{i=1}^{n} w_i x_i \leq W \quad (2)$$

$$W = \lceil 0.5 \sum_{i=1}^{n} w_i \rceil \quad (3)$$

where: v_{ip} is the value of the profit p of the item i,

w_i is the weight of the item i,

x_i is 1 if the item i selected, otherwise is 0, *W* is the weight capacity of the knapsack.

This work adopts the same manner as in [6] for generating test problems having three objectives. The values of w_i and v_{ip} are random integers in the interval [10, 100] where p is three objectives. In this study, item sizes of I were set ranging from 250, 500, 750, to 1,000. For the representation of the problem in NSGA-II and NSGA-III, chromosomes can be represented directly as a binary solution where the length of chromosome is equal to the number of items and the value in each slot can be either 0 or 1.

B. Experiment Setup

The comparison of performance of NSGA-II and NSGA-III is evaluated in term of Hypervolume which is the volume of space between solutions to the reference point. In this work, the larger volume is preferred. NSGA-II and NSGA-III are implemented by jMetal [8]. All parametric values of both algorithms followed those proposed in [1], [3]. They are summarized in Table I. Each experiment consists of four item sizes. Each item size comprises 20 instances and each instance was executed 10 runs. Results of Hypervolume of each item size are presented in minimum, maximum, and average values of the 10 runs of the 20 instances.

TABLE I. PARAMETRIC VALUES OF NSGA-II AND NSGA-III

Parameter	**Value**
Item sizes	250, 500, 750, and 1,000
Item instances	20 instances of each size
Reference points	91 for NSGA-III
Population size	92
Crossover operation	Simulated Binary Crossover (SBX) - the distribution index = 30 - the probability = 1.0
Mutation operation	Polynomial Mutation - the distribution index = 30 - the probability = 1/the length of chromosome
Stopping criteria	300 generations

III. RESULTS AND DISCUSSIONS

Fig. 1 presents the Hypervolume of NSGA-II and NSGA III on four item sizes. The minimum, maximum, and average cases of Hypervolume are depicted. Blue and red bars are used to represent values of NSGA-II and NSGA-III respectively.

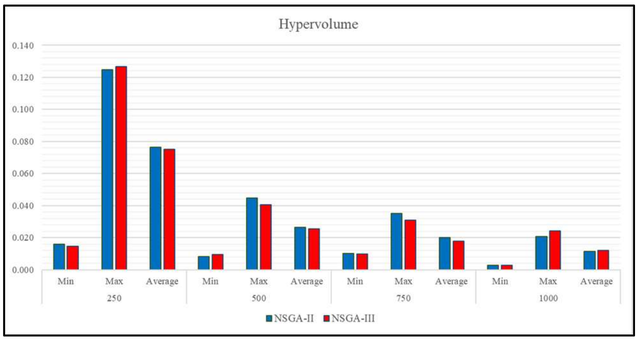

Fig. 1. Hypervolume of NSGA-II and NSGA-III

With the respect to Hypervolume in Fig. 1, NSGA-II yields slightly better Hypervolume than those of NSGA-III in 250, 500, and 750 items sizes but not in 1,000 item sizes. In overall, NSGA-II and NSGA-III gained 0.0336 and 0.0325 in the average Hypervolume respectively. Moreover, this study also observes the number of Pareto solutions where both algorithms yielded equally 14 of 92 solutions in the average value. This result reveals that both algorithms having the difficulty to generate a well distribution Pareto solutions.

In addition, NSGA-III is unable to generate solutions reaching the best reference points of each objective. This causes NSGA-III gained a lower number of Pareto solutions and poor diversity of solutions. By observing in the performance, it suggest that additional of some extra solutions that can guide the algorithm to a possible best reference point of each objective is advantageous. This strategy ought to increase the possibility of NSGA-III to generate Pareto solution with higher diversity.

IV. CONCLUSIONS AND FUTURE WORKS

In the discrete multi-objective optimizations, NSGA-III is still facing to generate Pareto solutions with higher diversity. This problem leads to poor performance in term of Hypervolume. This study compares and evaluates the performance of NSGA-III and NSGA-II in multi-objective knapsack problems. The results reaffirm that the improvement of NSGA-III is required in such problems. The observation in this study suggests that adding some extra solutions may help NSGA-III producing solutions which may reach reference areas of each objective. Consequently, NSGA-III might gain better the diversity of Pareto solutions.

Therefore, this study can be further extended by implementing the strategy mentioned above in NSGA-III. The result of the implementation ought to be applicable to various types of problems as well.

REFERENCES

[1] K. Deb and H. Jain, "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints," *IEEE Transactions on Evolutionary Computation,* vol. 18, no. 4, pp. 577-601, 2014.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation,* vol. 6, no. 2, pp. 182-197, 2002.

[3] H. Ishibuchi, R. Imada, Y. Setoguchi, and Y. Nojima, "Performance comparison of NSGA-II and NSGA-III on various many-objective test problems," in *2016 IEEE Congress on Evolutionary Computation (CEC)*, 2016, pp. 3045-3052.

[4] M. Tavana, Z. Li, M. Mobin, M. Komaki, and E. Teymourian, "Multi-objective control chart design optimization using NSGA-III and MOPSO enhanced with DEA and TOPSIS," *Expert Systems with Applications,* vol. 50, pp. 17-39, 2016.

[5] G. Campos Ciro, F. Dugardin, F. Yalaoui, and R. Kelly, "A NSGA-II and NSGA-III comparison for solving an open shop scheduling problem with resource constraints," *IFAC-PapersOnLine,* vol. 49, no. 12, pp. 1272-1277, 2016.

[6] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach," *IEEE Transactions on Evolutionary Computation,* vol. 3, no. 4, pp. 257-271, 1999.

[7] T. Lust and J. Teghem, "The multiobjective multidimensional knapsack problem: a survey and a new approach," *International Transactions in Operational Research,* vol. 19, no. 4, pp. 495-520, 2012.

[8] A. J. Nebro, J. J. Durillo, and M. Vergne, "Redesigning the jMetal Multi-Objective Optimization Framework," presented at the Proceedings of the Companion Publication of the 2015 Annual Conference on Genetic and Evolutionary Computation, Madrid, Spain, 2015.