



# **A Statistical FEA Method for Predicting Glass Fracture in Consumer Electronic Products**

*Marc Zampino, Ph.D.  
Product Integrity Organization  
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# Consumer Electronic Products



# A Complex Highly Integrated Assembly



## So why do build reliable products?

*Because our customers value it.*

Customers will always value:

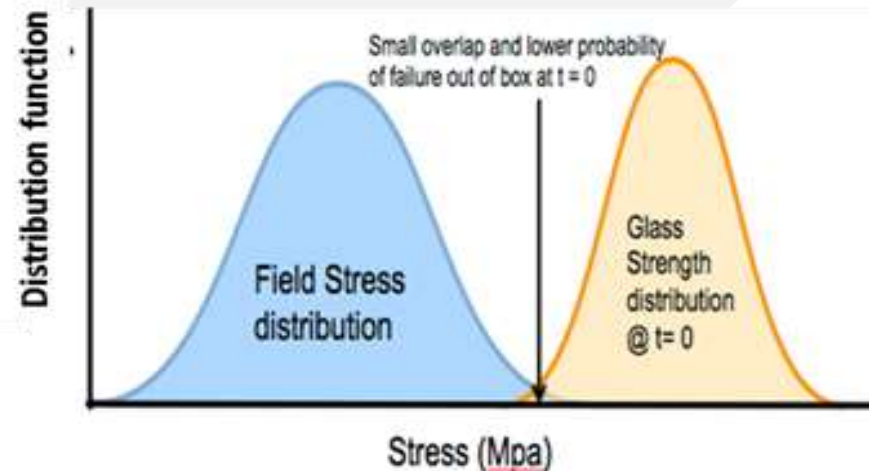
- Getting their stuff cheaper.
- **Getting it delivered faster.**
- Having it last longer.

**We strive to engineer premium product at a non-premium price.**

# Reliable Design: Balancing Stress and Strength

- The stress-strength interference concept is useful to illustrate the basic tenets of robust design
  - Parts have an inherent strength to specific applied loads.
  - The design of the product modifies the loads that are passed to the part.
  - The overlap of these two things gives us the risk of failure.
- When possible, we try to push the stress curve to the left and the strength curve to the right to minimize the overlap region.
  - Unfortunately, competitive CE products tend to live right in the cross over region.
  - The reality is that we design for an acceptable amount of risk.
- For assessing probabilistic risk, such as glass failure in drop impact:
  - Structural FEA can provide the stress distribution
  - Materials testing provides the strength distribution
- Looks and sounds simple....

*.....the devil always lies in the details*

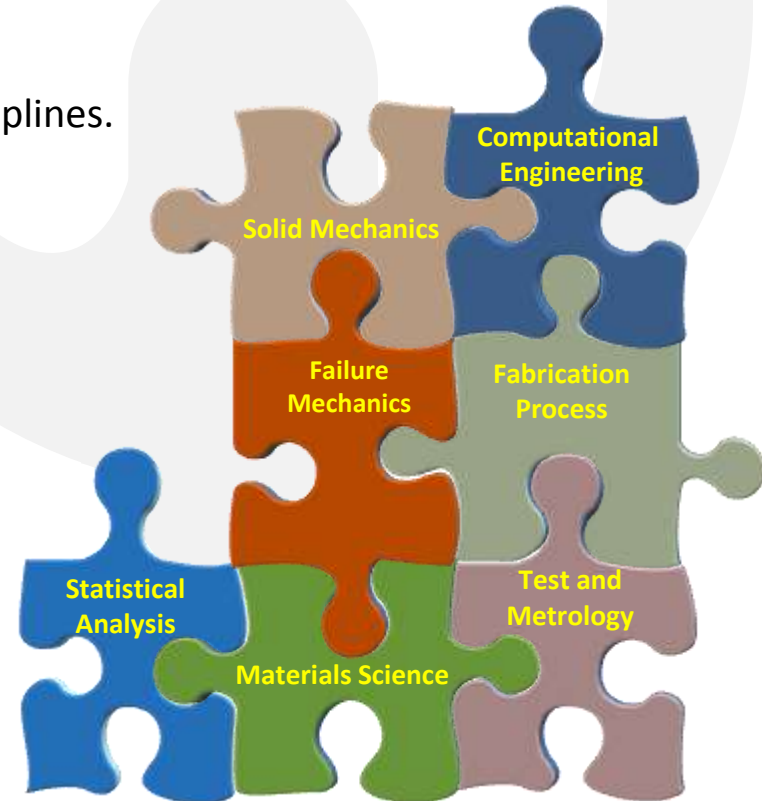
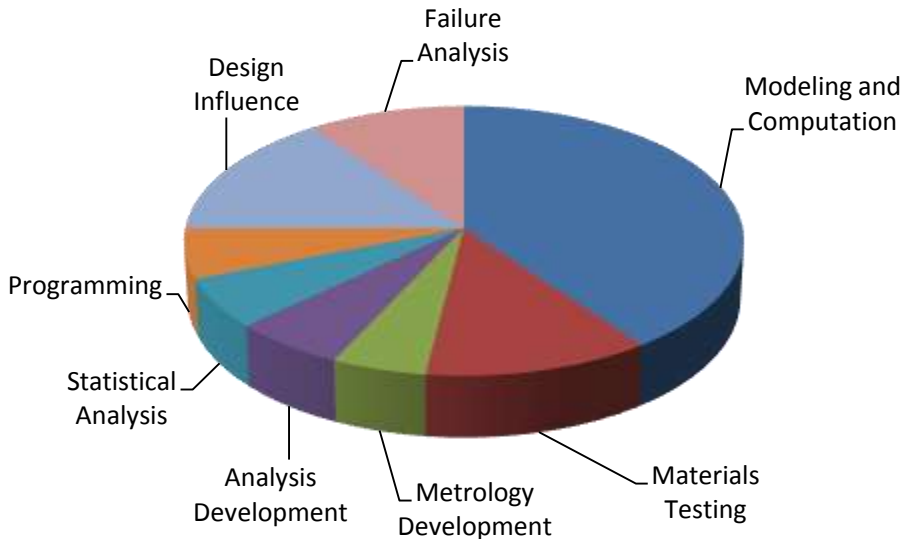




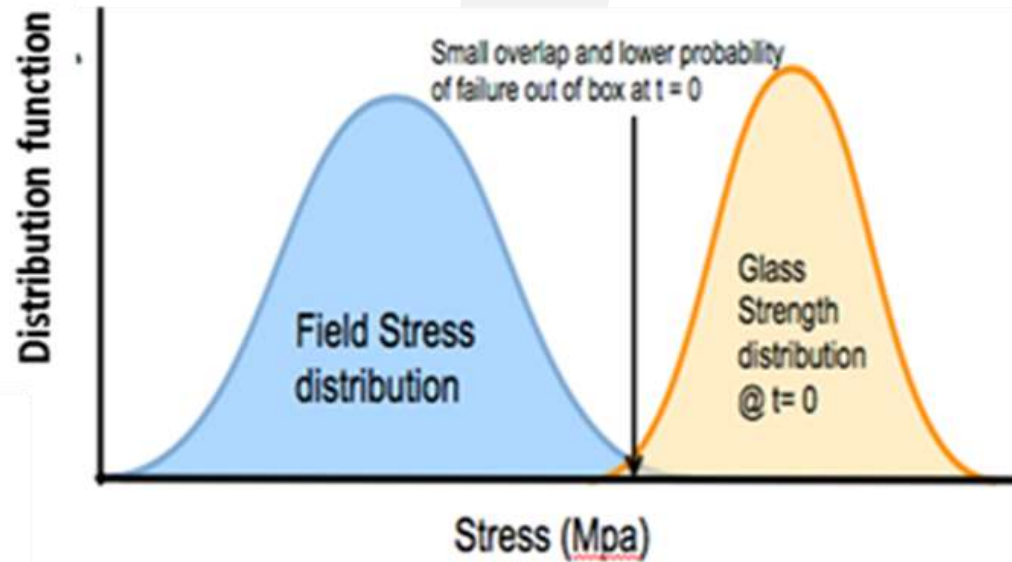
# So how do we achieve reliable design?

- A data driven iterative design process that is:
  - leverages “tribal knowledge”, historical data, lessons learned
  - reliant on *best-in-class* simulation engineering
  - up front testing and failure analysis
- Product development process that builds-in design integrity:
  - not an option or a nice-to-have
  - substantiates design decisions with data before “locking in”
- Product Integrity is the convergence of expertise in many disciplines.

*Design Integrity engineers have broad skill sets and responsibilities.....  
.....it's more than running code*



*Structural FEA provides the field stress distribution*



*....so let's look at how that is done*

# Behind the Scenes: What is Finite Element Analysis?

A numerical technique for finding approximate solutions to boundary value problems for PDE's.

- Solves the governing equations of motion, solid, fluid, thermal and electrical mechanics.
- For assemblies of diverse 3D parts with non-linear materials, the governing equations are not solvable without numerical methods.

It subdivides (discretizes) a large problem into smaller, simpler, parts, called finite elements.

- Within elements, the mechanics can be described by reduced forms of the governing equations.
- Creates a system of equations to be solved simultaneously.
- More elements, more accuracy.

Uses calculus of variations to approximate a solution by minimizing an associated error function.

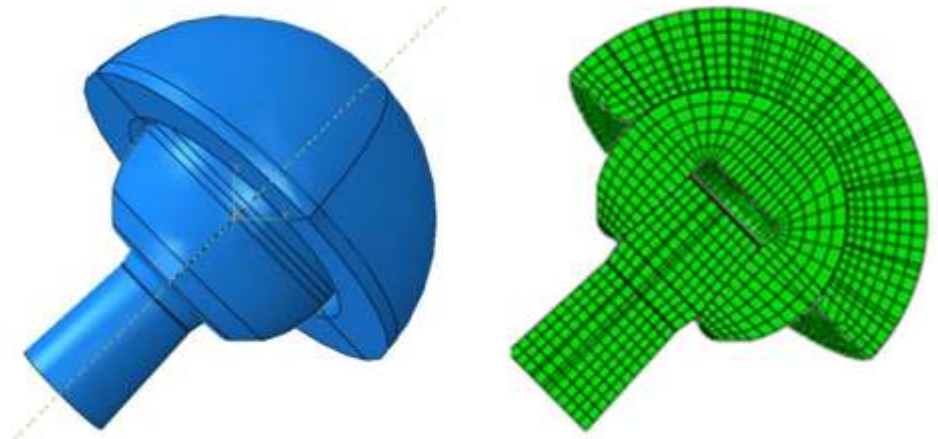
- Iterates to ensure conservation of mass and energy across the system.

$$\sigma_{ji,j} + F_i = \rho \partial_{tt} u_i \quad \begin{array}{l} \text{Eqn of Motion} \\ 3 \text{ eqn; } 6 \text{ ind terms} \end{array}$$

$$\epsilon_{ij} = \frac{1}{2} (u_{j,i} + u_{i,j}) \quad \begin{array}{l} \text{Strain-displacement} \\ 6 \text{ eqn; } 9 \text{ ind terms} \end{array}$$

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad \begin{array}{l} \text{Constitutive (Hooke's Law)} \\ 6 \text{ eqns; } 21 \text{ ind terms} \end{array}$$

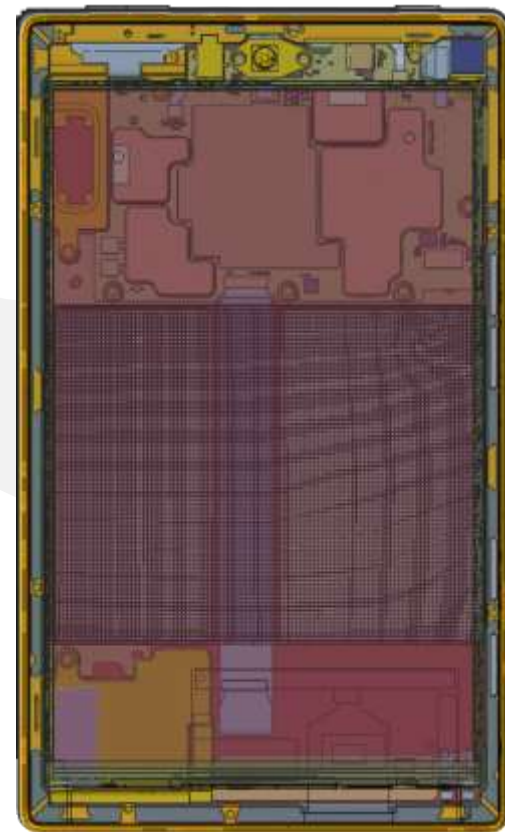
↳  $F = Kx$



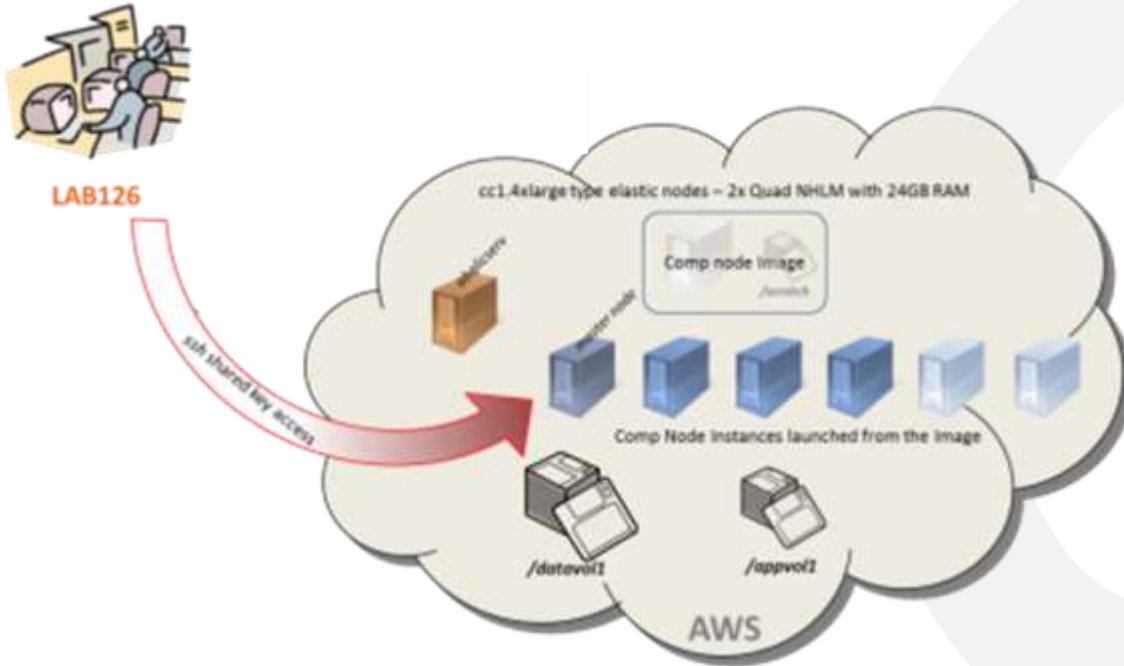


## Behind the Scenes: Modeling

- Fully detailed models that capture dynamic effects, contact, material and geometric nonlinearities
- Custom developed multi-threaded batch meshing scripts to reduce modeling time.
- Typical models range from 80-150 parts
- Model size: 6-10M DOFs



# Behind the Scenes: Compute



Full top assembly drop simulations require parallel processing compute systems.

- Ability to go 'Wide or Deep' to maximize throughput for the team.
- Jobs require >24 cpus for reasonable run times
- Scalability is limited.
- Average of 10 GB of data generated per run.

Results are provided in several ways to quantify, visualize and trend the response of the system.

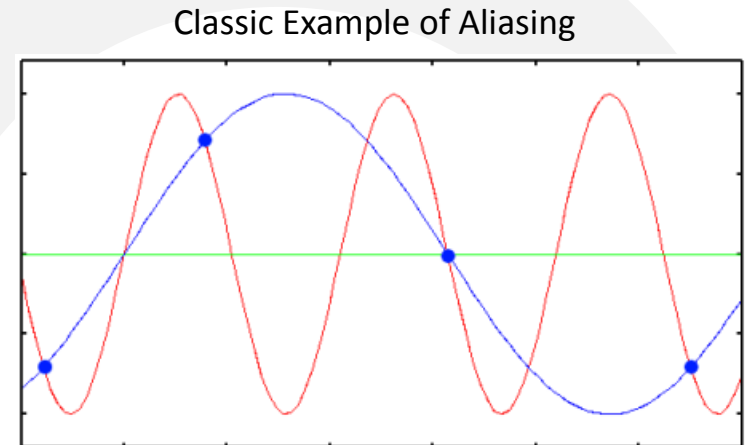
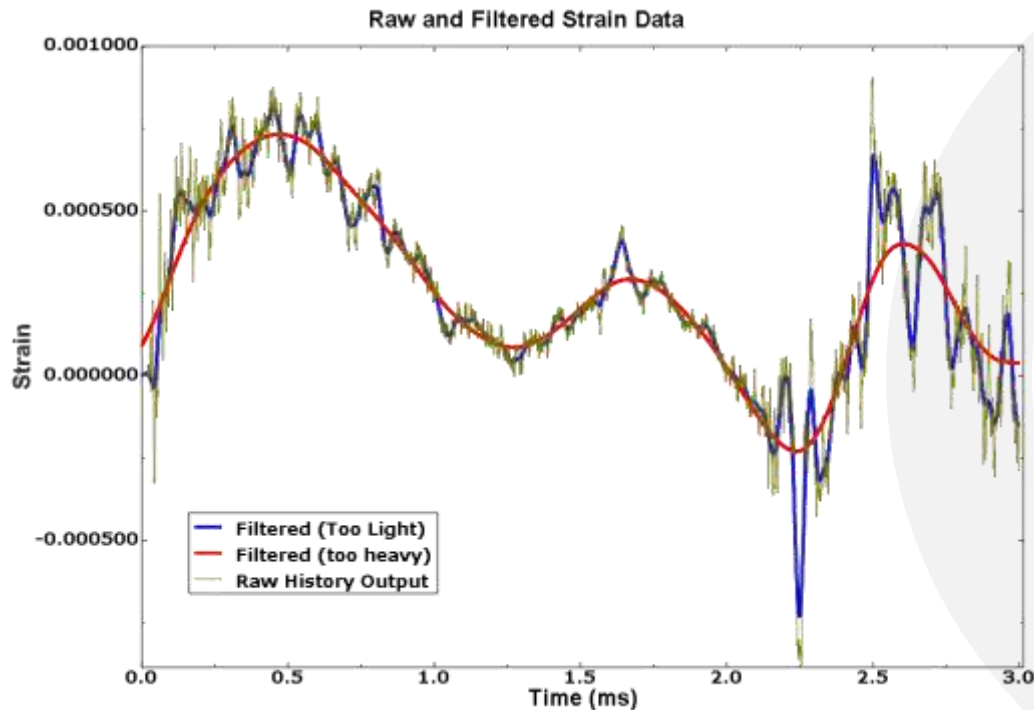
- Animations provide spatial, temporal and quantitative metrics
- External views, cross sections, transparent overlays
- XY Plots: time history, force vs. displacement, etc.
- Contour images of spatial results. Raw and processed.

Results are externally processed for more detailed analyses such as predicting glass fracture.

Typical simulation passes generate large numbers of outputs which are too time consuming to process manually.

- Custom post-processing scripts for automating the process
- Faster, consistent, more accurate

## Behind the Scenes: Data isn't always clean and simple



- Digital signal processing (DSP) is an integral part of drop simulation data capture.
  - *Resampling and regularization, filtering (unidirectional and bidirectional), frequency response, FFT*
- Huge scale: ten's of thousands of data streams simultaneously

Drop impact: results that are highly spatial and temporal in nature

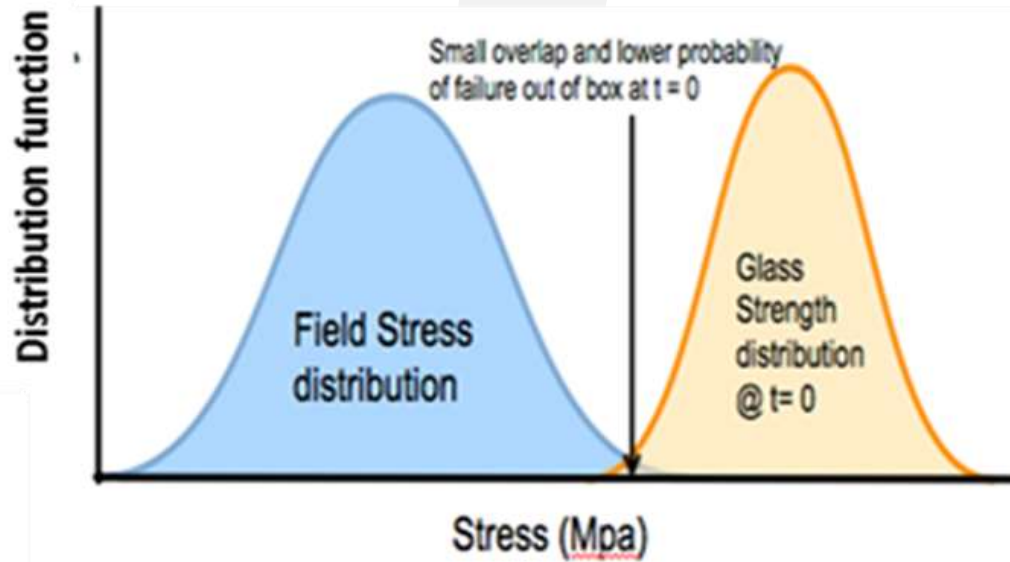
→ you have to deal with issues such as *aliasing, sampling errors, and digital noise.*



2013 KINDLE FIRE HD 7"

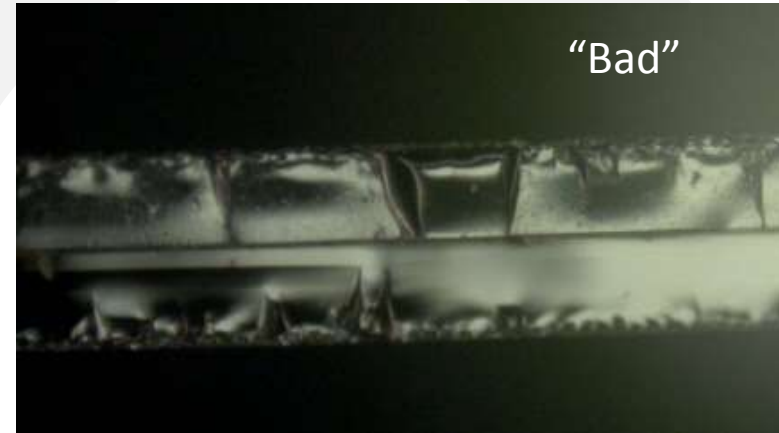


*Now it's time to discuss the glass strength distribution*

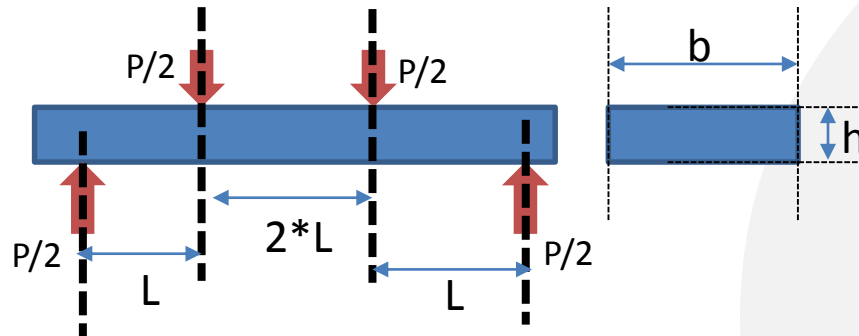


# Glass Failure is Due to Surface and Edge Defects

- Failure in glass occurs because of pre-existing defects (stable cracks) in the glass becoming unstable and propagating across the glass causing catastrophic failure.
- Inherent defects: flaws in the polycrystalline structure of the glass are inherent in the glass at the time of manufacture
- The defects that really effect design and reliability are created during the many steps to create displays, touch sensors, and cover glass components.
  - photolithography for electrical functionality, adhesive application, filling with liquid crystal material, joining processes, application of polymer layers, driver and flex attachment, cosmetic paint, and optical coatings.
- The **scribe and break process** is the single largest source of defects which affect glass strength.
  - Additional damage may be caused by handling during processing.



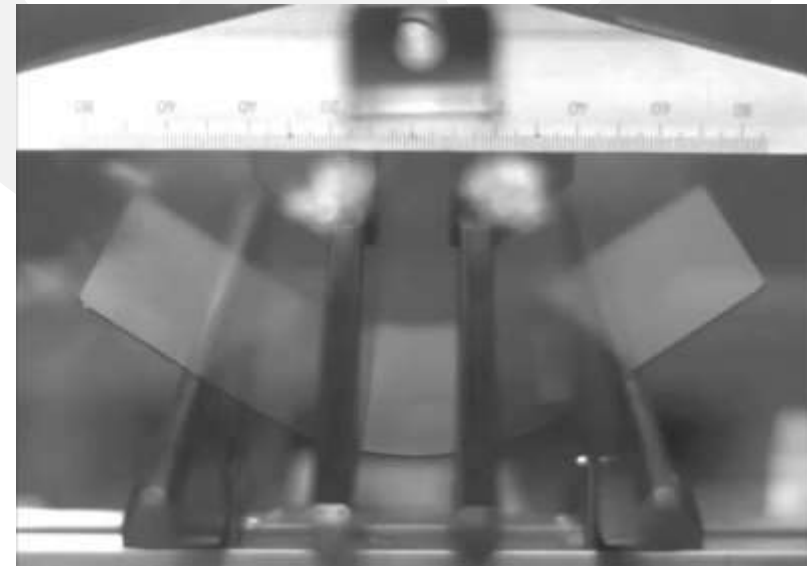
# Characterizing Glass Strength



$$\text{Strain} \propto \frac{3}{8} * \frac{w * h}{L^2}$$

$$\text{Stress} \propto \frac{3}{4} * \frac{P * L}{b * h^2}$$

- Four-point Bend Test is traditionally used for brittle materials
  - uniform curvature / quasi-steady loading rate
- Defects of varying size are randomly dispersed in the glass
  - frequency follows a statistical distribution
  - defects have a critical stress at which they fail
- The largest defect in the test section will fail first (weakest link concept).
- Easily determines critical failure metrics (stress or strain)
  - pure bending mechanics
  - small displacement assumption



## Characterizing Glass Strength (con't)

Brittle materials are traditionally fitted with a 2-P Weibull distribution

- Provides a shape ( $m$ ) and strength parameter ( $\sigma_o$ )
- Strength related to a geometry metric ( $V_o$ )
- Max Likelihood Estimation typically used

The Weibull parameters obtained are related ONLY to:

- geometric size of the sample in the loading span
- the observed failure mode locations

Strength is effected by:

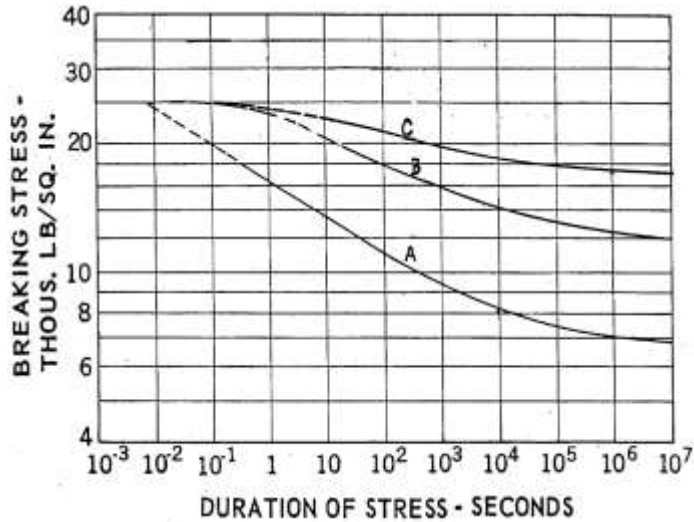
- Size Effects, Loading Rate Effects

$$F(\sigma) = 1 - \exp \left[ -\frac{V}{V_o} \left( \frac{\sigma}{\sigma_o} \right)^m \right]$$

### Underlying assumptions (Weibull, 1931)

- Statistical homogeneity and isotropy of the material: the strength of a specimen is independent on its position within the specimen.
- Statistical independence: The reliabilities of subvolumes/subsurfaces of a specimen are to be multiplied in order to obtain the reliability of the whole specimen.
- The weakest link concept: The weakest part (subvolume/subsurface) of a specimen determines its strength.

# Loading Rate Effects on Glass Strength



From George W. McLellan and E.B. Shand, "Glass Engineering Handbook"; Mc Graw Hill Company, 1984

With typical strain rates for drop impact (10<sup>-1</sup> to 10<sup>1</sup> /s), fracture mechanics theory tells us that **strength can be increased by 1.5 to 2.0 times over static testing strength.**

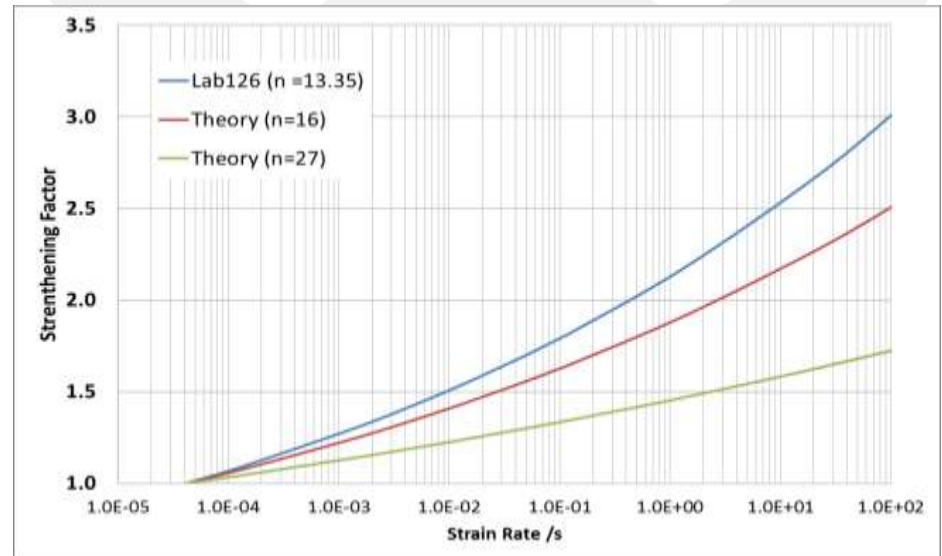
The strength of glass is related to the rate at which it is loaded.

- *Stress corrosion* is related to humidity effects on crack propagation, both by the chemical interaction between the water and the glass, and by the transport of water to the crack tip.

From fracture mechanics, a basic relationship for the effect of loading rate and time on the strength can be derived:

$$\sigma = \sigma_o (\dot{\sigma} / \dot{\sigma}_o)^{(1/(n+1))} \quad \text{and} \quad \sigma = \sigma_o (t_o / t)^{(1/n)}$$

for soda-lime glass  $n \approx 16$  to 27 (Vandebroek, 2009)





# Size Effects on Glass Strength

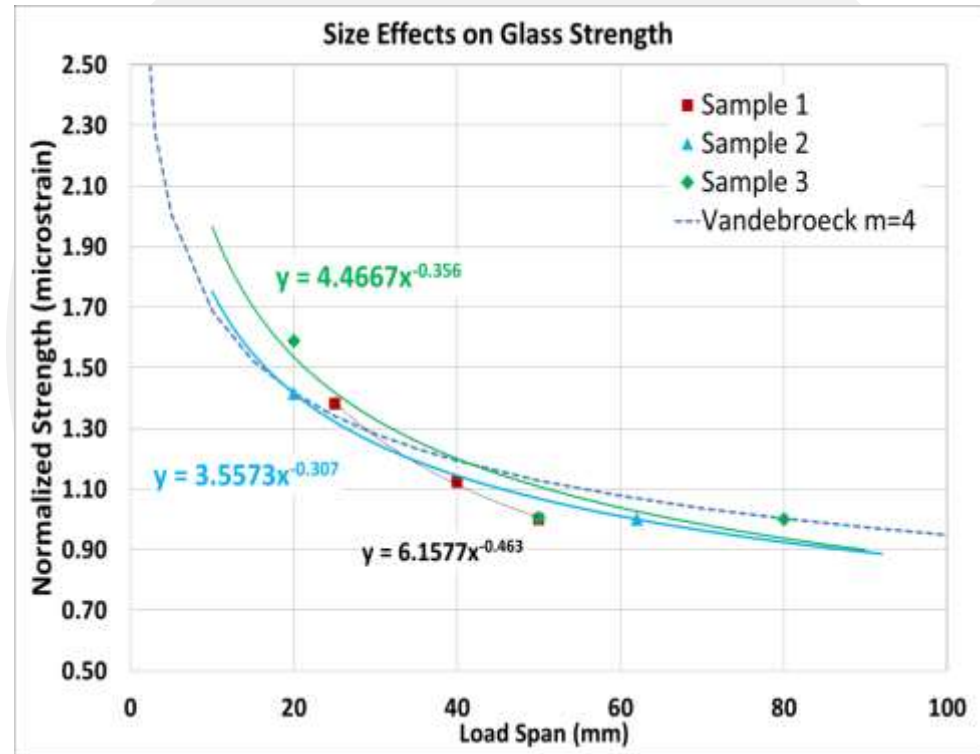
Weibull's formulation can be used directly to calculate the strength at other geometric sizes, with scaling relying solely on the shape parameter:

$$\sigma_n = \sigma_o \left( \frac{L_n}{L_o} \right)^{(-1/m)}$$

*It tells us that larger sections under test will have lower strength.*

Weibull's work infers that the shape parameter is independent of geometric size.

- *Is this really true?*



Observed issues with scaling:

- Cut glass always scale with an effective shape parameter smaller than the large sample tested value.
- The same glass tested at two spans never have the same shape parameter and scale poorly at either tested value.
- Shape parameter appears too erratic for small sample sets taken from the same population of glass.

# Deviations from Weibull Scaling?

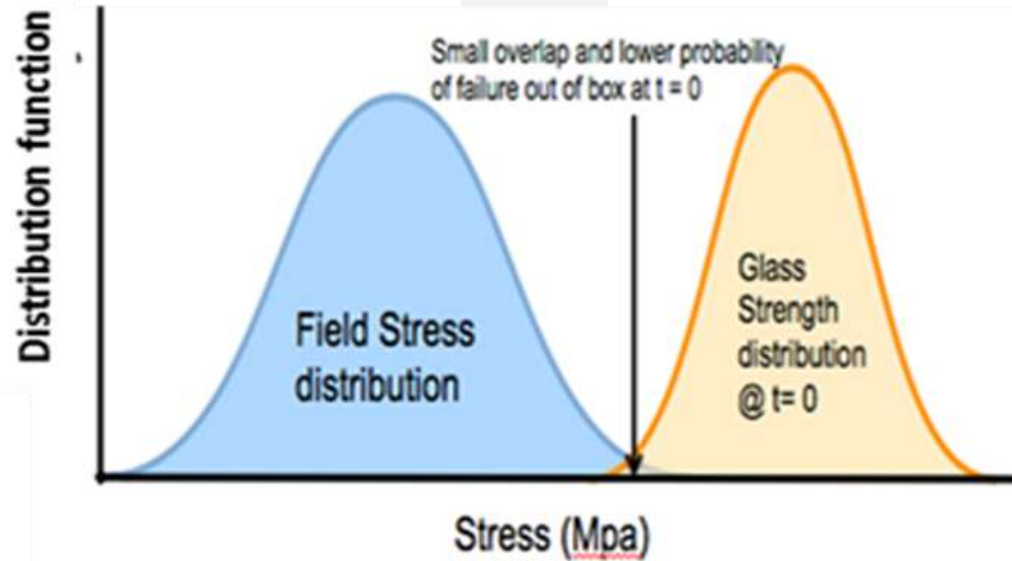
## Engineered defects versus naturally occurring:

- Weibull's Assumption: Statistical homogeneity and isotropy of the material: the strength of a specimen is independent on its position within the specimen.
- Weibull's work was directed at intrinsic defects in the material, and the theoretical handling was based on volume centric flaw distributions.
- It is not unreasonable to assume the extrinsic defects from the scribe and break and other handling may violate the assumption for homogeneity and isotropy in the material.

## Does cut grass really follow a Weibull Distribution? (Todinov (2008), Danzer (2006), Danzer (2007))

- The assumption of non-interacting flaws and no new crack creation may be too conservative.
- Monte Carlo simulations show that brittle fracture is not necessarily follow a Weibull distribution.
- It can be shown there are conditions where it may not be possible to clearly distinguish if the data truly follows a Weibull, Guassian, or similar distribution:
  - Testing with small sample sets ( $\approx 30$ )
  - Multimodal flaw size distributions or relatively high flaw density.
  - Volume, surface, and edge defect distributions may not follow similar distributions or have the same sensitivity to geometric size.

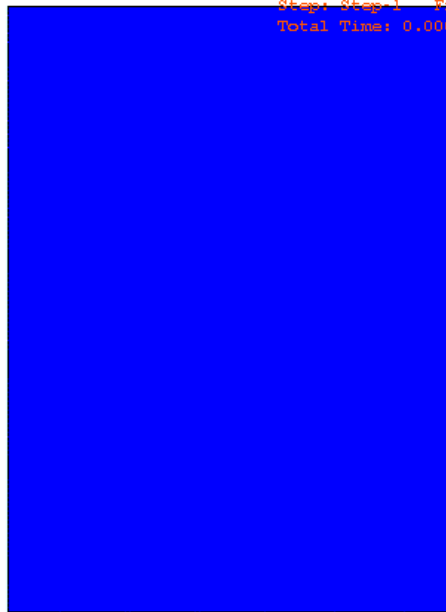
*Let's just see how the numbers work out using the tested values.....*



*Now, we need to reduce a temporal and spatially varying stress distribution and use the strength distribution to determine risk.*

# Temporal and Spatially Varying Stress Distributions

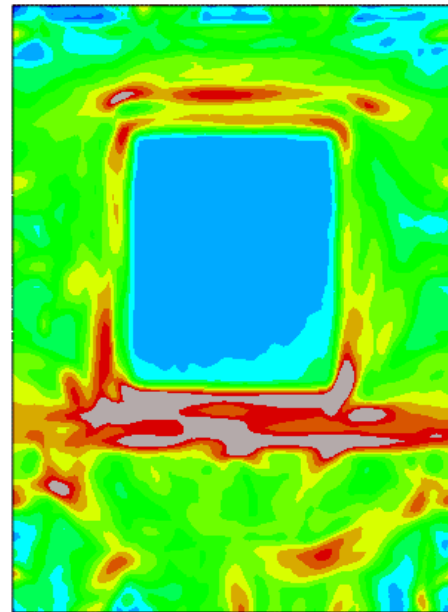
Temporal/Spatially Result



Max Principal Strain

→

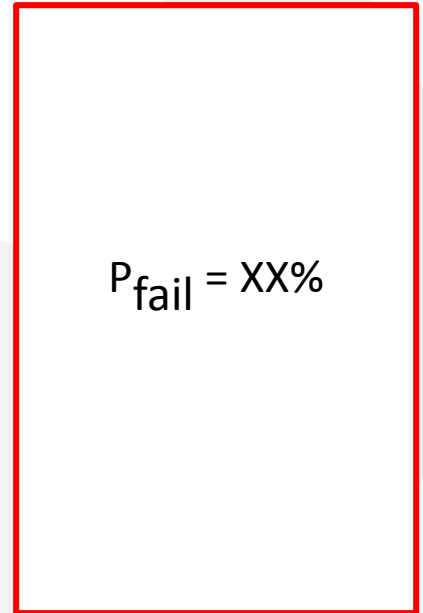
Temporal Compression



Max Principal Strain

→

Spatial Compression



Quantitative Risk

Goal: Go from raw results, compress time and spatial variations to obtain a quantitative risk metric.

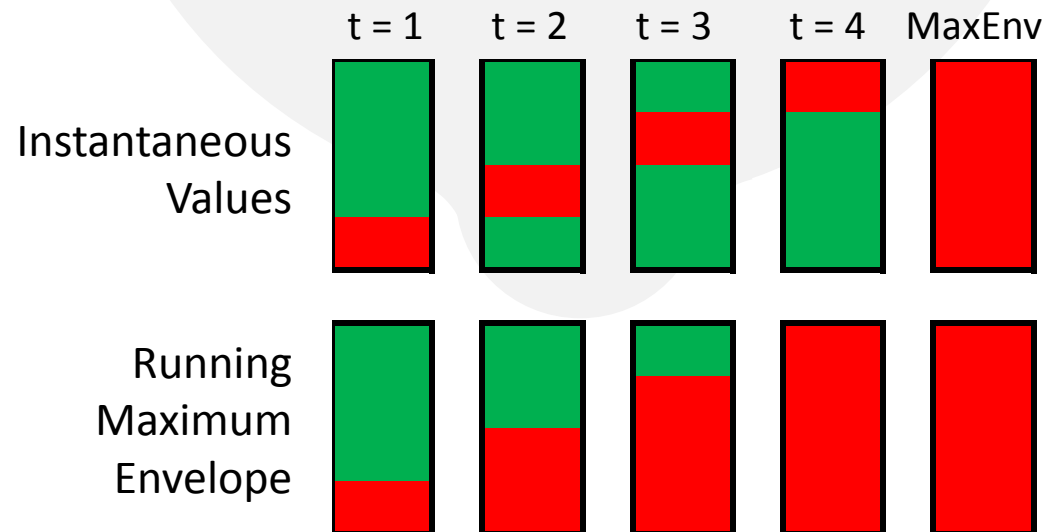
# Temporal Compression

- We rely on the condition that brittle materials do not have cumulative damage, nor fatigue failure modes by repeated loadings below critical stress values. (*technically only for short periods of time*)
- From Weibull: Strength is based on pre-existing defects in the material; no new defects are created, defects do not interact, defects don't grow.

***Repeated loadings ARE NOT independent events!!***

- From weakest link concept, we are only concerned about the largest defect that can be loaded at every location over the duration of the entire loading event.

- This is achieved by enveloping the maximum values at each element (i.e. spatial) location over the duration of the drop event.





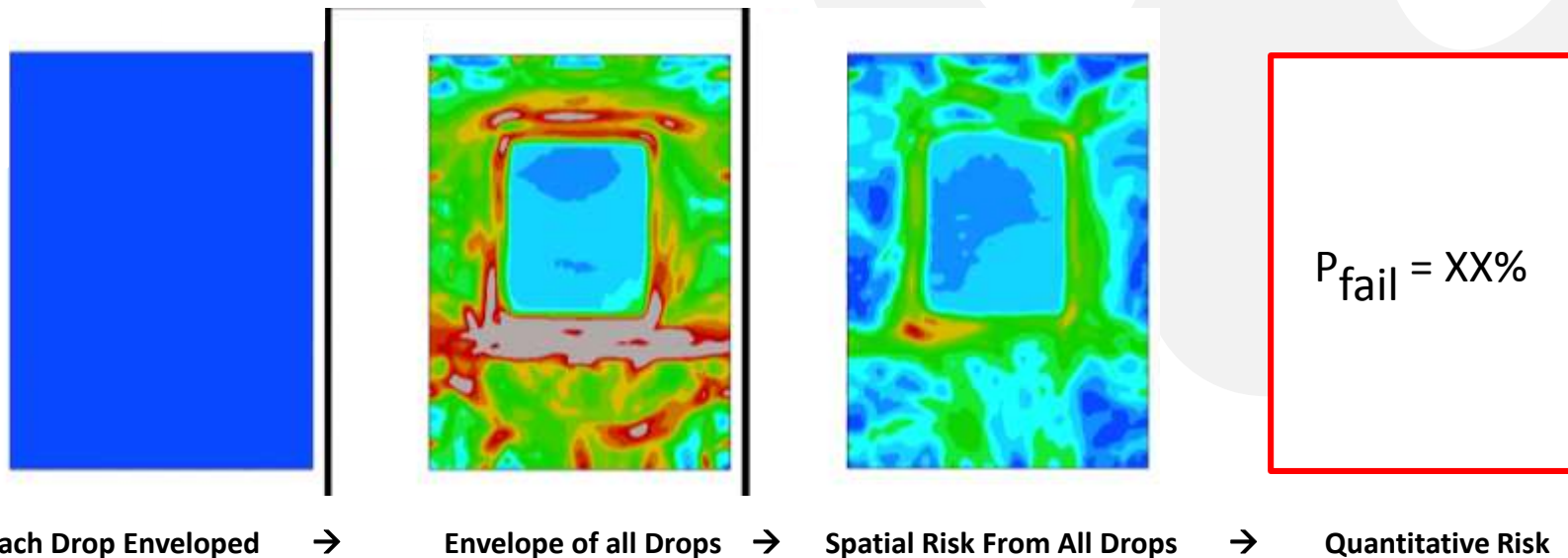
# Temporal Compression Across Multiple Drop Impacts

## Repeated loadings are NOT Independent Events

- Multiple loadings only alters the stress distribution used to assess risk.
  - Defects are not created, altered, or move around on the glass between loadings.

## Risk for a Multiple Drop Sequence

- Assumption: there is no a major degradation/damage to the structural system
- Multiple drops are just an extended temporal variation of the stress distribution
- Handled by enveloping maximum values at each spatial location over all the drop impacts.



## Spatial Compression (Getting a single risk from a varying stress distribution)

- To determine the total risk of failure from a spatially varying stress state we start with Weibull's 2<sup>nd</sup> assumption and use the **independent subvolumes/subsurfaces**:

$$P_{fail} = 1 - \prod_{i=1}^n R_i \quad R_i = \exp \left[ -1 \left( \frac{\sigma_i}{\sigma_o} \right)^m \right]$$

- Strength has to be scaled first to the size of the subvolume/subsurface.
  - Total risk is found by multiplying the reliabilities together:
- When the stress is spatially varying, then the integrated form of the Weibull distribution can be used within the subvolume/subsurface, or for the entire part (scaling is actually built into the integrated form):

$$P_{fail} = 1 - \exp \left[ \frac{-1}{A_0} \int_0^A \left( \frac{\sigma(A)}{\sigma_o} \right)^m dA \right]$$

- If used for subvolumes, strength has to be scaled to subvolume, multiply together.
  - If used for the whole part, the form will scale between the test and part sizes.
- Both forms are equivalent and will deliver the same results.

*The temporal and spatially varying problem presented here is different than most structural risk assessments in that:*

- *Risk is determined by the probability of having a defect and its critical stress in the same place at any point in time.*
- *Risk only increases by an increase in the area under elevated stress.*

## Revisit: Enveloping Across Drop Orientations

**Goal: To saturate the stressor distribution so that it accurately reflects the environmental conditions of interest.**

Correlation to test results means capturing the test.....with all its inherent variations/errors.

Testing orientations are actually probabilistic. *(Simulate what you tested, or test what was simulated)*

Small variations in drop orientations are often meaningful for the most critical drop orientations.

- Concept of 'cones/rings of death' and the projected surface patch on a "sphere of impact"
- Products have sweet-spots, hot-spots, and systemic issues

Simulating/testing only: sweet spots → under estimate risk  
hot-spots → over estimate risk.

Generally, roll-off is handled by running additional simulations capturing rolled-off orientations.....

*but how these runs are combined (prior to enveloping) will affect the final risk assessment*

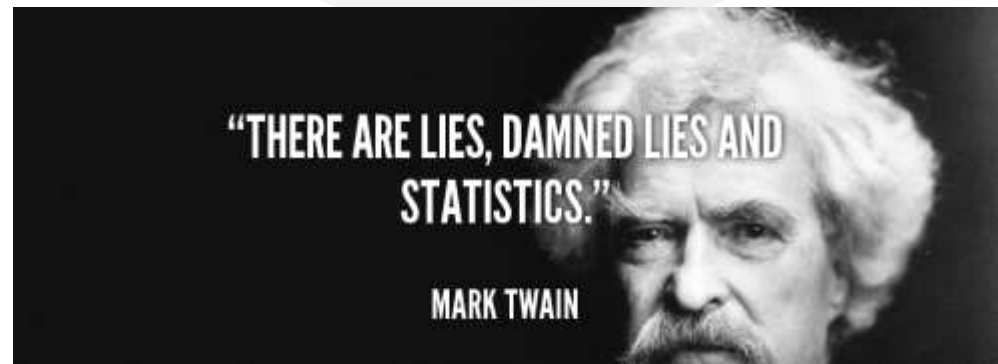
Concepts: "Overloading" and "underloading" the stress distributions

**The prediction method should account for the probabilistic nature of tested orientations if small amounts of roll-off cause measurable changes in the predicted risk.**

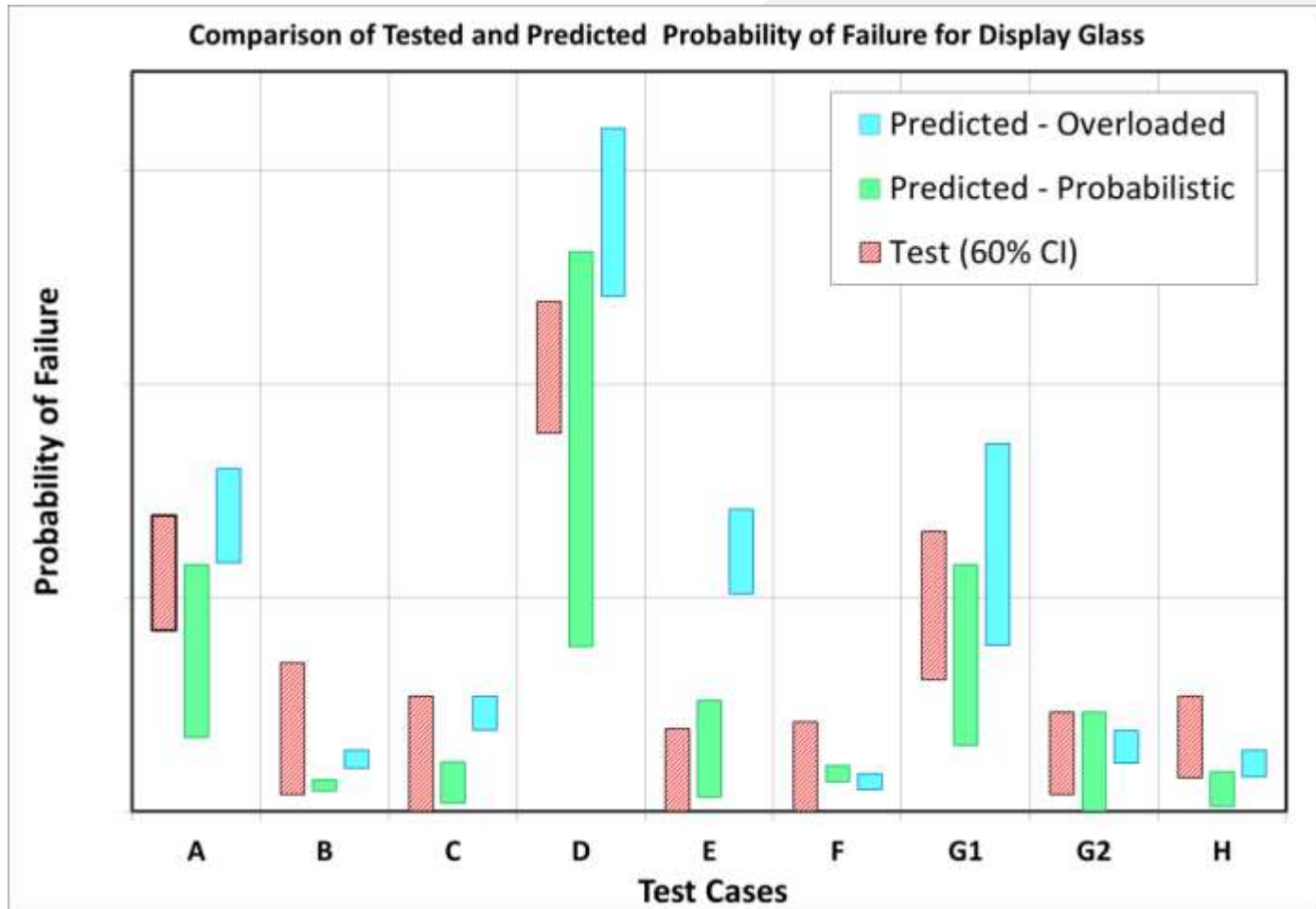
beware



metrics & statistics still require  
**reasoning** and **visual** examination



# Predicted and tested risk compared.....why are their bars?



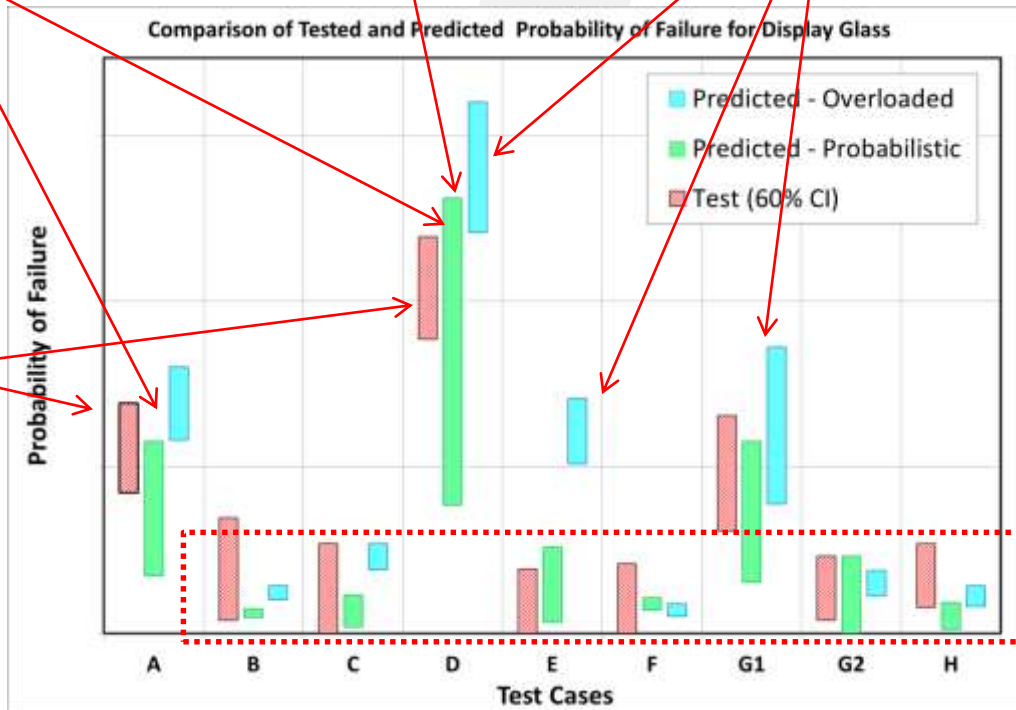
- Tested results shown by their 60% binomial confidence interval.
- Predicted results are shown for two methods for combining drops; interval reflects bounding rate factor.
  - Overloaded: all orientations combined, target plus roll-off orientations
  - Probabilistic: *Monte Carlo* method used to combine orientations to prevent overloading the distribution.

# Results: Details to discuss

As risk increases, intervals naturally grow (and quickly)

Large interval caused by scheme to randomize orientations combined with rate correction interval using bounding values.

Cases where over loading created large/excessive error in the prediction



Dialing in the rate correction can optimize both methods to 'center up' on the tested risk

In general, overloading shows upper (conservative) bound for risk.

As low levels of risk, the prediction methods will (should) converge.



# Statistical FEA

Finite element analysis is a solution method for a boundary value problem....

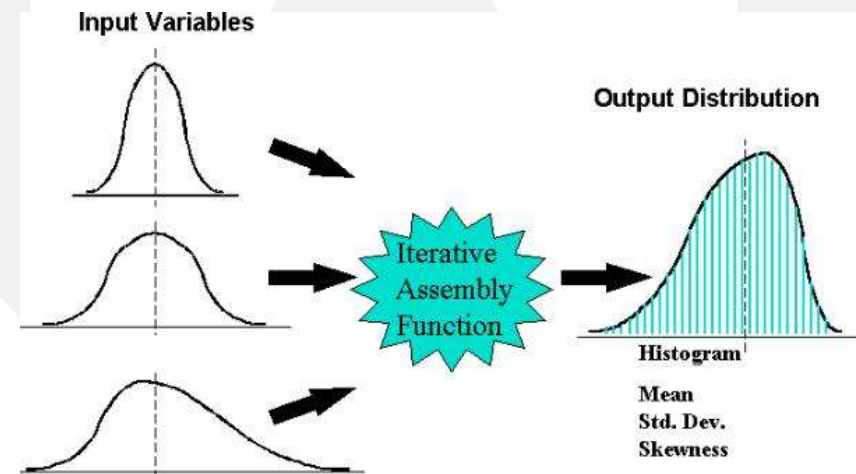
*...each case is a solution for a single set of boundary conditions*

*Statistical FEA* is the combination of results from multiple boundary conditions using a statistical framework to account for the variations in the following:

- glass strength test sample size
- glass strength during production, vendors, wear in field
- strain rate for the reported stressors
- orientation in testing and/or random drop scenarios
- orientation (if modeling tumble or random user drop).
- drop height (for random drop scenarios)

To account for these variations we can employ:

- Bounding (limiting) sets (→ large intervals)
- Monte Carlo methods (→ proper distributions)
- Other analytical approaches (?)



**Goal: To minimize the number of (expensive) runs needed, scale results for structural effects (if possible/reliable), and account for strength effects in post processing**

**Does it bring value?**

**Does it work?**

**Can we deploy it reliably?**

**Are results clear or interpretative?**

**Is there still work to be done?**





**Thank You!**