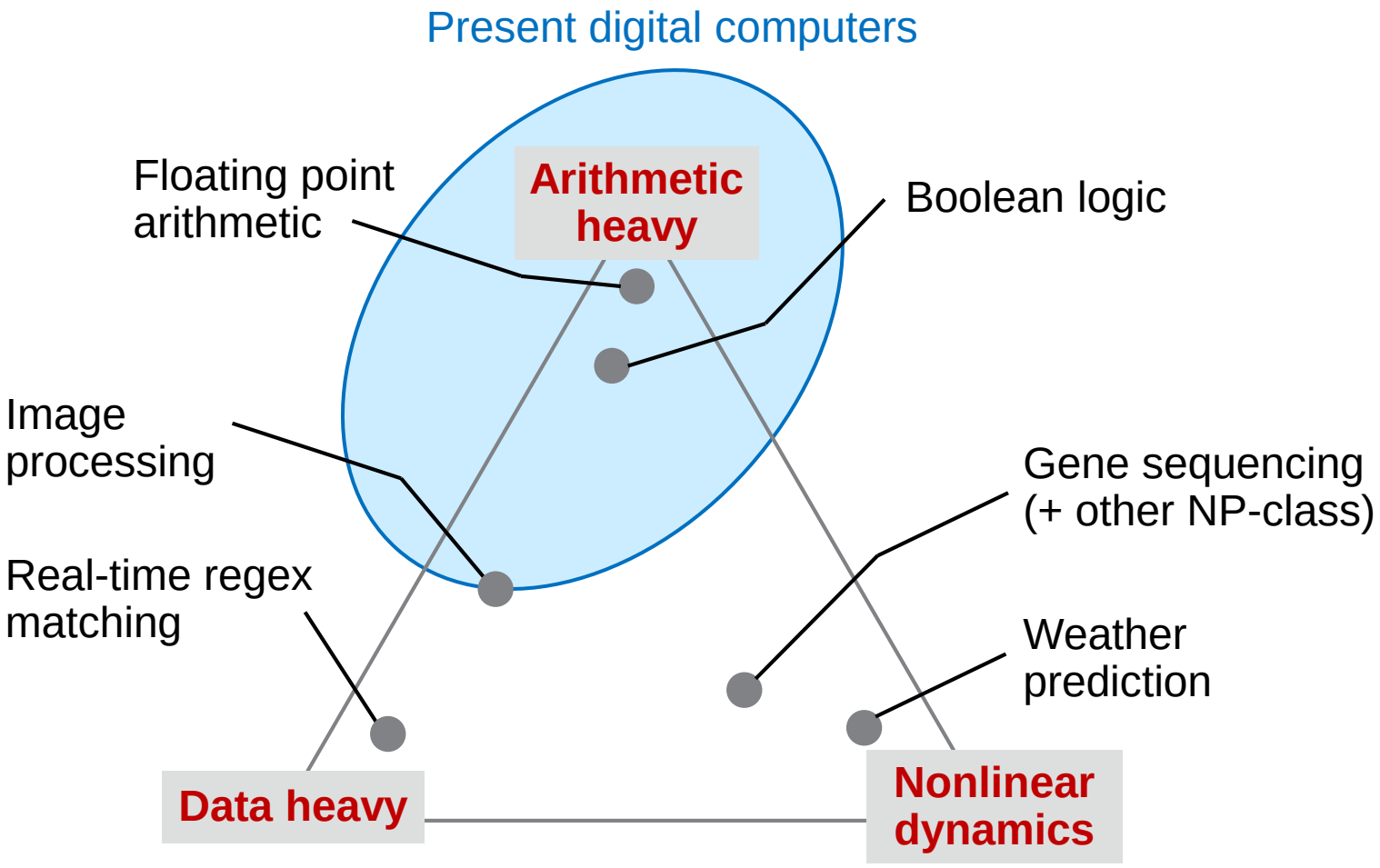
Nonlinear oxide devices and brain-like analog computing

Suhas Kumar

Hewlett Packard
Labs

Thanks for the funding and user facilities

The voids of computing

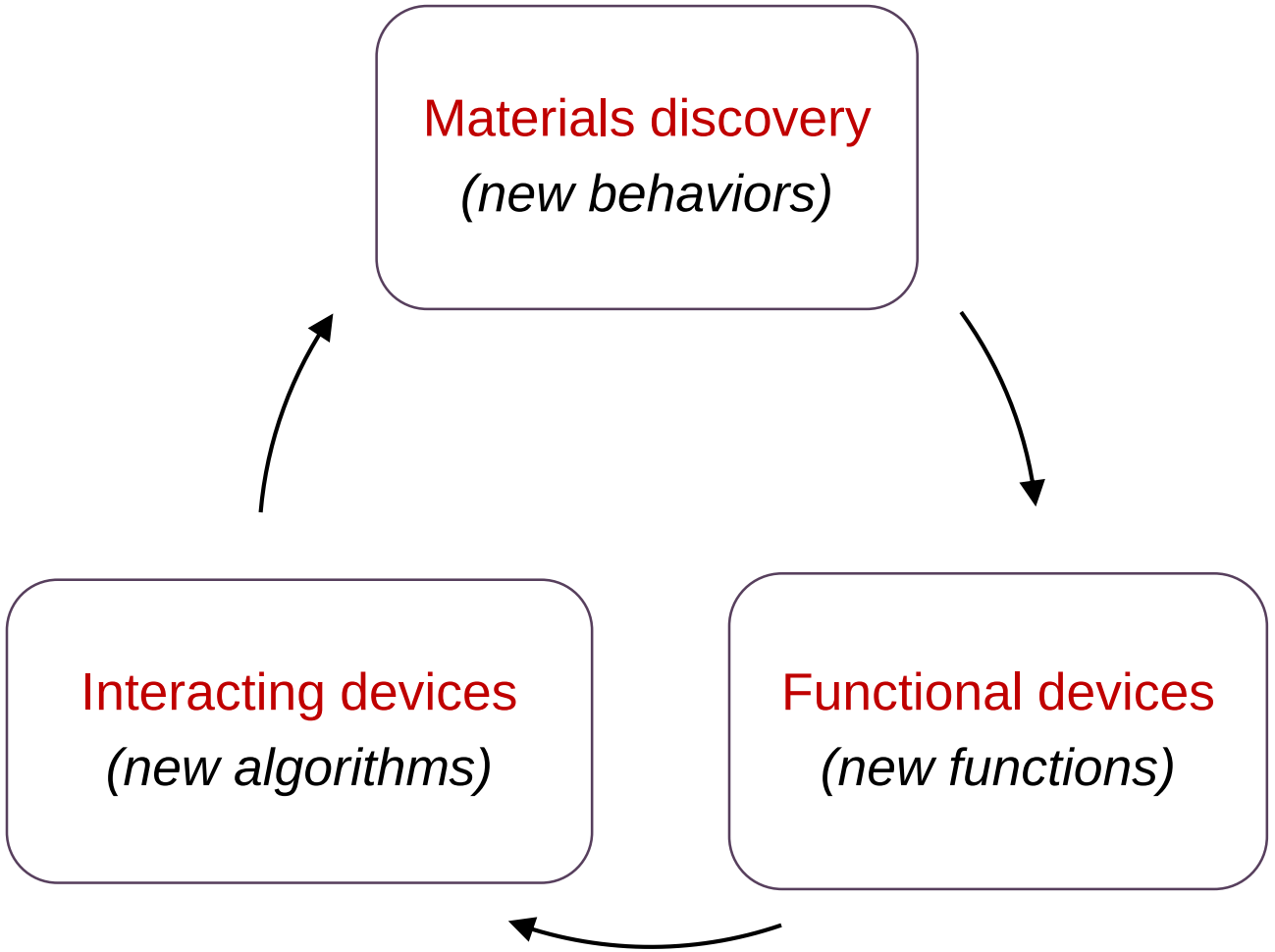


Major limitations of digital computers:

- End of Moore's law

- Von Neumann architecture
- Boltzmann tyranny
- Boolean logic
- Turing limit

The post-Moore's law physics-driven computer



Outline of the talk

1. Neuronic devices – information processing or communication

2. Synaptic devices – information storage

3. The fundamental connection between synaptic and neuronic devices

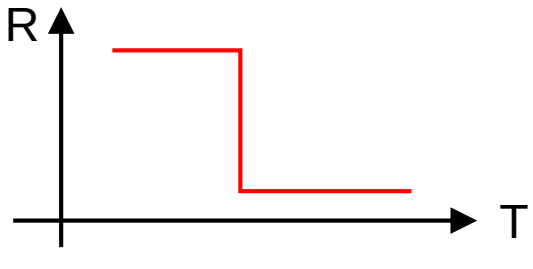
4. Synaptic + neuronic NP-hard accelerators

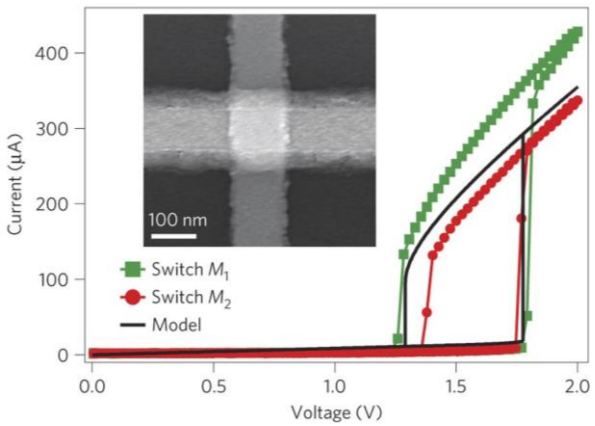
Neuronic devices

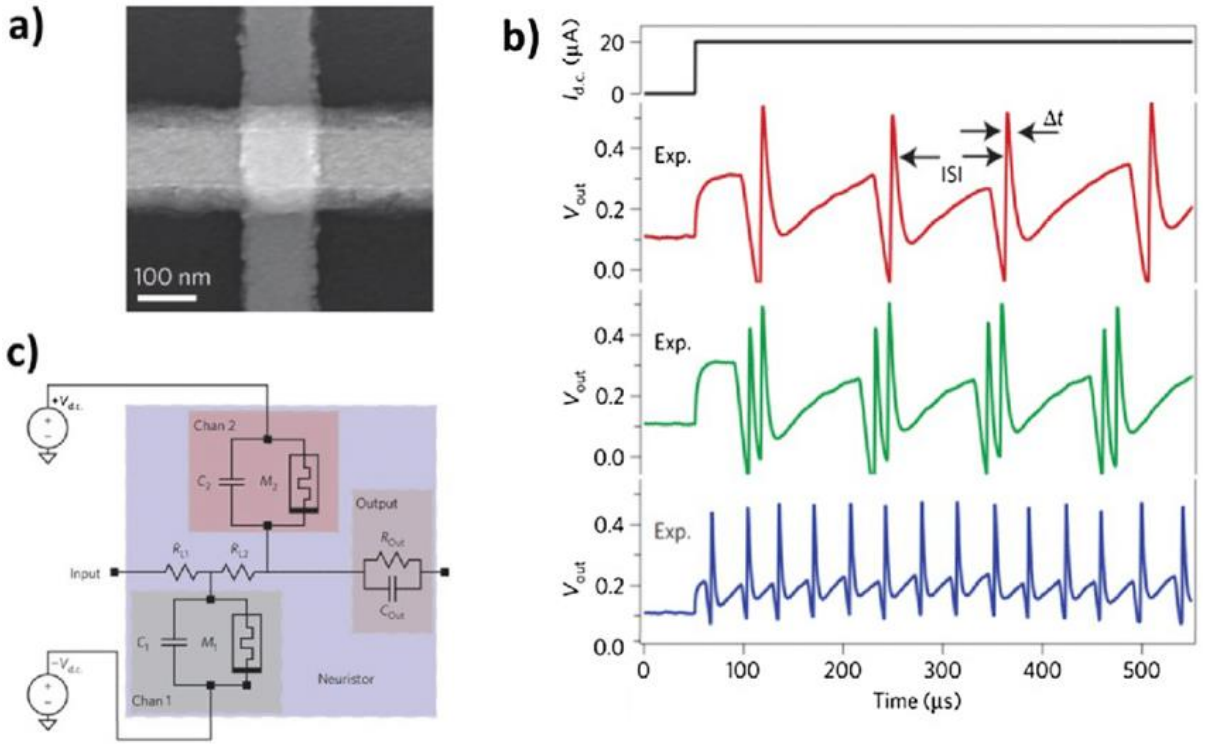
Mott Memristors

Volatile resistive switching can emulate neuron-like spiking!

NbO2 → Mott insulator



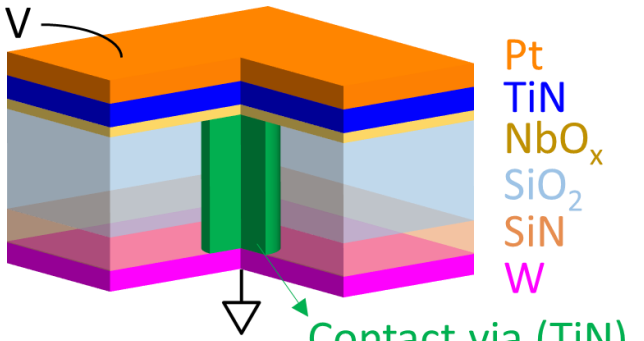




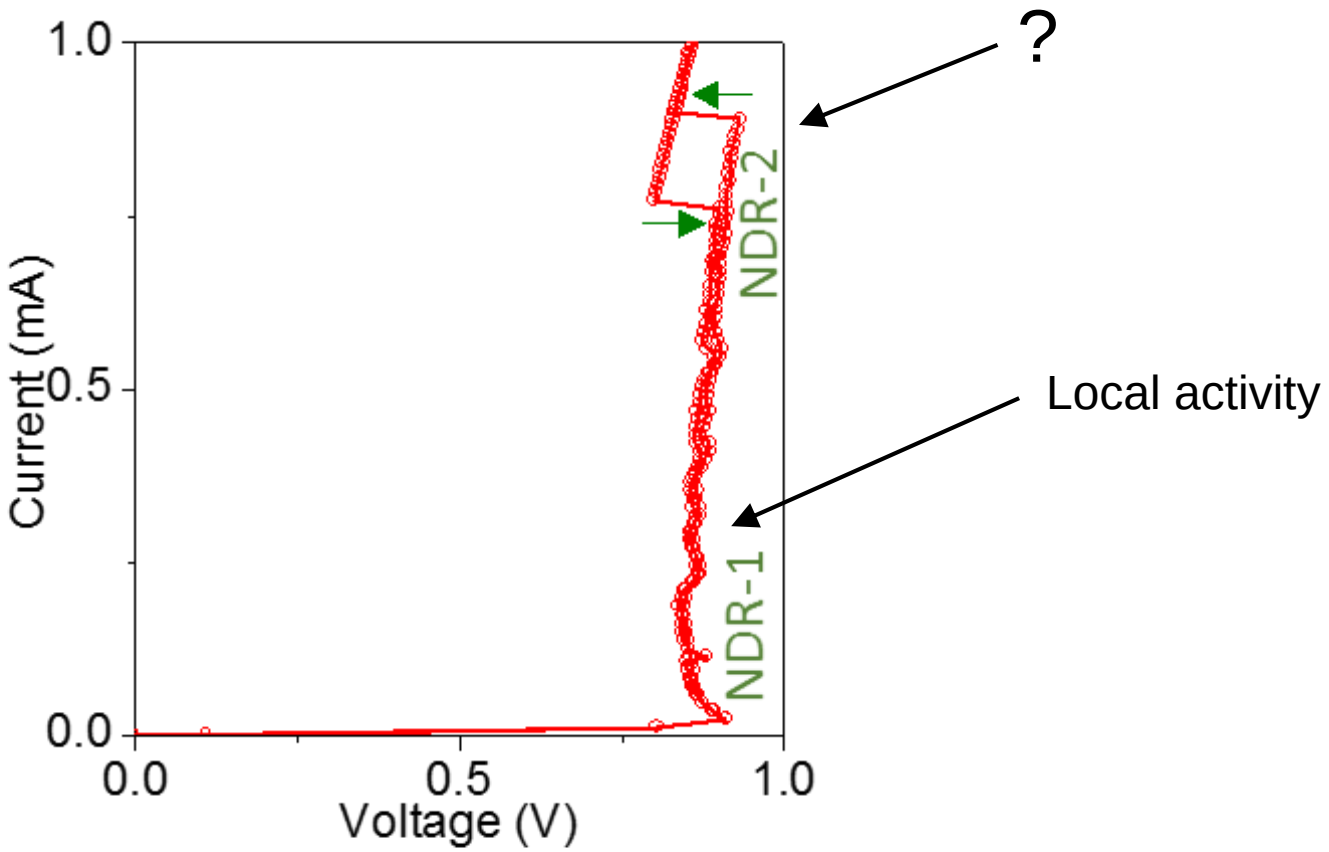
Nature Materials **12**, 114 (2013)

Devices and static behavior

NbO_2 devices

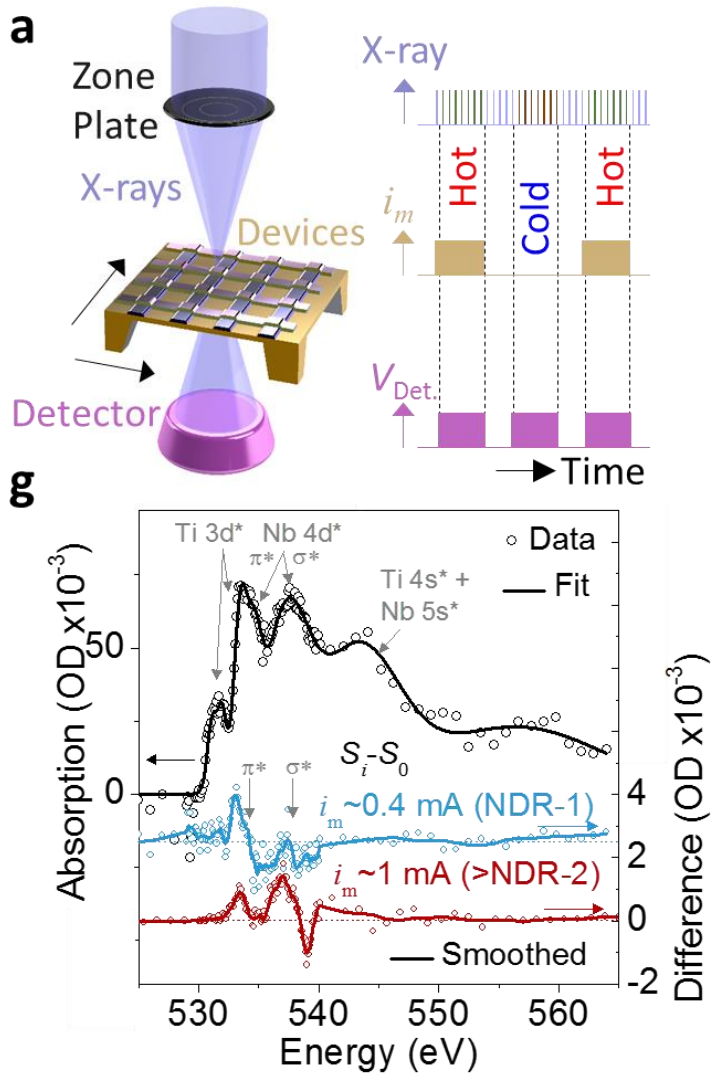


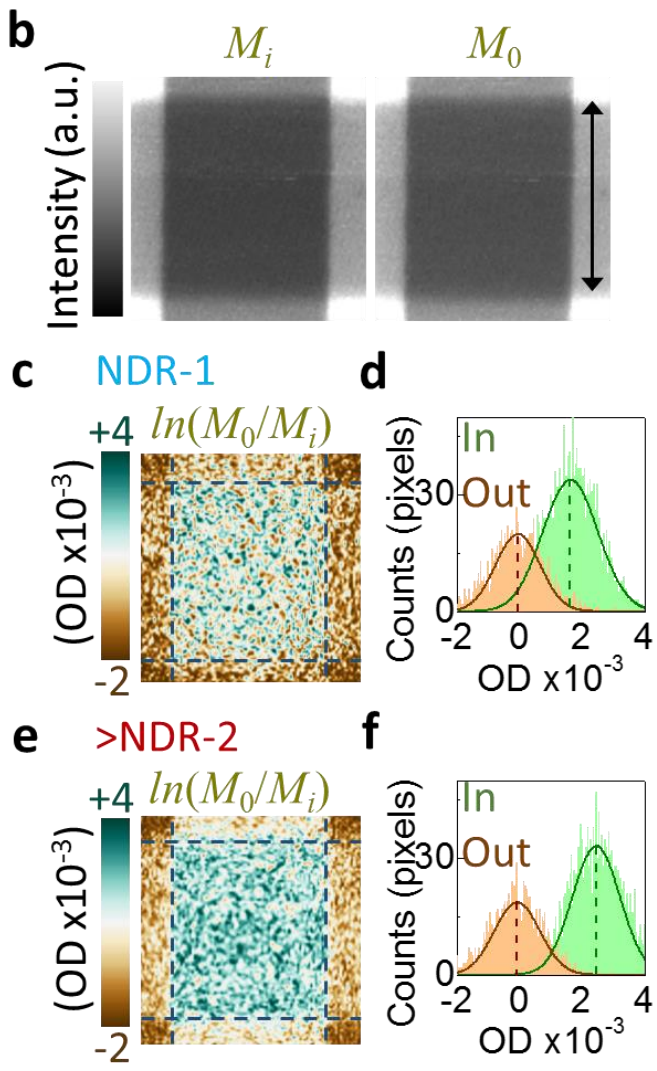
Diameter ~70 nm

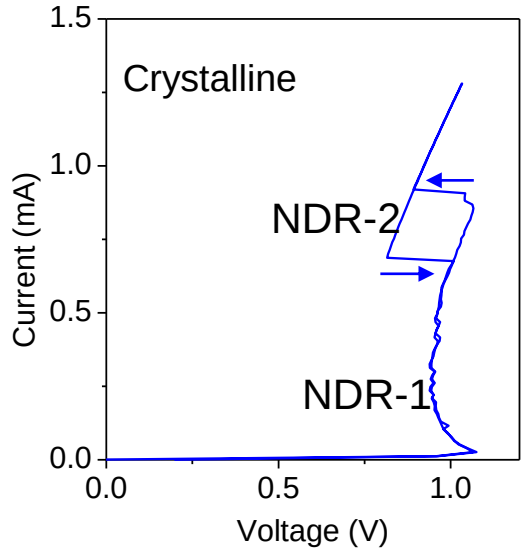


Kumar *et al., Nature Communications*, 8, 658 (2017)

The cause of NDR in NbO_2

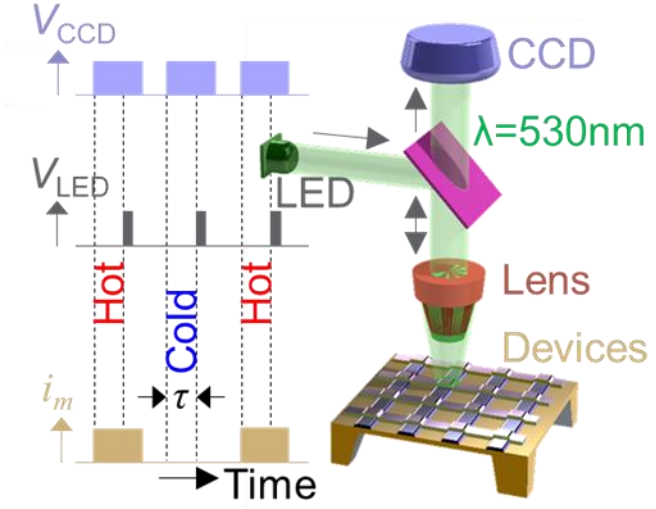


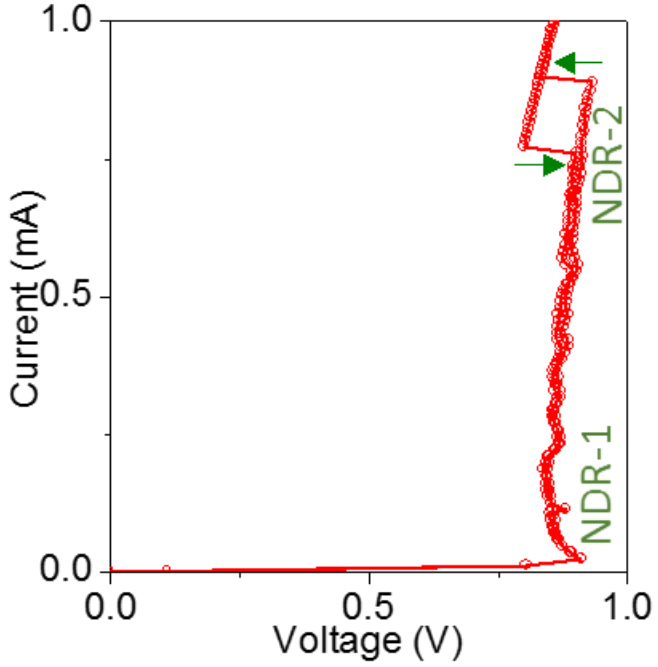


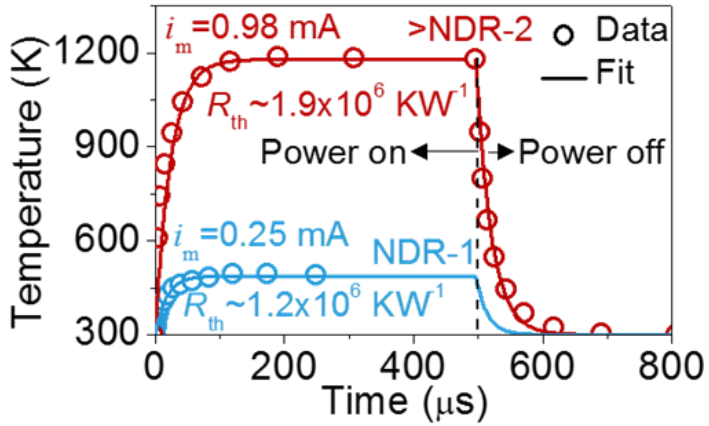


Kumar, Nature Comms. 8, 658 (2017)

Thermal anomaly in NbO_2







$$\sigma_{th}^{Metal} < \sigma_{th}^{Insulator}$$

Counter Wiedemann Franz postulate

Kumar *et al*., *Nature Communications* 8, 658 (2017)

Lee *et al., Science* 355, 371 (2017)

Model for the electrical behavior

Modified 3D Poole-Frenkel Equation

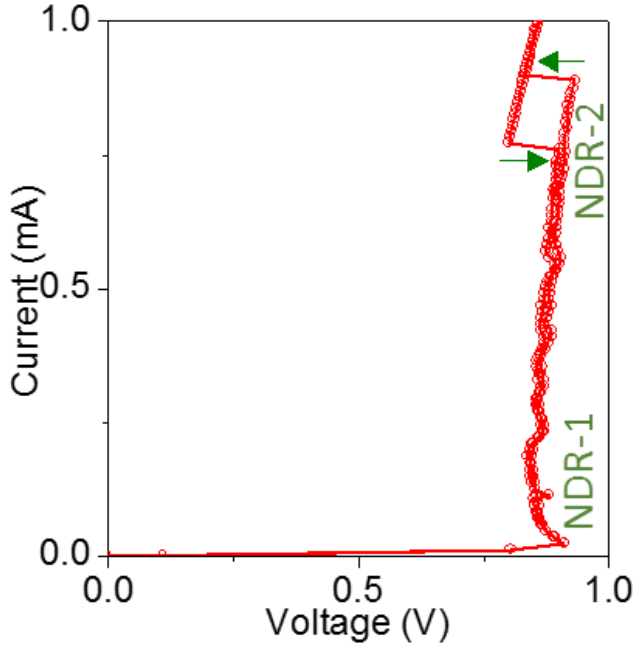
$$i_m = \left[\alpha e^{-\frac{E_a}{2k_B T}} \left(\frac{k_B T}{\beta\sqrt{v_m}}\right)^2 \left\{1 + \left(\frac{\beta\sqrt{v_m/t}}{k_B T} - 1\right) e^{\frac{\beta\sqrt{v_m/t}}{k_B T}}\right\} + \frac{\alpha e^{-\frac{E_a}{k_B T}}}{2t}\right] v_m \quad \ldots \text{Equation (1)}$$

Temperature dynamics

$$\frac{dT}{dt} = \frac{i_m v_m}{C_{th}} - \frac{T - T_{amb}}{C_{th} R_{th}(T)} \quad \ldots \text{Equation (2)}$$

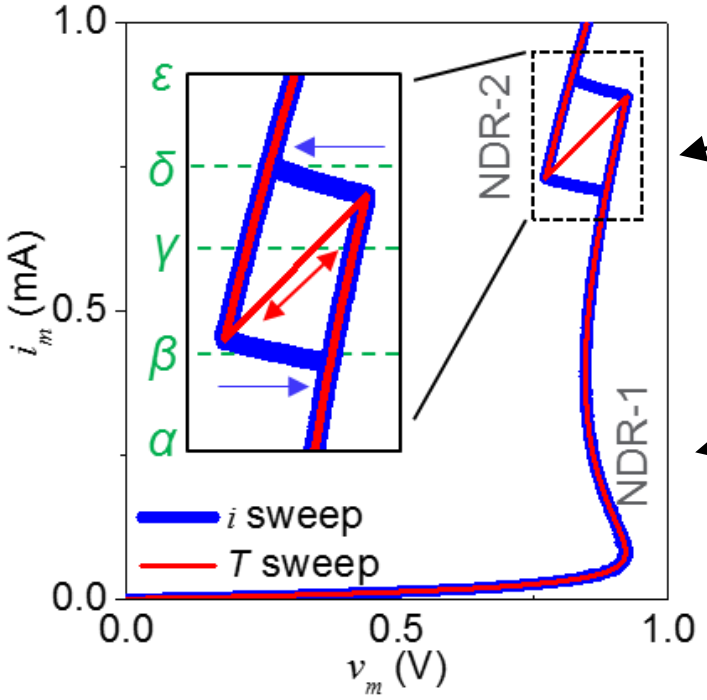
Mott transition in R_{th}

$$R_{th}(T) = \begin{cases} 1.4 \times 10^6 \ (\text{for } T \le T_C) \\ 2 \times 10^6 \ (\text{for } T > T_C) \end{cases} \quad \ldots \text{Equation (3)}$$



Kumar *et al.*, *Nature*, 548, 318 (2017)
Kumar *et al.*, *Nature Communications*, 8, 658 (2017)

Temperature-controlled instability (Chua Corsage)



Mott transition +
Local activity
(Chua Corsage)

NDR +
Local activity

New type of current-voltage behavior!

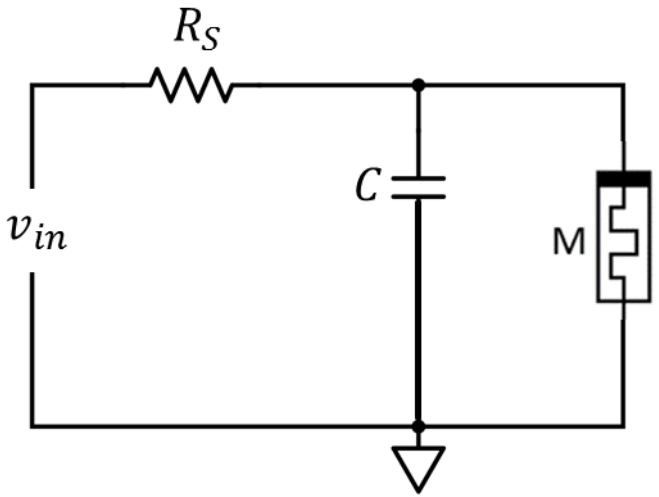
Theorem: Chua corsage leads to chaos in dynamics

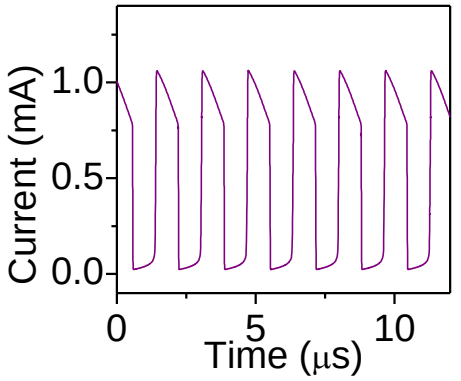
Kumar *et al*, *Nature Communications,* (2018)
Kumar *et al*., *Nature Communications*, 8, 658 (2017)
Mannan, *Int. J. Bifurcation Chaos,* 27, 1730011 (2017)

Modeling the dynamical behavior

$$\frac{dT}{dt} = \frac{v_m^2}{R(T)C_{th}} - \frac{T - T_{amb}}{R_{th}C_{th}} + \eta(T)$$

$$\frac{dv_m}{dt} = \frac{v_{in}}{R_S C} - v_m\left(\frac{1}{R(T)C} + \frac{1}{R_S C}\right)$$





What does it take to produce chaos?

1. Local activity
2. A minimum of three state variables; OR two state variables + a coupled dynamic signal

For chaos, where is the third state variable or oscillating force?

$$\eta(\mathrm{T}) = T\left(\frac{k_B}{C_{th}}\right)^{\frac{1}{2}} \frac{4\pi}{R_{th}C_{th}} \cos\left[\frac{2\pi t}{R_{th}C_{th}}\right]$$

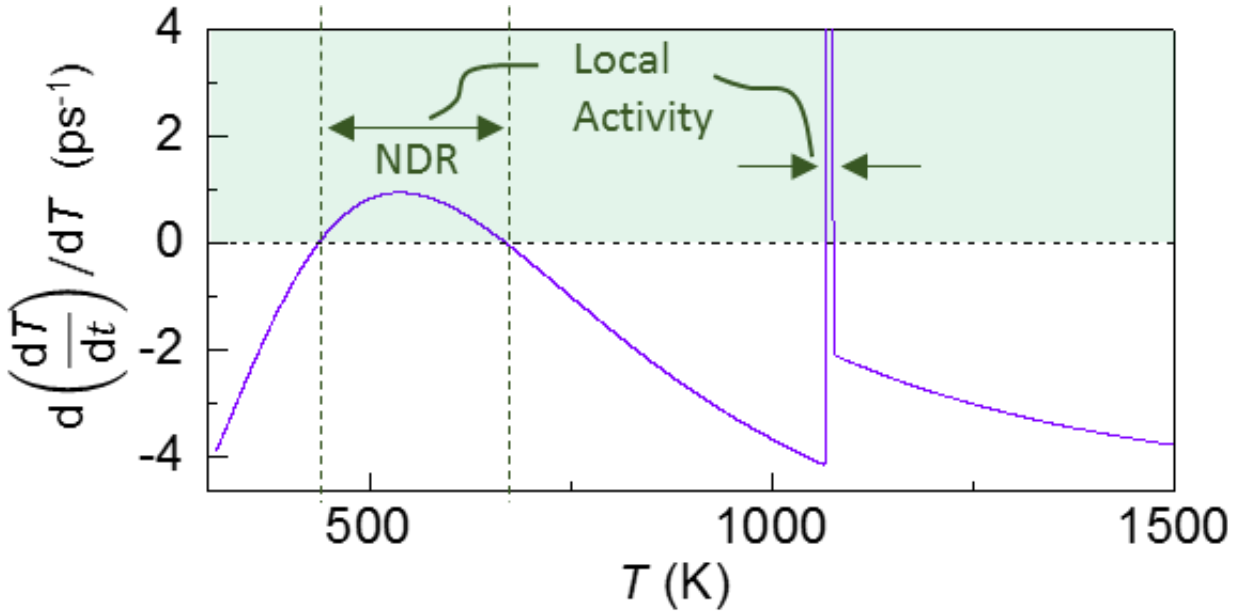
Kumar *et al.*, *Nature*, 548, 318 (2017)

Fluctuation-dissipation theorem of local activity

Local Activity:

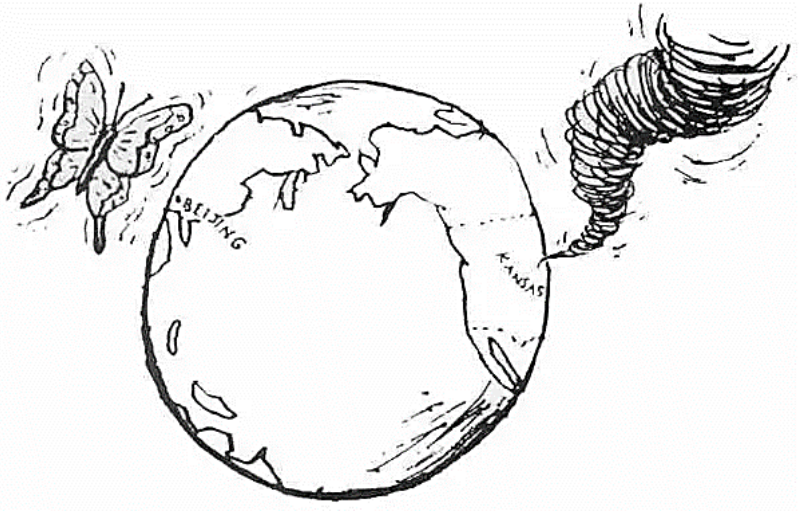
The biology-based theory that now allows us to predict dynamics of electronic devices using their static behavior.

Local activity leads to chaos



Kumar *et al.*, *Nature Communications,* (2018)

What is chaos?



– Extreme sensitivity to variations in initial conditions

– In between perfectly ordered and completely random behavior

– Very difficult to predict (but not impossible)

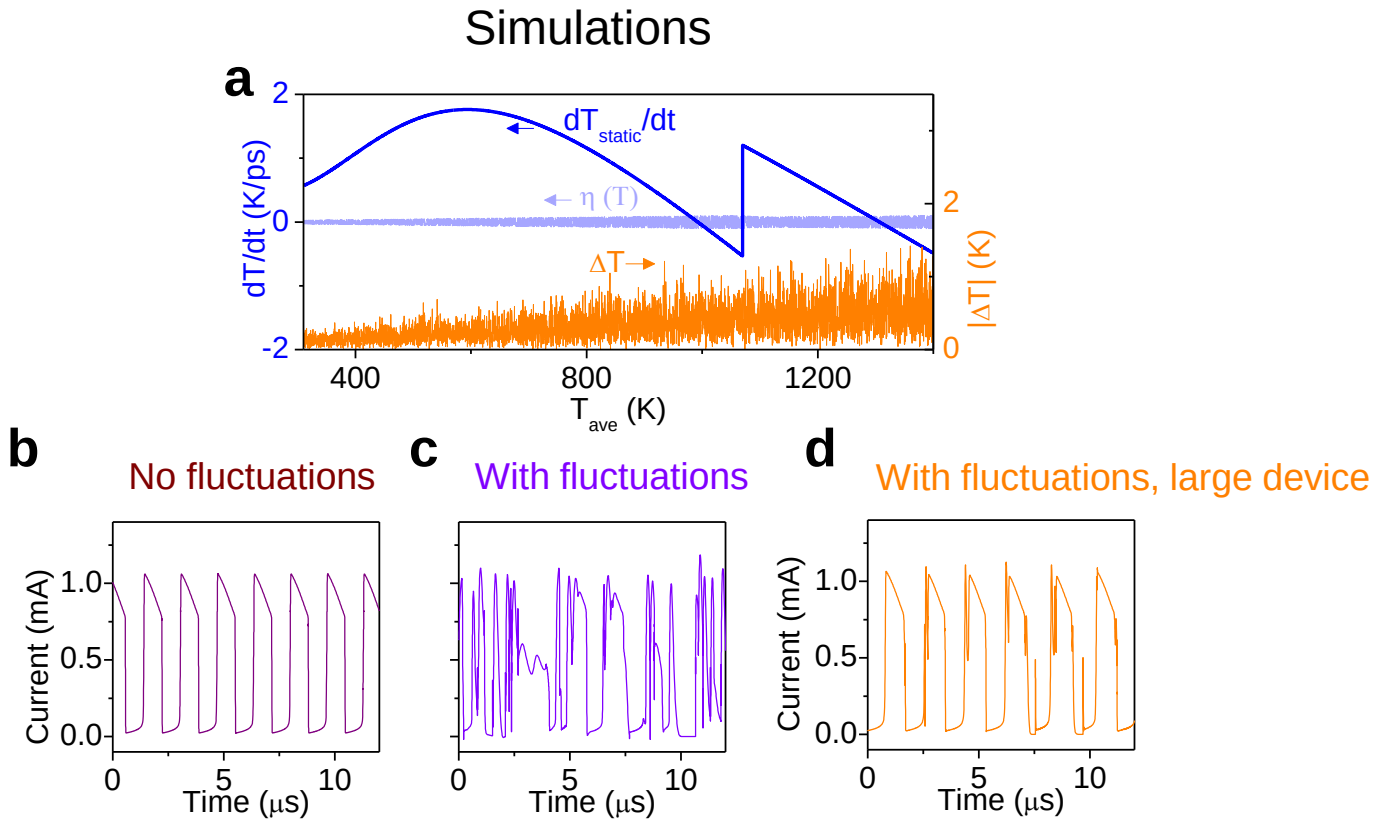
– Why is edge of chaos behavior favored in computing systems?
  – Units communicate/cooperate with to compute
  – But no global synchronization
  – Applies to neurons in a brain

Thermal fluctuations

$$\eta(T) = T\left(\frac{k_B}{C_{th}}\right)^{\frac{1}{2}} \frac{4\pi}{R_{th}C_{th}} \cos\left[\frac{2\pi t}{R_{th}C_{th}}\right]$$

Double pendulum with a large driving force!

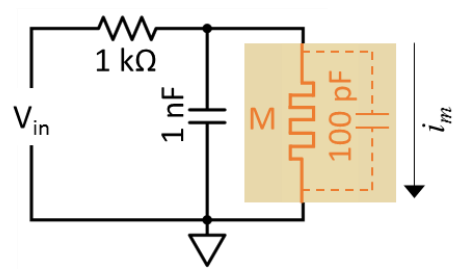
Takes hundreds of transistors to emulate!



Kumar *et al.*, *Nature*, 548, 318 (2017)

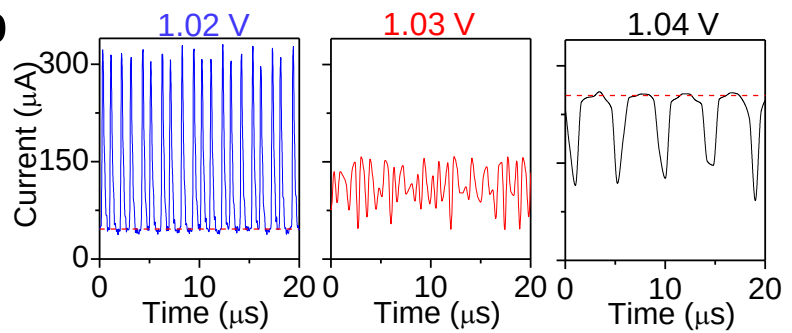
Chaotic oscillations

a

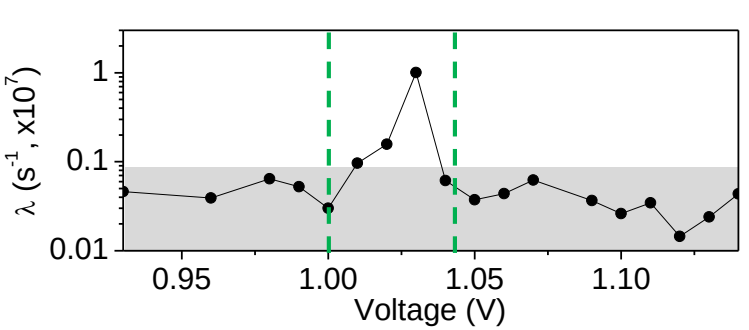


Controlled chaos in a single electronic device!

b



c



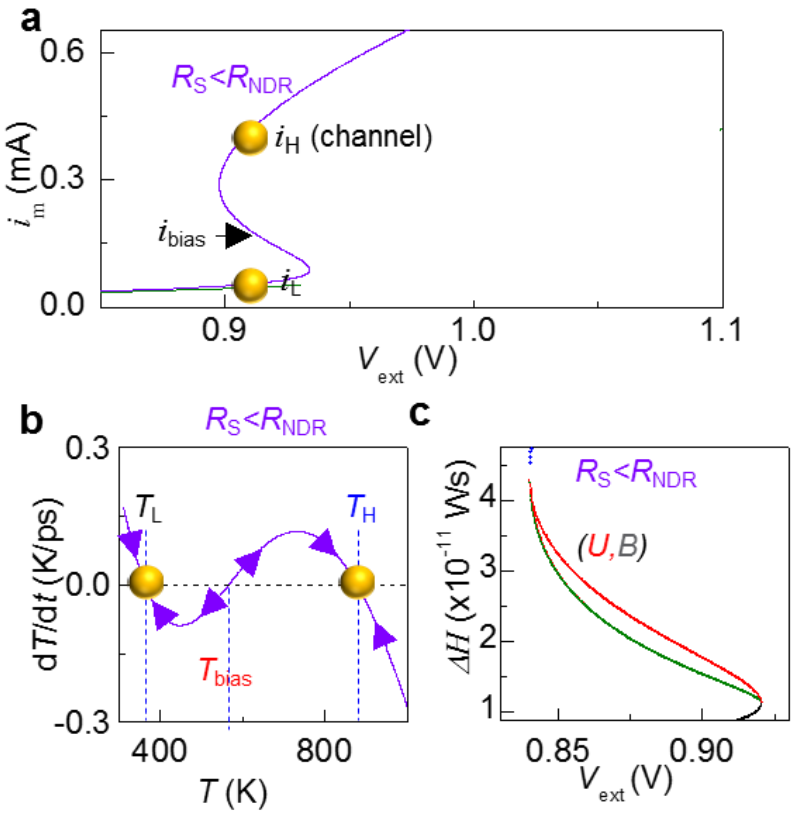
$$\lambda = \lim_{t\to\infty} \lim_{\delta \mathbf{Z}_0 \to 0} \frac{1}{t} \ln \frac{|\delta \mathbf{Z}(t)|}{|\delta \mathbf{Z}_0|}$$

Are temperature fluctuations the only route to chaos?

Kumar *et al.*, *Nature*, 548, 318 (2017)

Nonlinearity and bifurcations

Bifurcations due to instabilities



$$j_U.A = j_L.(A - x.A) + j_H.(x.A)$$

A different route to chaos in nonlinear devices.

The missing pieces of device models:

1. Minimization of internal energy or ΔH

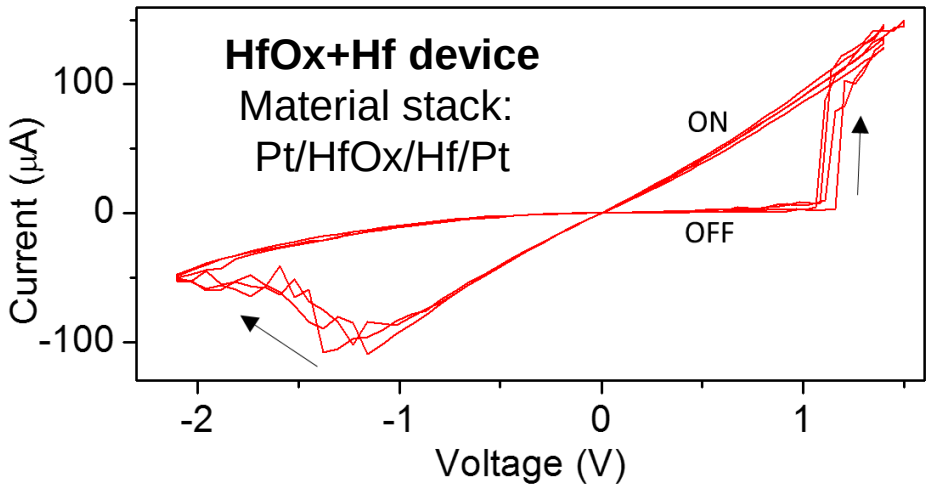
2. Spontaneous symmetry breaking during nonlinearity

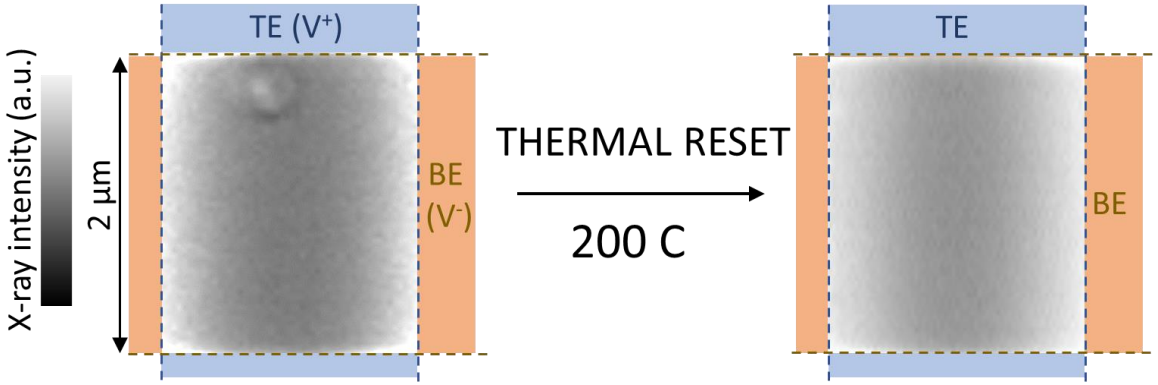
3. Amplification and coupling of ambient thermal fluctuations in nanoscale devices

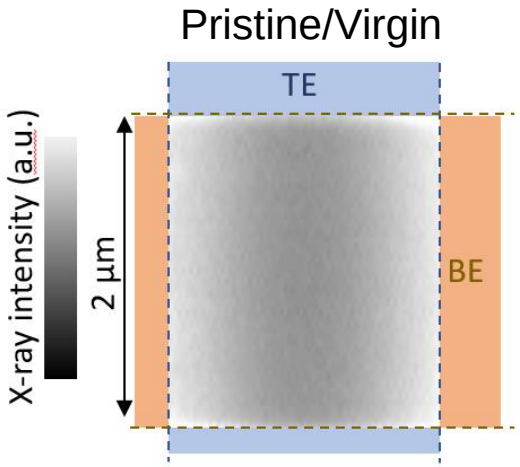
Kumar *et al.*, *Nature Communications,* (2018)
Kumar *et al., Advanced Materials,* 28, 2772 (2016)
Ridley, *Proc. Phys. Soc.,* 82, 954 (1963)

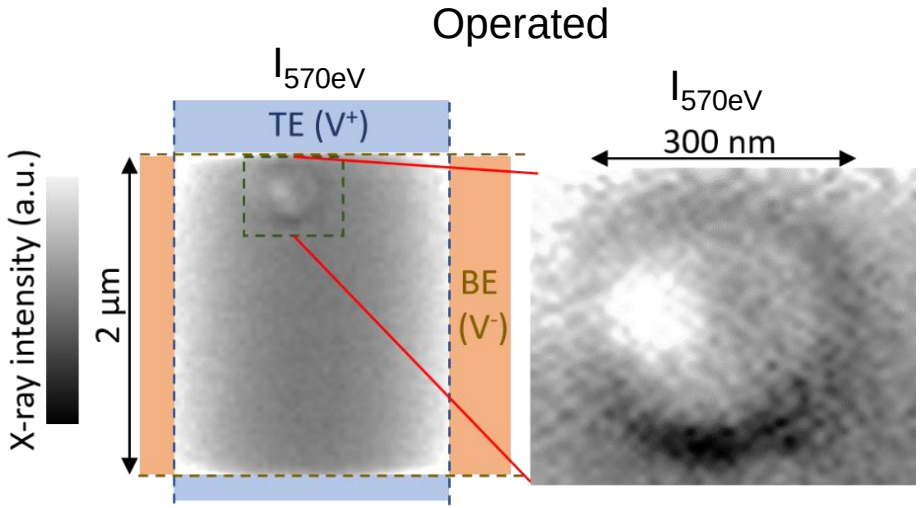
Synaptic devices

Ion migration in Hf and Ta oxides



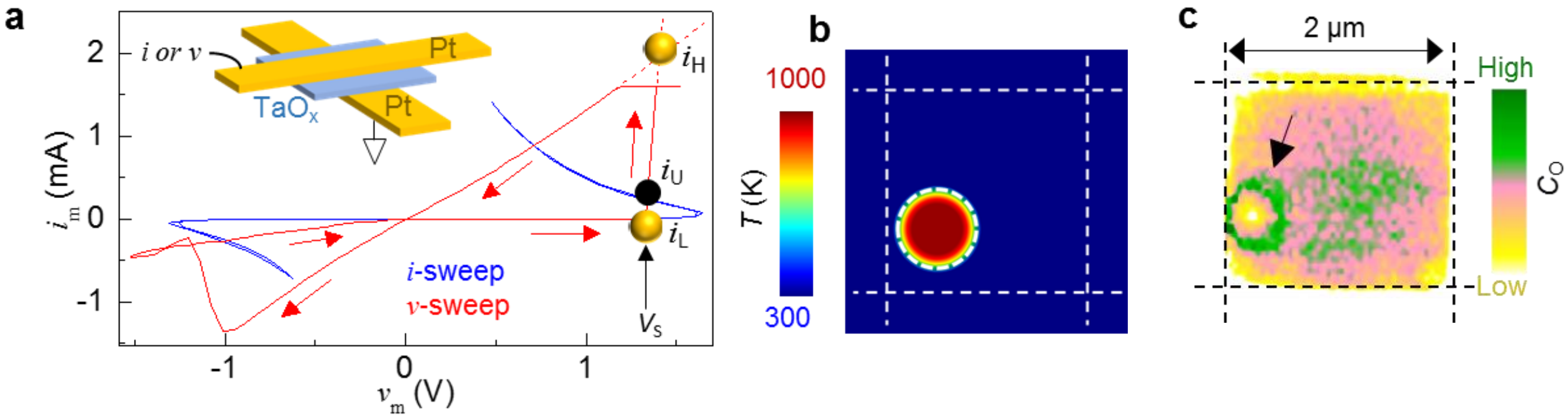






Non-volatile switching and neuronic devices

The origin of nonvolatile storage in ReRAM



The origin of non-volatile storage

Fundamental requirement for S-NDR → non-volatility

$$i = Gv$$
$$G = \psi T^{\xi}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{iv}{C_{th}} - \frac{T - T_{\mathrm{amb}}}{C_{th}R_{\mathrm{th}}}$$

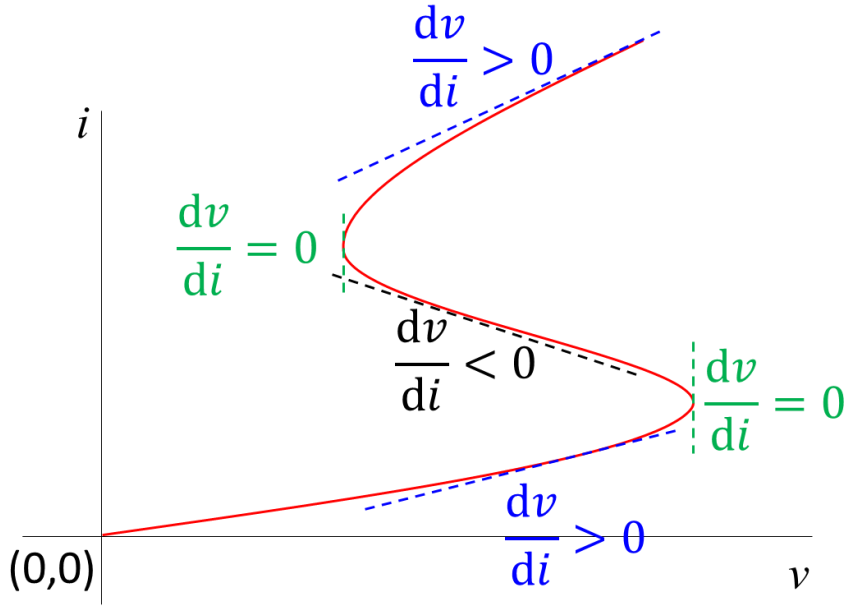
Steady state: $\frac{\mathrm{d}T}{\mathrm{d}t} = 0$

$$T = T_{amb} + R_{th}v^2\psi T^{\xi}$$

$$\frac{\mathrm{d}T}{\mathrm{d}i} = R_{\mathrm{th}}v^2\psi\xi T^{\xi-1}\frac{\mathrm{d}T}{\mathrm{d}i} + 2R_{\mathrm{th}}v\psi T^{\xi}\frac{\mathrm{d}v}{\mathrm{d}i}$$

$$R_{\mathrm{th}}v^2\psi\xi T^{\xi-1} = 1$$

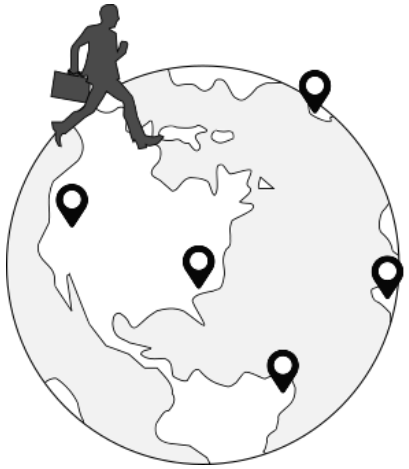
$$T = T_{\mathrm{NDR}} = \frac{\xi\, T_{\mathrm{amb}}}{\xi - 1} \qquad \rightarrow \xi > 1$$



Estimate only this material parameter and one can predict most of the nonlinear and information storage behaviors.

A brain-like computer using synaptic and neuronic oxide memristors

The traveling salesman problem



Objective:

Find the shortest path

Constraints:

1. Visit every city once
2. Visit every city no more than once
3. Do not visit more than one city in a given stop

“Hard” problems

It is non-deterministic polynomial (NP) complete.

Other NP-complete/hard problems:

Gene sequencing/traveling salesman

Sudoku

Vehicle routing

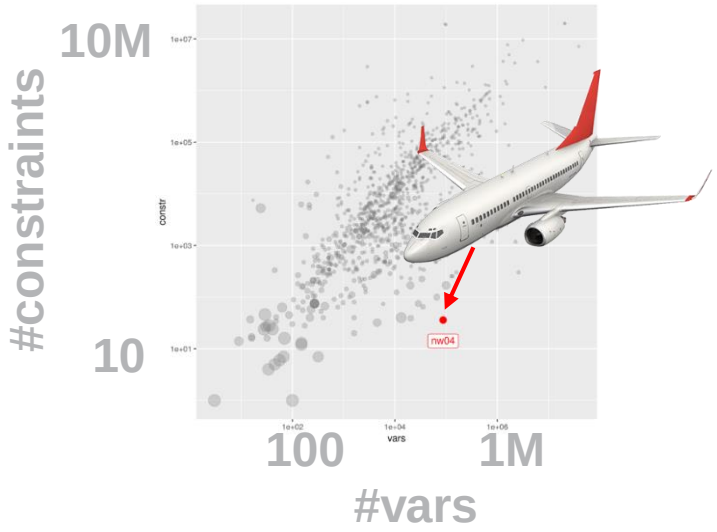
Open shop scheduling

Bandwidth allocation

NFL scheduling

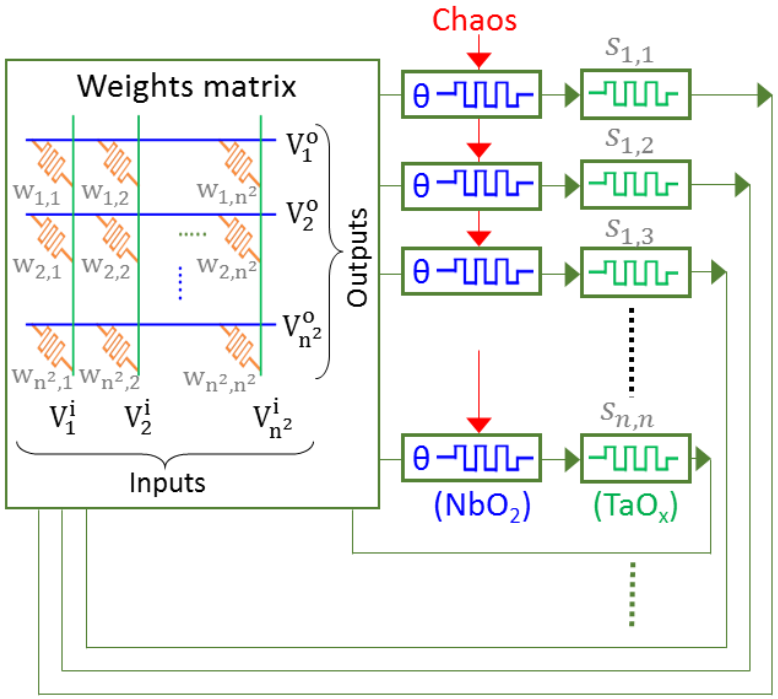
256 games
20k variables
50k constraints

Takes ~3 months on a 1000-core system to solve!



Hopfield network

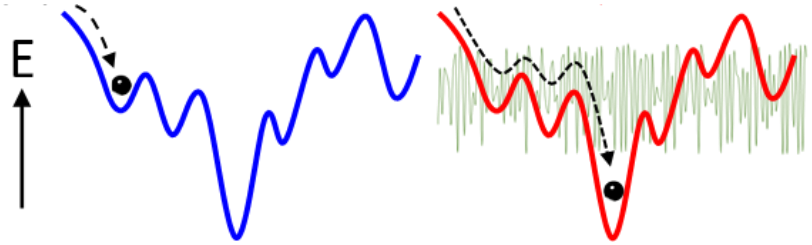
Recursive feedback neural network



Synapses + neurons

Energy function:

$$E = -\frac{1}{2}\sum_i \sum_j s_{i,j} \sum_k \sum_l s_{k,l} w_{(i,j),(k,l)} + \sum_i \sum_j s_{i,j}\theta$$

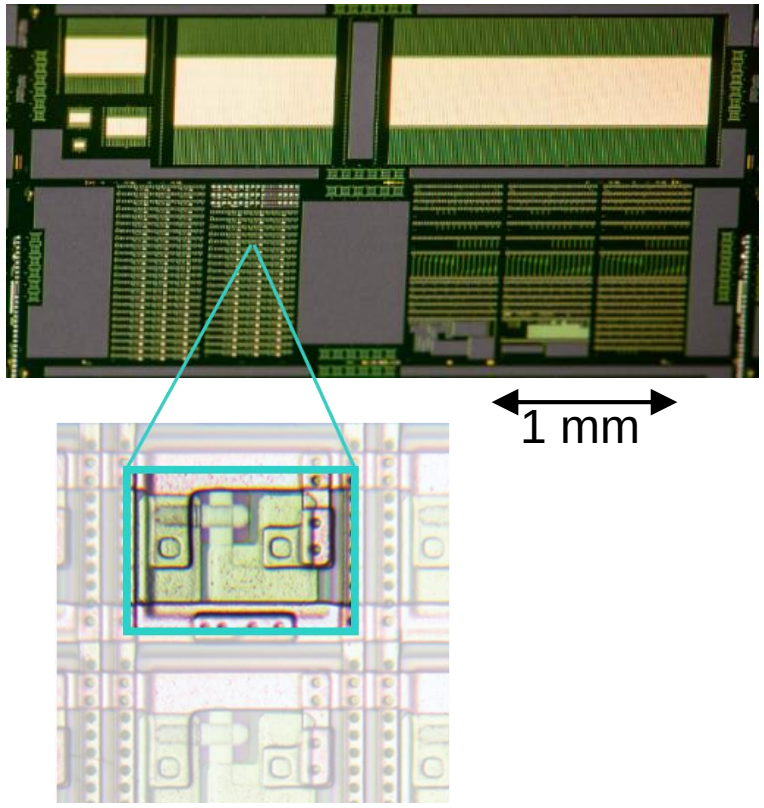


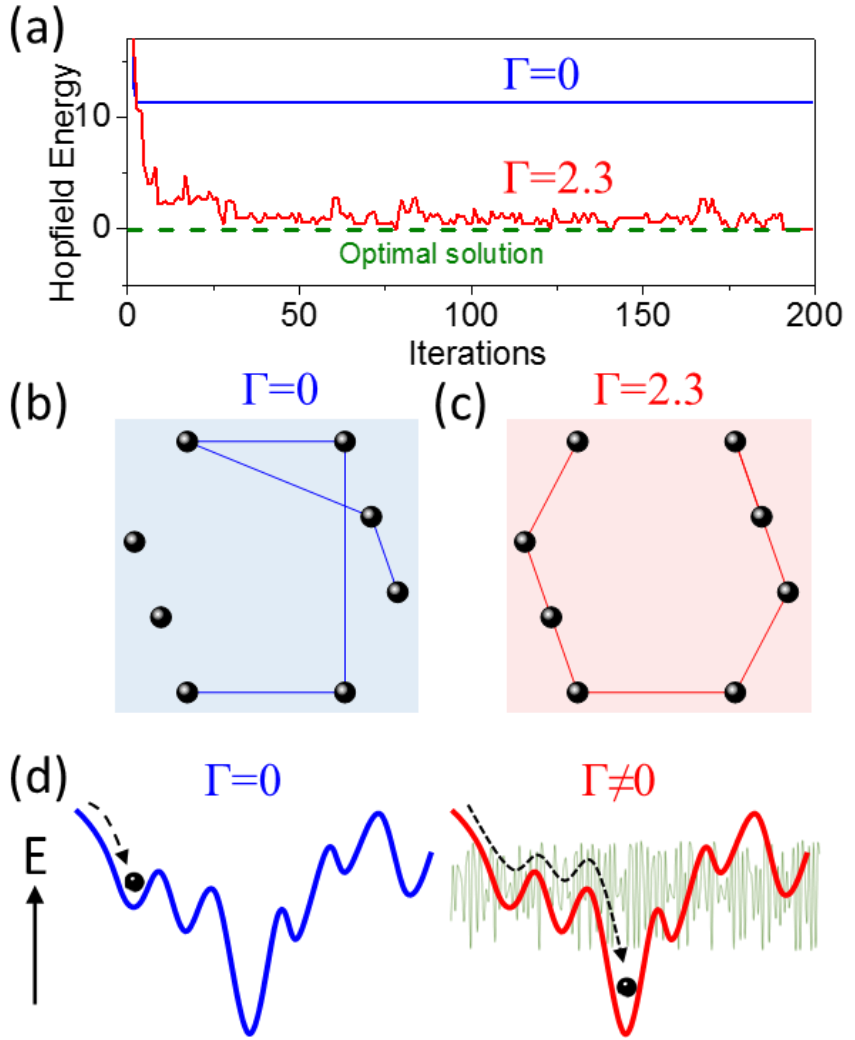
An incredibly compact transistorless all-analogue implementation of a Hopfield network

Kumar *et al.*, *Nature*, 548, 318 (2017)
US Patent App. 15/141,410

Example of one solution with chaos

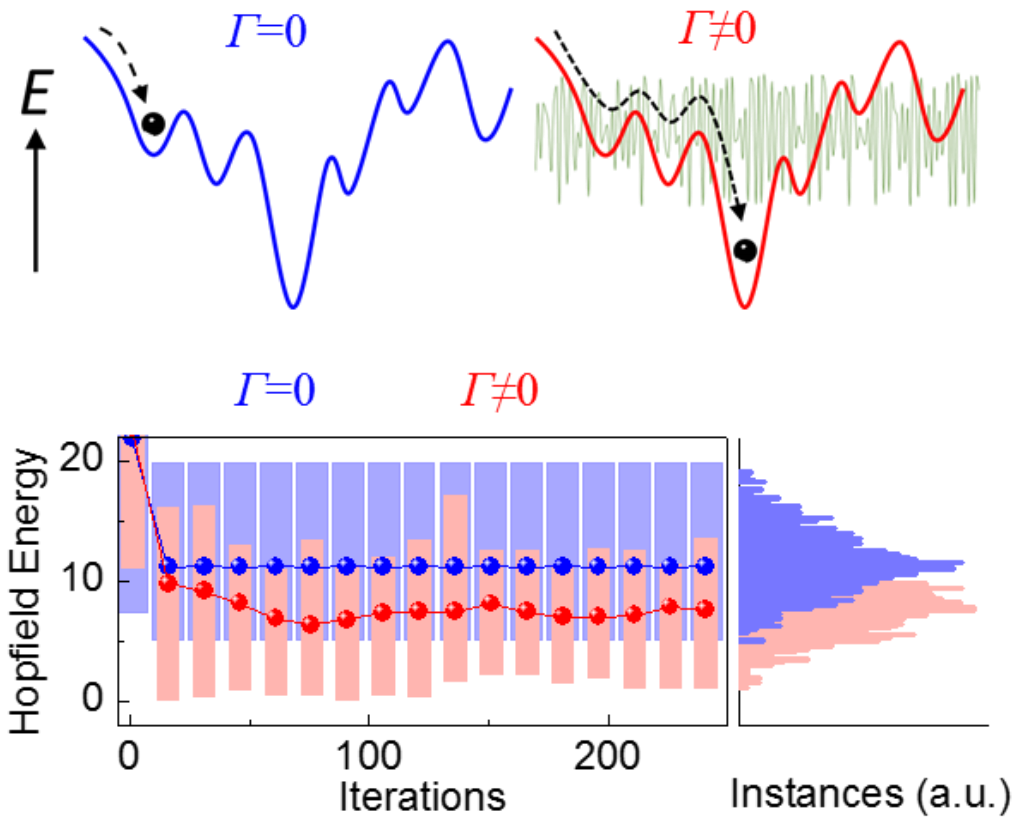
The traveling salesman problem





Kumar *et al.*, *Nature*, 548, 318 (2017)
US Patent App. 15/141,410 (2017)

Statistics of many solutions with and without chaos



We only want *better* solutions quickly.

High precision → prohibitive slow downs

Literally annealing the system into its solution!

Room temperature analogue of quantum adiabatic annealing

Kumar *et al.*, *Nature*, 548, 318 (2017)

Mem-HNN outperforms digital hardware and quantum annealing

Benchmarking on a suite of 60 node max-cut problems

	Analog	**Digital**	**Quantum**
	Mem-HNN	GPU	D-wave 2000Q
Clock frequency	1 GHz	1.5 GHz	
Time-to-solution	0.3 µs	10 µs	10^4 s
Power	792 mW	<250 W	25,000 W
Solutions/s/Watts	4.6 x 10^6	>400	4 x 10^{-9}

10,000x better Solutions/sec/Watt !!

arXiv:1903.11194: ***Kumar et. al.*** **Harnessing Intrinsic Noise in Memristor Hopfield Neural Networks for Combinatorial Optimization**

Major Caveat: Circuit level analysis only – no scalable architecture analyzed yet

Reason for optimism: NP-hard problems are *compute-intensive*, not *data-intensive*

Summary

- Analog, brain-like computing can outperform any digital alternative in solving certain classes of intractable problems

- But we need new device physics to do that
  - Controlled chaos
  - Nano-size coupling with ambient thermal fluctuations
  - Enthalpy minimization and symmetry breaking
  - Super-linearity and non-volatility (the connection between neurons and synapses)