Improvements in the CE+EPSO method for the transmission expansion planning problem

Leonel Carvalho, Vladimiro Miranda, Armando Leite da Silva, Carolina Marcelino, Elizabeth Wanner
leonel.m.carvalho@inesctec.pt

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CE+EPSO Method

• Combination of two heuristic-based optimization methods
  – Cross-Entropy (CE) Method for space exploration
  – Evolutionary Particle Swarm Optimization (EPSO)
    for space exploitation
    • A special solution generation scheme was used in parallel with EPSO

• Switching between the two methods is done when the best fitness stops improving above a certain threshold over a certain number of consecutive iterations
  – Experiments were carried out to find the optimal number of fitness evaluations to switch between the two methods
  – Binomial variables were used, so no possibility of using the variance of the sampling distributions...
CE Method

- **Heuristic** method proposed for combinatorial and continuous non-linear optimization based on the Kullback–Leibler divergence

- Based on the concept that **optimization problems can be defined as a rare-event estimation problem**
  - The probability of locating the optimal solution using random sampling is a rare-event

- The **CE Method** is an **adaptive iterative sampling process**
  - It starts by defining a sampling distribution for the variables (Bernoulli, Binomial, Gaussian, etc.) followed by an iterated adjustment of the distribution parameters (e.g. mean, standard deviation) according to the performance of an elite subset of the samples
CE Method

Select the success probability $p_{0,j}$ and the number of trials $N_j$ for each variable $j$, the number of samples per iteration $N_{\text{samples}}$, the rarity parameter $\rho$, the smoothing parameter $\alpha$, $k := 0$

Do

$k := k + 1$

Generate a sample of $X_1, ..., X_N$ from the sampling distribution $B(N_j, p_{k-1,j})$

Compute $S(X_1), ..., S(X_N)$ and order the samples from the best to the worst performing ones, i.e. $S(X_1) < S(X_2) < ... < S(X_N)$

Compute $\gamma_k$ as the $\rho$th quantile of the performance values and select $N_{\text{elite}} = \rho N_{\text{samples}}$; let $\Psi$ be the subset from the ordered set of samples that contains all the $N_{\text{elite}}$ samples, i.e., the samples $S(X) < \gamma_k$

For $j = 1$ to $n$

$$p_{k,j} := \frac{1}{N_j} \sum_{i \in \Psi} \frac{X_{i,j}}{N_{\text{elite}}}$$

End For

Apply smoothing

$$p_{k,j} := \alpha p_{k,j} + (1-\alpha)p_{k-1,j}$$

Until $k < k^{\text{MAX}}$
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$

![Graph showing CE Method Iteration 1 (200 Samples)](image)

# Elite Samples = 20
Example for the Garver - Case 1

• **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$

![Graph showing CE Method Iteration 3 (200 Samples)]

- Elite Samples
- Total Samples

# Elite Samples = 20
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$

![CE Method Iteration 5 (200 Samples)](image)

# Elite Samples = 20
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$

![CE Method Iteration 9 (200 Samples)](image)

# Elite Samples = 20
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$

![Graph showing CE Method Iteration 11 (200 Samples) with Elite Samples and Total Samples]
Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial $p = 0.5$
EPSO

- **Evolutionary Particle Swarm Optimization (EPSO)** is a hybrid between **Evolutionary Strategies** and **Particle Swarm Optimization**
  - **Replication**: each individual is replicated $r$ times
  - **Mutation**: the $r$ clones have their weights $w$ mutated
  - **Recombination**: the $r+1$ individuals generate one offspring
  - **Evaluation**: each offspring has its fitness evaluated
  - **Selection**: the best particle out of the $r+1$ survives to be part of a new generation
Recombination in EPSO

• Movement Rule

\[ \mathbf{X}^{\text{new}} = \mathbf{X} + \mathbf{V}^{\text{new}} \]

\[ \mathbf{V}^{\text{new}} = w_I^* \mathbf{V} + w_M^*(\mathbf{X}_M - \mathbf{X}) + w_C^* \mathbf{P}(\mathbf{X}_G^* - \mathbf{X}) \]

– **Inertia**: movement in the same direction

– **Memory**: attraction towards the individual best solution

– **Cooperation**: attraction to a region near the global best position
Recombination in EPSO

• Weights are mutated and selected

\[ w^* = w[1 + \sigma N(0,1)] \]

• The individuals are attracted to a region near the best solution found

\[ X_G^* = X_G [1 + w_{GB}^* N(0,1)] \]

• Matrix \( P \) acts as a communication barrier

\[
V_{\text{new}} = w_I^* V + w_M^* (X_M - X) + w_C^* P (X_G^* - X)
\]

\[
P = \begin{bmatrix}
1 & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

If \( i \neq j \) then \( P_{ij} = 0 \)

Elseif \( U(0,1) > p \) then \( P_{ij} = 0 \)

Else \( P_{ij} = 1 \)
Special Solution Generation Scheme for TEP

- Given the nature of the TEP problem, as special solution generation scheme was used in parallel with the recombination operator of EPSO
  - 50% chance of using this scheme or EPSO (no particular reason for choosing this value...)

\[ i := 0 \]

Do

\[ i := i + 1 \]

Create a New Solution by copying the Current Solution
Randomly select one variable of the New Solution and subtract its value by 1
Enforce space limits in the New Solution

Until New Solution = Current Solution and \( i \leq \# \) variables
Special Solution Generation Scheme for TEP

- **Mean Best Fitness (31 trials)**
EPSO Parameter Tuning

• To improve performance, EPSO parameters were tuned using an iterative optimization algorithm based on a $2^2$ factorial design.

• Parameters tuned
  – Mutation Rate $\tau$ - [0.2, 0.8]
  – Communication Probability $P$ - [0.2, 0.8]

• Iterative methodology
  – 4 x 10 trial runs with combinations of limit values of the intervals defined for $\tau$ and $P$
  – Two-way ANOVA tells the parameter that most impacts the response
  – Main effect tells which limit of the interval should be reduced
## Example of EPSO Parameter Tuning

### Two-way ANOVA results

<table>
<thead>
<tr>
<th>Factor</th>
<th>F-test Statistic</th>
<th>P-value</th>
<th>Main Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Iteration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>3.474</td>
<td>0.064</td>
<td>-350.73</td>
</tr>
<tr>
<td>$P$</td>
<td>6.675</td>
<td>0.011</td>
<td>486.14</td>
</tr>
<tr>
<td>$\tau \times P$</td>
<td>0.225</td>
<td>0.636</td>
<td>89.17</td>
</tr>
<tr>
<td>2nd Iteration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>5.469</td>
<td>0.021</td>
<td>-548.35</td>
</tr>
<tr>
<td>$P$</td>
<td>6.022</td>
<td>0.015</td>
<td>575.42</td>
</tr>
<tr>
<td>$\tau \times P$</td>
<td>1.496</td>
<td>0.223</td>
<td>286.79</td>
</tr>
<tr>
<td>3rd Iteration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>5.508</td>
<td>0.020</td>
<td>-448.7</td>
</tr>
<tr>
<td>$P$</td>
<td>1.448</td>
<td>0.231</td>
<td>230.01</td>
</tr>
<tr>
<td>$\tau \times P$</td>
<td>0.958</td>
<td>0.329</td>
<td>187.14</td>
</tr>
<tr>
<td>4th Iteration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>2.849</td>
<td>0.093</td>
<td>-306.78</td>
</tr>
<tr>
<td>$P$</td>
<td>2.497</td>
<td>0.116</td>
<td>287.21</td>
</tr>
<tr>
<td>$\tau \times P$</td>
<td>1.807</td>
<td>0.181</td>
<td>244.34</td>
</tr>
</tbody>
</table>

**Intervals for $\tau$ and $P$**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$ (low)</td>
<td>Start 1st 2nd 3rd</td>
</tr>
<tr>
<td>$\tau$ (high)</td>
<td>0.2 0.2 0.2 0.5</td>
</tr>
<tr>
<td>$P$ (low)</td>
<td>0.2 0.2 0.2 0.2</td>
</tr>
<tr>
<td>$P$ (high)</td>
<td>0.8 0.5 0.35 0.35</td>
</tr>
</tbody>
</table>

**Box-plots for the final range of $\tau$ and $P$**

Greater than the threshold level of 0.05

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**Example of EPSO Parameter Tuning**

*Note: The image includes box-plots and tables that illustrate the tuning process.*
Results 31 Trials

• Garver – Case 1
Results 31 Trials

- Garver – Case 2
Results 31 Trials

- IEEE 24 bus
Results 31 Trials

• IEEE 118 bus
# Results 31 Trials

<table>
<thead>
<tr>
<th>Fitness</th>
<th>Garver Case 1</th>
<th>Garver Case 2</th>
<th>IEEE 14 bus</th>
<th>IEEE 118 bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>110</td>
<td>268</td>
<td>70</td>
<td>396</td>
</tr>
<tr>
<td>Worst</td>
<td>110</td>
<td>322</td>
<td>70</td>
<td>396</td>
</tr>
<tr>
<td>Mean</td>
<td>110</td>
<td>269.74</td>
<td>70</td>
<td>396</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0</td>
<td>9.70</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Final Remarks

• **Combination of methods for different stages** of the search process can **improve accuracy** and **robustness**

• **Parameter tuning** can **reduce the information** required from the **user**
  – The procedure described must be done offline

• **Tailor-made recombination rules** can lead to **better performances**
  – Requires specific information about the problem being solved
Thank you for your attention!

Leonel Carvalho
leonel.m.carvalho@inesctec.pt