Improvements in the CE+EPSO method for the transmission expansion planning problem

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CE+EPSO Method

- **Combination of two heuristic-based optimization methods**
  - **Cross-Entropy (CE) Method** for space exploration
  - **Evolutionary Particle Swarm Optimization (EPSO)** for space exploitation
    - A special solution generation scheme was used in parallel with EPSO

- **Switching between the two methods is done when the best fitness stops improving above a certain threshold over a certain number of consecutive iterations**
  - Experiments were carried out to find the optimal number of fitness evaluations to switch between the two methods
  - Binomial variables were used, so no possibility of using the variance of the sampling distributions...

CE Method

- **Heuristic** method proposed for combinatorial and continuous non-linear optimization based on the **Kullback–Leibler divergence**

- Based on the concept that **optimization problems can be defined as a rare-event estimation problem**
  - The probability of locating the optimal solution using random sampling is a rare-event

- The **CE Method** is an **adaptive iterative sampling process**
  - It starts by defining a sampling distribution for the variables (Bernoulli, Binomial, Gaussian, etc.) followed by an iterated adjustment of the distribution parameters (e.g. mean, standard deviation) according to the performance of an elite subset of the samples

CE Method

Select the success probability $p_{0,j}$ and the number of trials N_j for each variable j, the number of samples per iteration $N^{samples}$, the rarity parameter ρ, the smoothing parameter α, $k := 0$

Do

$k := k + 1$

Generate a sample of $\mathbf{X}_1, \ldots, \mathbf{X}_N$ from the sampling distribution $B(N_j, p_{k-1,j})$

Compute $S(\mathbf{X}_1), \ldots, S(\mathbf{X}_N)$ and order the samples from the best to the worst performing ones, i.e. $S(\mathbf{X}_1) < S(\mathbf{X}_2) < \ldots < S(\mathbf{X}_N)$

Compute γ_k as the ρth quantile of the performance values and select $N^{\text{elite}} = \rho N^{samples}$; let ψ be the subset from the ordered set of samples that contains all the N^{elite} samples, i.e., the samples $S(\mathbf{X}) < \gamma_k$

For $j = 1$ to n

$$p_{k,j} := \frac{1}{N_j} \sum_{i \in \Psi} \frac{X_{i,j}}{N^{elite}}$$

End For

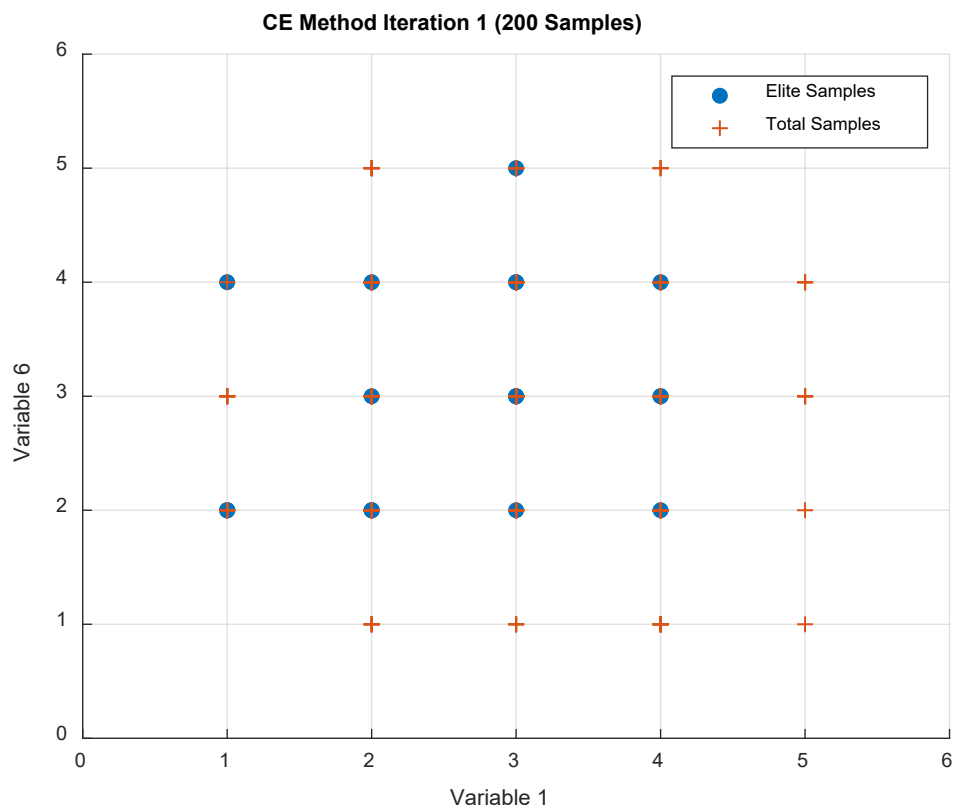
Apply smoothing

$$p_{k,j} := \alpha \, p_{k,j} + (1-\alpha) p_{k-1,j}$$

Until $k < k^{MAX}$

Example for the Garver - Case 1

- **Variables 1 & 6**: Binomial distribution with initial p = 0.5



Elite Samples = 20

Example for the Garver - Case 1

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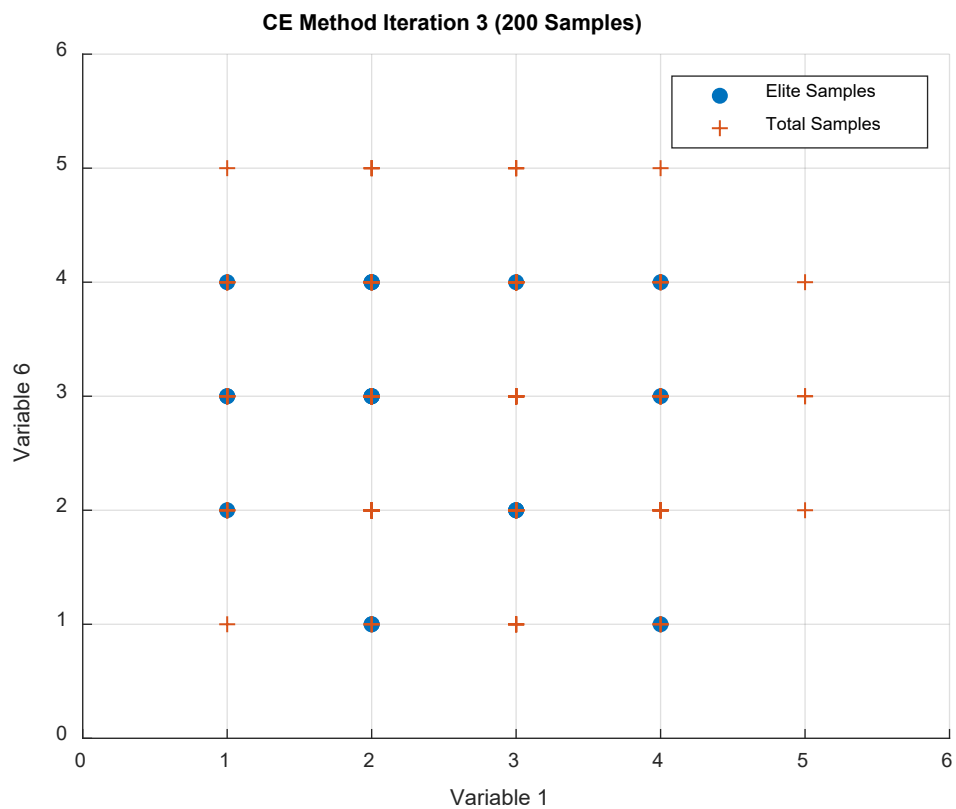

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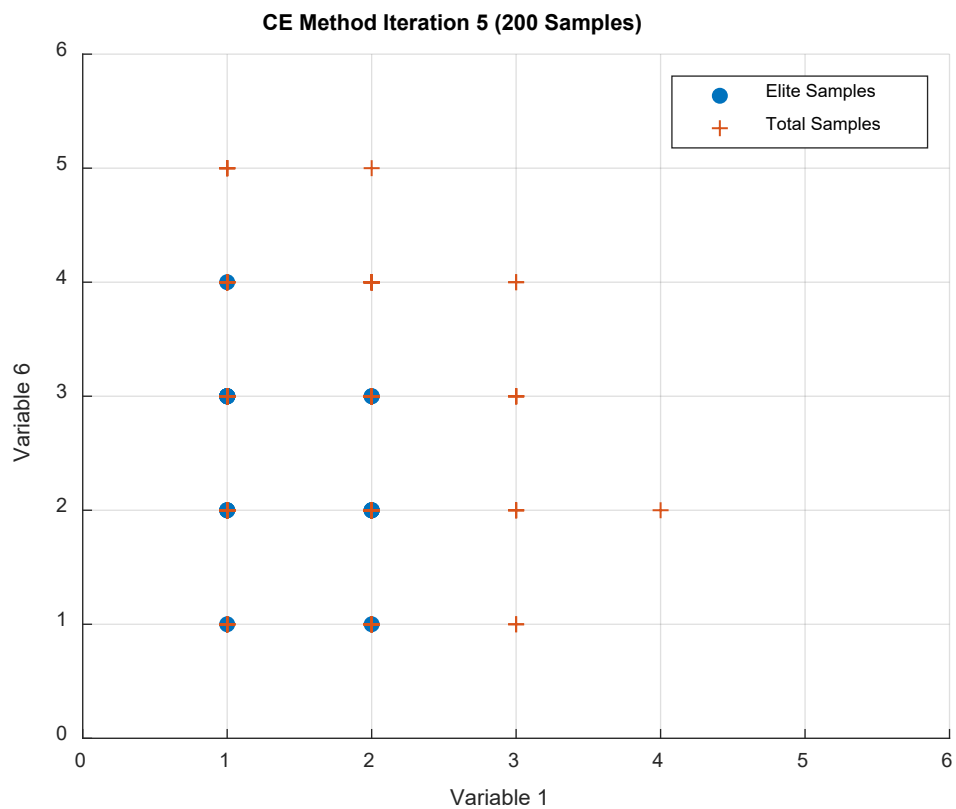

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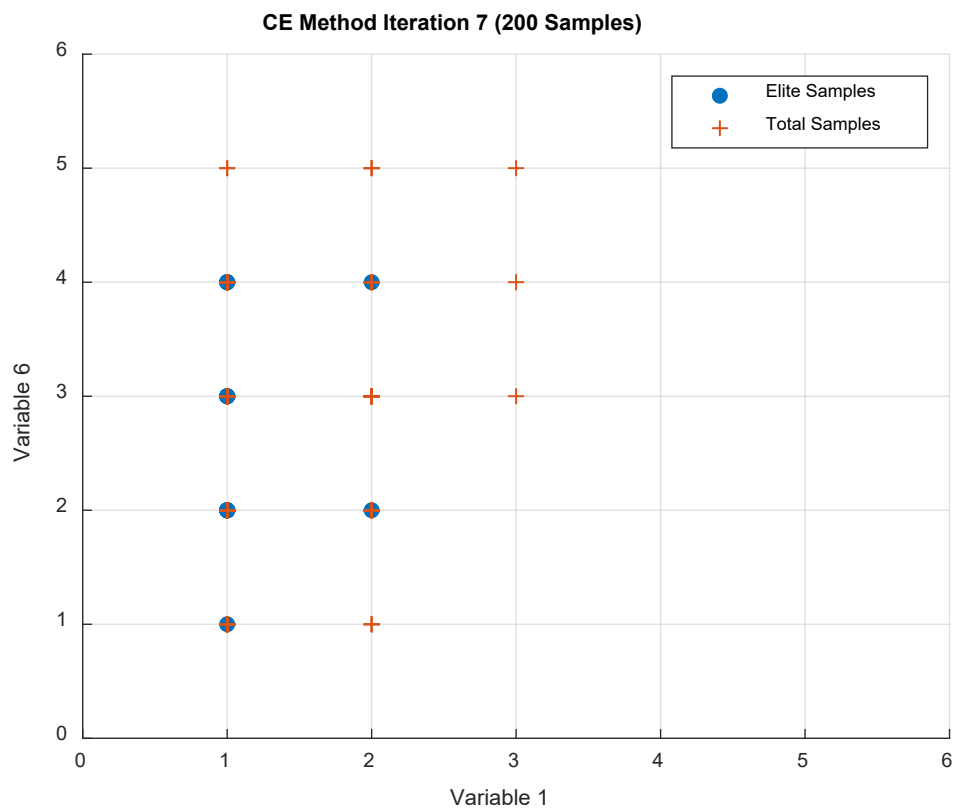

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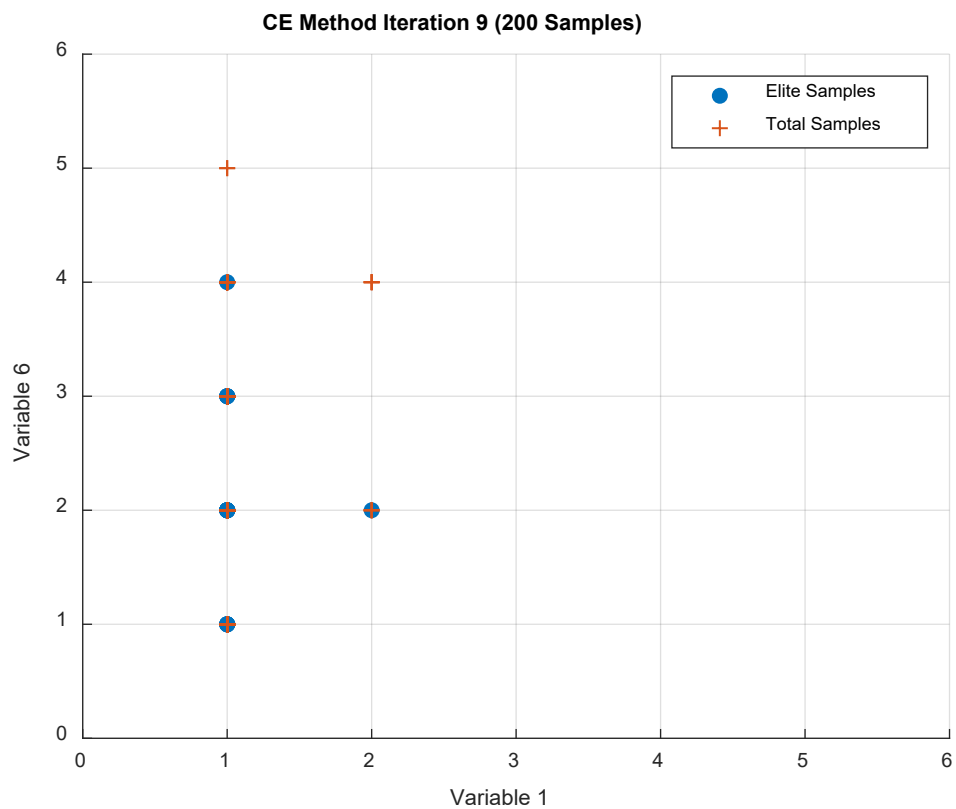

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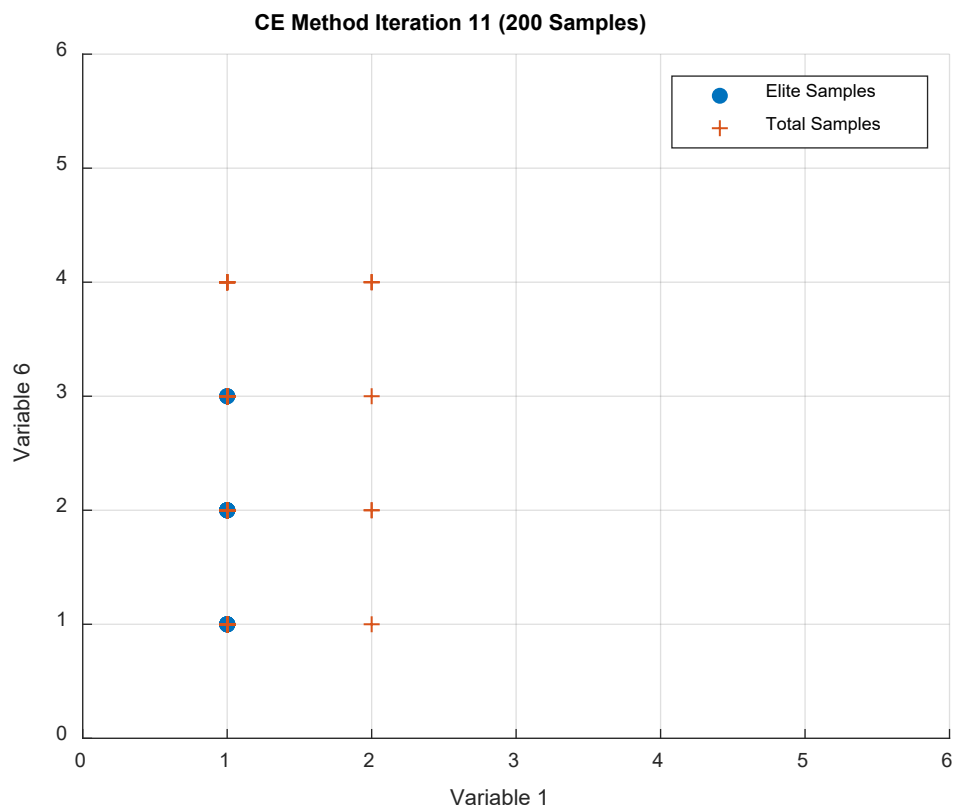

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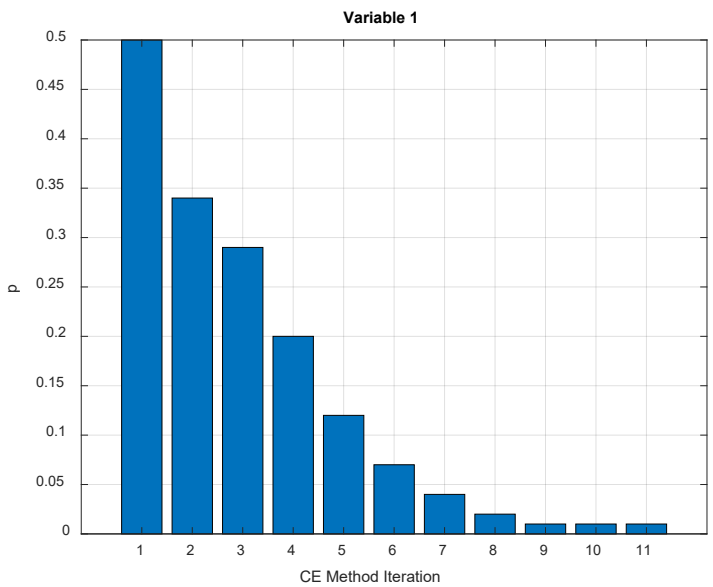
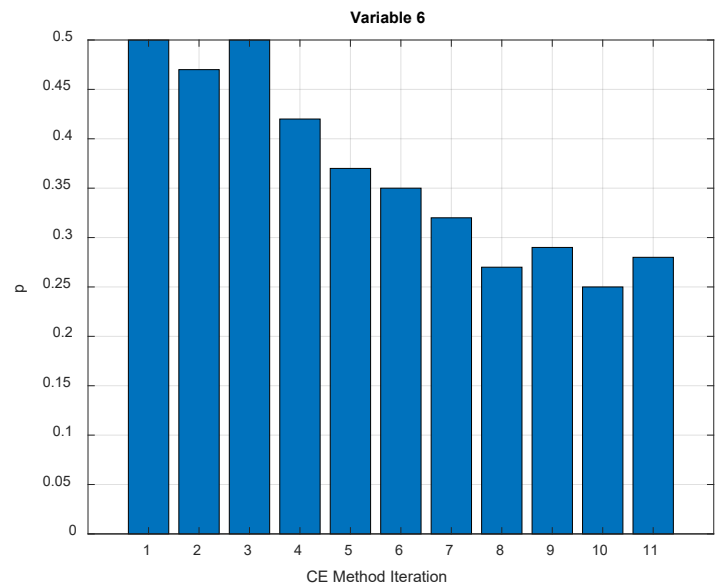




EPSO

- **Evolutionary Particle Swarm Optimization** (EPSO) is a hybrid between Evolutionary Strategies and Particle Swarm Optimization
  - **Replication**: each individual is replicated *r* times
  - **Mutation**: the *r* clones have their weights *w* mutated
  - **Recombination**: the *r*+1 individuals generate one offspring
  - **Evaluation**: each offspring has its fitness evaluated
  - **Selection**: the best particle out of the *r*+1 survives to be part of a new generation

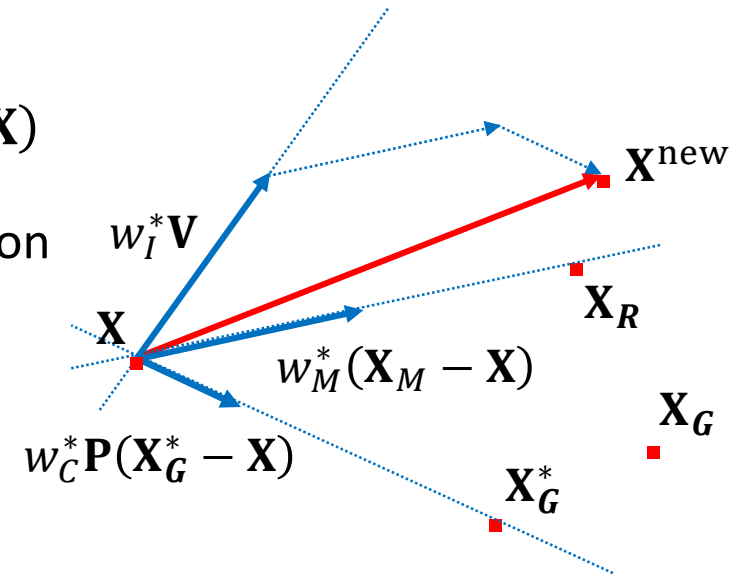
Recombination in EPSO

- Movement Rule

$$\mathbf{X}^{\text{new}} = \mathbf{X} + \mathbf{V}^{\text{new}}$$

$$\mathbf{V}^{\text{new}} = w_I^* \mathbf{V} + w_M^* (\mathbf{X}_M - \mathbf{X}) + w_C^* \mathbf{P}(\mathbf{X}_G^* - \mathbf{X})$$

  - **Inertia**: movement in the same direction
  - **Memory**: attraction towards the individual best solution
  - **Cooperation**: attraction to a region near the global best position



Recombination in EPSO

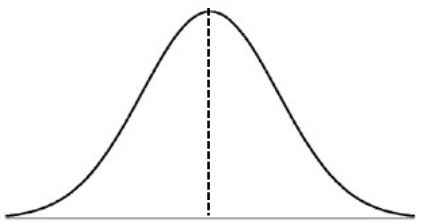
- Weights are mutated and selected

Mutation Rate

$$w^{*}=w[1+\sigma N(0,1)]$$

- The individuals are attracted to a region near the best solution found

$$\mathbf{X}_{\boldsymbol{G}}^{*}=\mathbf{X}_{\boldsymbol{G}}\,[1+w_{GB}^{*}N(0,1)]$$



$\mathbf{X}_{\boldsymbol{G}}$

- Matrix **P** acts as a communication barrier

$$\mathbf{V}^{\text{new}}=w_{I}^{*}\mathbf{V}+w_{M}^{*}(\mathbf{X}_{\boldsymbol{M}}-\mathbf{X})+w_{C}^{*}\mathbf{P}(\mathbf{X}_{\boldsymbol{G}}^{*}-\mathbf{X})$$

$$\mathbf{P}=\begin{bmatrix}1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & 0\end{bmatrix}$$

Communication Probability

If $i \neq j$ then $P_{ij} = 0$

ElseIf $U(0,1) > p$ then $P_{ij} = 0$

Else $P_{ij} = 1$

Special Solution Generation Scheme for TEP

- Given the nature of the TEP problem, as **special solution generation scheme** was used **in parallel** with the **recombination operator** of **EPSO**
  - 50% chance of using this scheme or EPSO (no particular reason for choosing this value...)

$i := 0$

Do

$i := i + 1$

Create a New Solution by copying the Current Solution

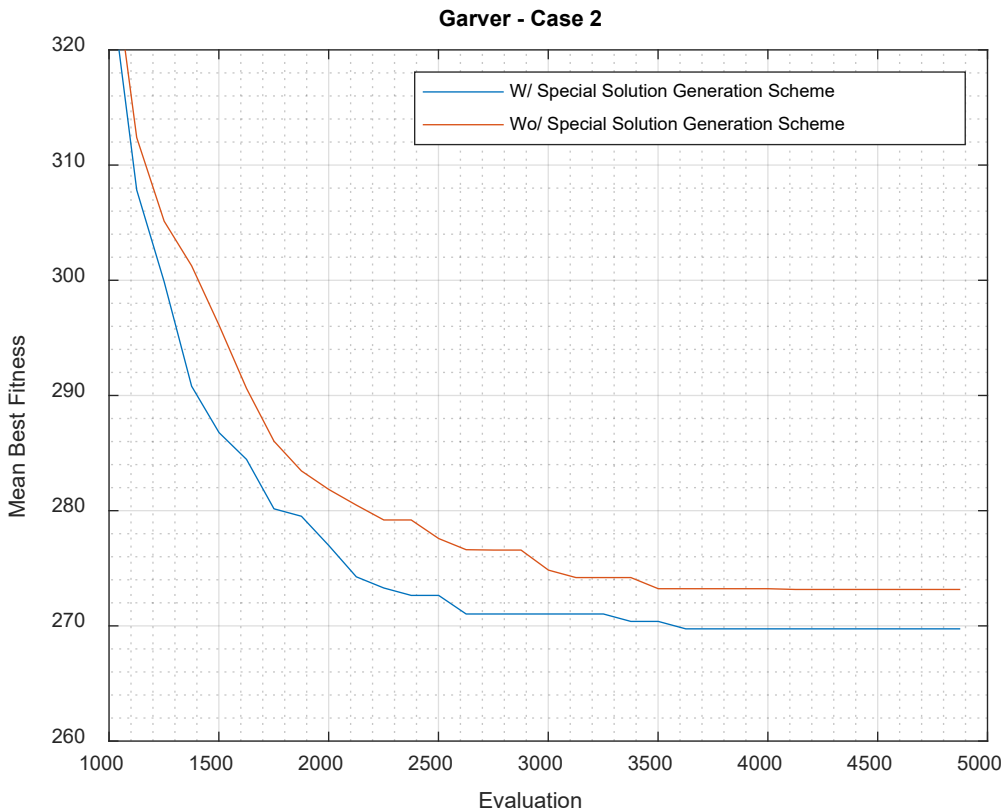
Randomly select one variable of the New Solution and subtract its value by 1

Enforce space limits in the New Solution

Until New Solution = Current Solution and $i \leq$ # variables

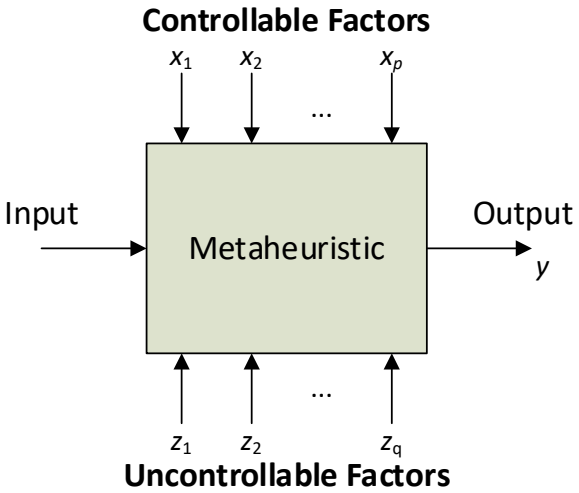
Special Solution Generation Scheme for TEP

- **Mean Best Fitness (31 trials)**



EPSO Parameter Tuning

- **To improve performance, EPSO parameters** were **tuned** using an iterative optimization algorithm based on a **2^2 factorial design**



- **Parameters tuned**
  - Mutation Rate $\boldsymbol{\tau}$ - **[0.2, 0.8]**
  - Communication Probability $\boldsymbol{P}$ - **[0.2, 0.8]**

- **Iterative methodology**
  - 4 x 10 trial runs with combinations of limit values of the intervals defined for $\boldsymbol{\tau}$ and $\boldsymbol{P}$
  - Two-way ANOVA tells the parameter that most impacts the response
  - Main effect tells which limit of the interval should be reduced

Example of EPSO Parameter Tuning

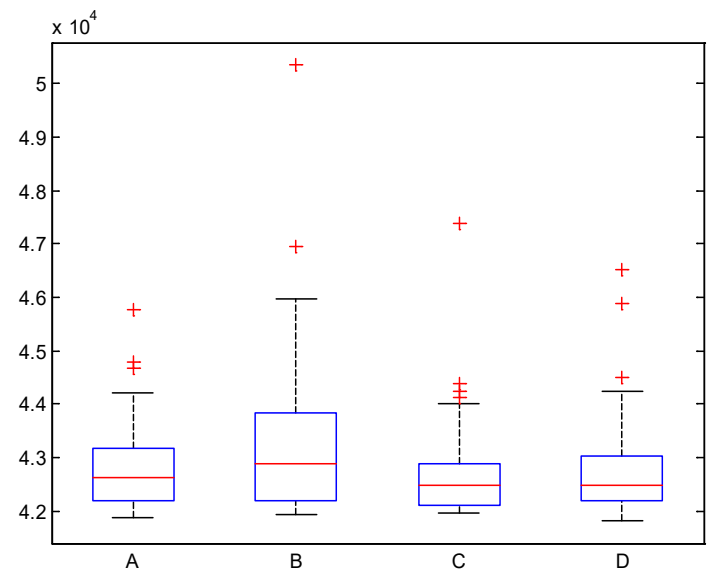
Two-way ANOVA results

Factor	F-test Statistic	P-value	Main Effect
	1st Iteration		
τ	3.474	0.064	-350.73
P	**6.675**	**0.011**	**486.14**
τ × P	0.225	0.636	89.17
	2nd Iteration		
τ	5.469	0.021	-548.35
P	**6.022**	**0.015**	**575.42**
τ × P	1.496	0.223	286.79
	3nd Iteration		
τ	**5.508**	**0.020**	**-448.7**
P	1.448	0.231	230.01
τ × P	0.958	0.329	187.14
	4th Iteration		
τ	2.849	**0.093**	-306.78
P	2.497	**0.116**	287.21
τ × P	1.807	**0.181**	244.34

Greater than the threshold level of 0.05

Intervals for *τ* and *P*

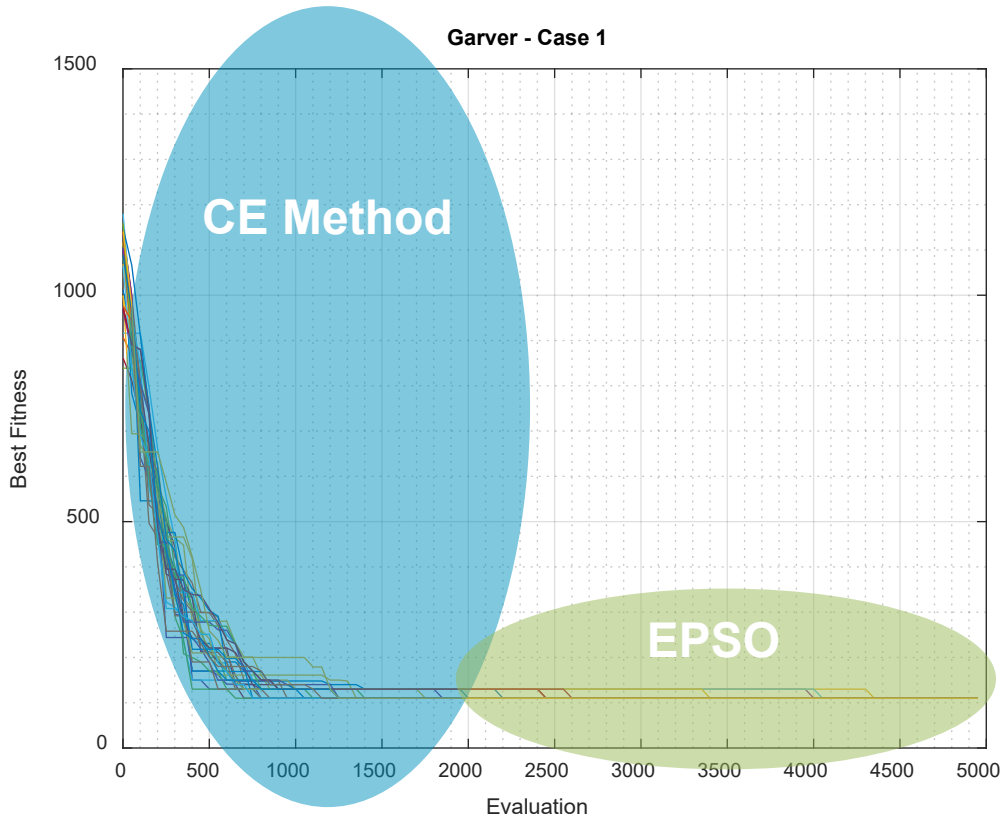
Factor	Iteration			
	Start	1st	2nd	3rd
τ (low)	**0.2**	0.2	0.2	**0.5**
τ (high)	**0.8**	0.8	0.8	0.8
P (low)	**0.2**	0.2	0.2	0.2
P (high)	**0.8**	**0.5**	**0.35**	0.35



Box-plots for the final range of *τ* and *P*

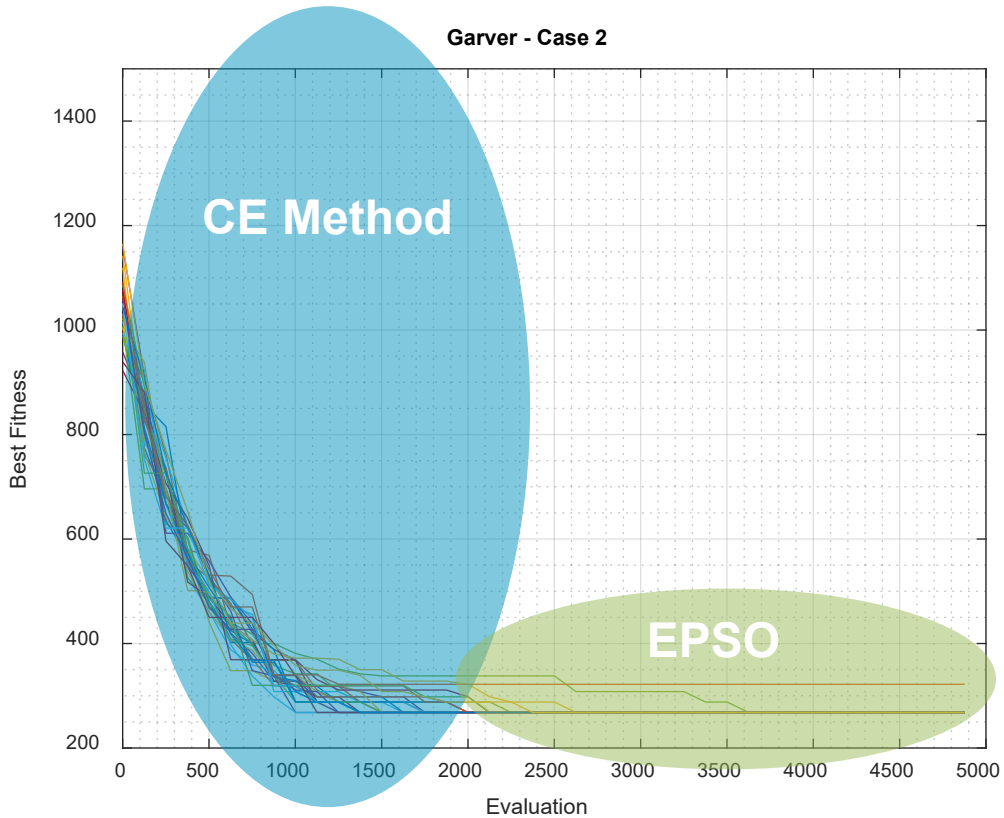
Results 31 Trials

- Garver – Case 1



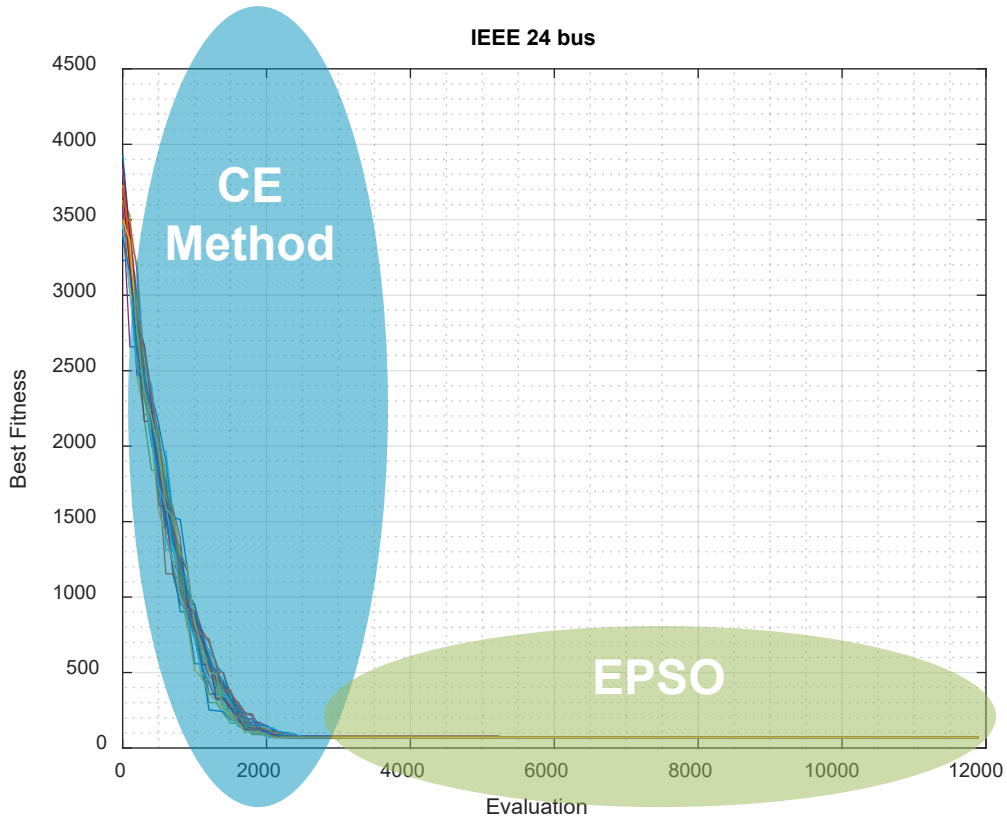
Results 31 Trials

- Garver – Case 2



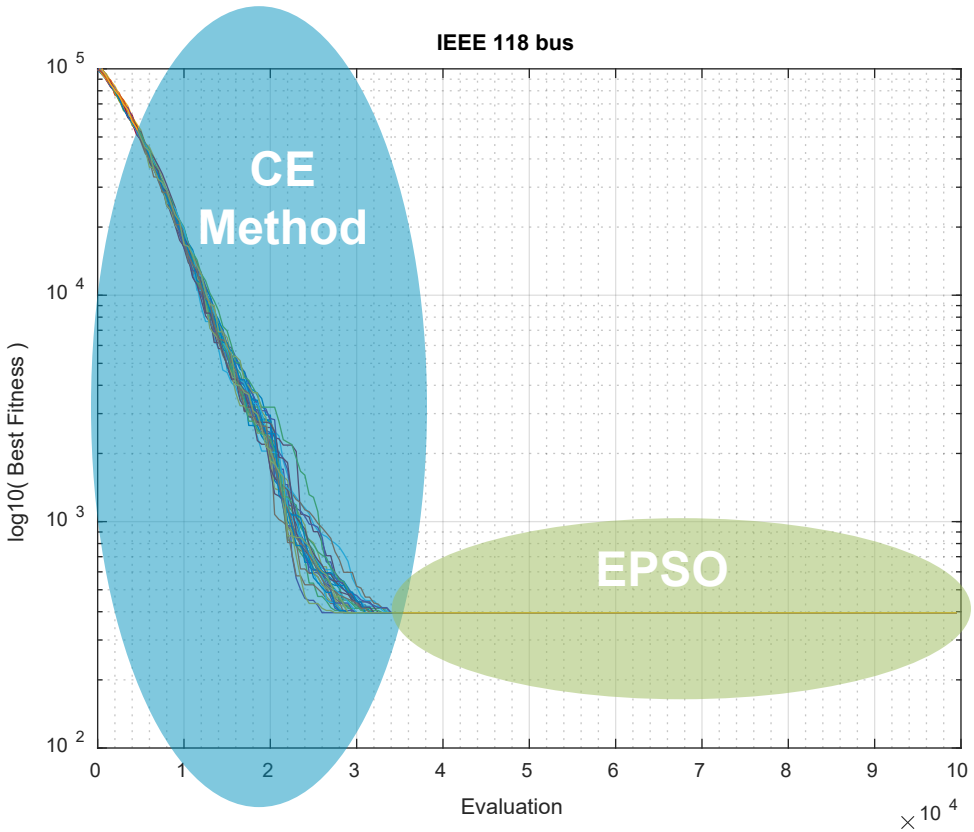
Results 31 Trials

- IEEE 24 bus



Results 31 Trials

- IEEE 118 bus



Results 31 Trials

Fitness	Garver Case 1	Garver Case 2	IEEE 14 bus	IEEE 118 bus
Best	110	268	70	396
Worst	110	322	70	396
Mean	110	269.74	70	396
Standard Deviation	0	9.70	0	0

Final Remarks

- **Combination of methods** for **different stages** of the search process can **improve accuracy** and **robustness**

- **Parameter tuning can** reduce the **information** required from the **user**
  - The procedure described must be done offline

- **Tailor-made recombination rules** can lead to **better performances**
  - Requires specific information about the problem being solved

Thank you for your attention!

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