Cooperative Combination of the Cross-Entropy Method and the Evolutionary Particle Swarm Optimization to Improve Search Domain Exploration and Exploitation

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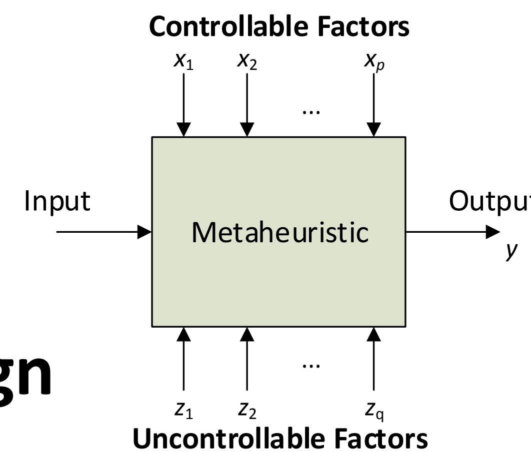
Methodological Approach

- **Combination** of two optimization methods
  - **Cross-Entropy (CE) Method** for exploration
  - **Evolutionary Particle Swarm Optimization (EPSO)** for exploitation



- **EPSO parameters** were **tuned** using an iterative optimization process based on a **2^2 factorial design**
  - Only the mutation rate $\boldsymbol{\tau}$ and the probability of communication $\boldsymbol{P}$ were optimized

CE Method for Optimization

- **Monte Carlo approach** to combinatorial and continuous non-linear optimization proposed by Reuven Rubinstein

- Starts by defining a **sampling distribution** for the **optimization variables** and gradually adjust the parameters (e.g. mean, standard deviation) according to the performance of part of the samples
  - Example of distributions: Bernoulli, Binomial, **Gaussian**

CE Method for Optimization

Select μ_0 and σ_0^2, the number of samples per iteration N, the rarity parameter ρ, the smoothing parameter α, $k := 0$

Do

$k := k + 1$

Generate a sample of $\mathbf{X}_1, \ldots, \mathbf{X}_N$ from the sampling distribution $N(\mu_{k-1}, \sigma_{k-1}^2)$

Compute $S(\mathbf{X}_1), \ldots, S(\mathbf{X}_N)$ and order the samples from the worst to the best performing ones, i.e. $S(\mathbf{X}_1) < S(\mathbf{X}_2) < \ldots < S(\mathbf{X}_N)$

Compute γ_k as the ρ^{th} quantile of the performance values and select $N^{elite}=\rho N$; let ψ be the subset from the ordered set of samples that contains all the N^{elite} samples, i.e., the samples $S(\mathbf{X})<\gamma_k$

For $j = 1$ to n

$$\mu_{kj} := \sum_{i \in \Psi} \frac{X_{ij}}{N^{elite}} \qquad \sigma_{kj}^2 := \sum_{i \in \Psi} \frac{\left(X_{ij} - \mu_{kj}\right)^2}{N^{elite}}$$

End For

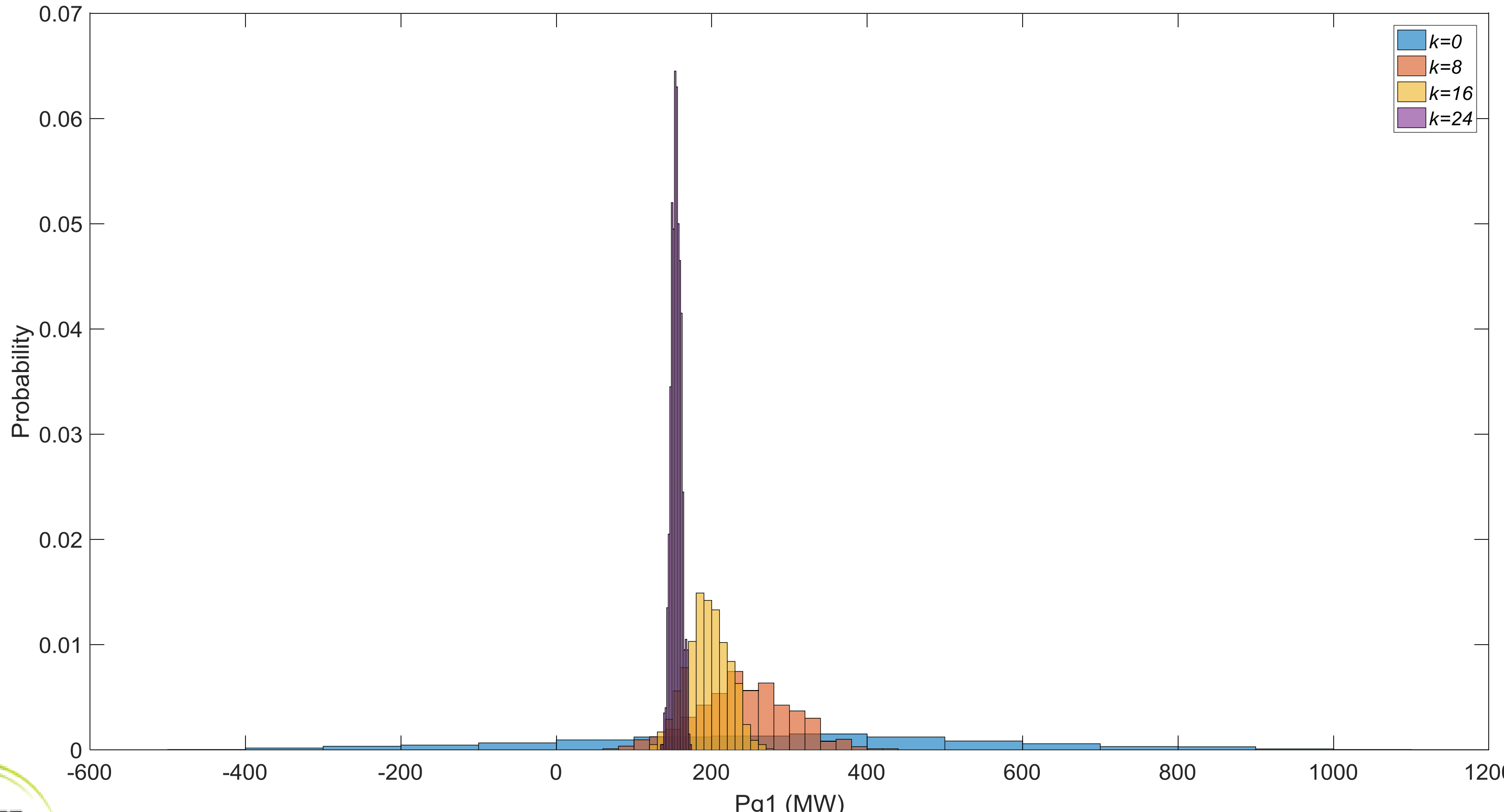
Apply smoothing

$$\boldsymbol{\mu}_k := \alpha\, \boldsymbol{\mu}_k + (1-\alpha)\, \boldsymbol{\mu}_{k-1} \qquad \boldsymbol{\sigma}_k^2 := \alpha\, \boldsymbol{\sigma}_k^2 + (1-\alpha)\, \boldsymbol{\sigma}_{k-1}^2$$

Until $k < k^{MAX}$

CE Method for Optimization

- Test Bed OPF: Case 1



EPSO

- **Evolutionary Particle Swarm Optimization** (EPSO) is an hybrid between Evolutionary Strategies (ES) and Particle Swarm Optimization (PSO) proposed by Vladimiro Miranda
  - **Replication**: each individual is replicated *r* times
  - **Mutation**: the *r* clones have their weights *w* mutated
  - **Recombination**: the *r*+1 individuals generate one offspring
  - **Evaluation**: each offspring has its fitness evaluated
  - **Selection**: the best particle out of the *r*+1 survives to be part of a new generation

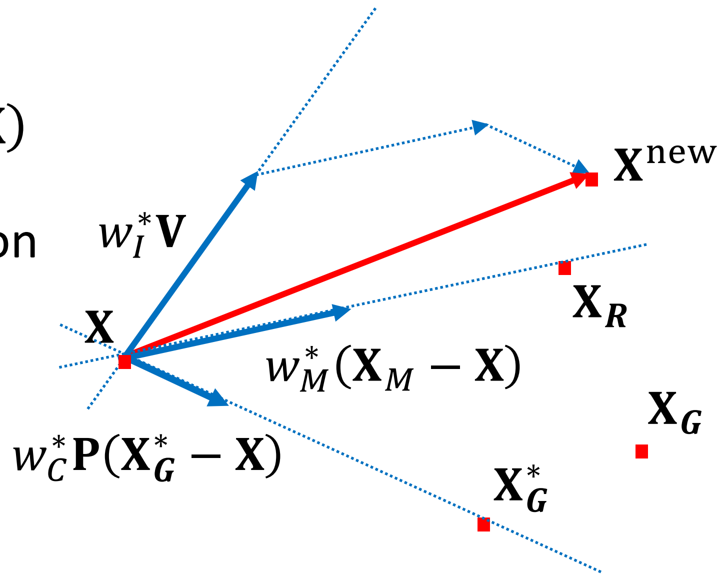
EPSO

- Movement Rule

$$\mathbf{X}^{\text{new}} = \mathbf{X} + \mathbf{V}^{\text{new}}$$

$$\mathbf{V}^{\text{new}} = w_I^* \mathbf{V} + w_M^* (\mathbf{X}_{\boldsymbol{M}} - \mathbf{X}) + w_C^* \mathbf{P}(\mathbf{X}_{\boldsymbol{G}}^* - \mathbf{X})$$

  - **Inertia**: movement in the same direction
  - **Memory**: attraction towards the individual best solution
  - **Cooperation**: attraction to a region near the global best position



EPSO

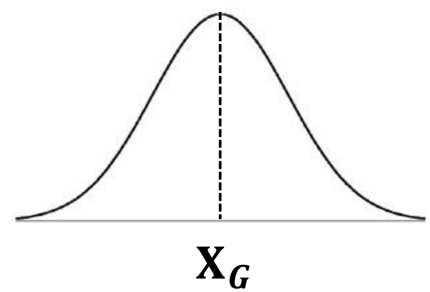
- Weights are mutated and subjected to selection

Mutation Rate

$$w^* = w[1 + \sigma N(0,1)]$$

- The individuals are attracted to a region near the best solution found

$$\mathbf{X}_{\boldsymbol{G}}^* = \mathbf{X}_{\boldsymbol{G}}\ [1 + w_{GB}^* N(0,1)]$$



- Matrix **P** acts as a communication barrier

Communication Probability

$$\mathbf{V}^{\text{new}} = w_I^* \mathbf{V} + w_M^* (\mathbf{X}_{\boldsymbol{M}} - \mathbf{X}) + w_C^* \mathbf{P} (\mathbf{X}_{\boldsymbol{G}}^* - \mathbf{X})$$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

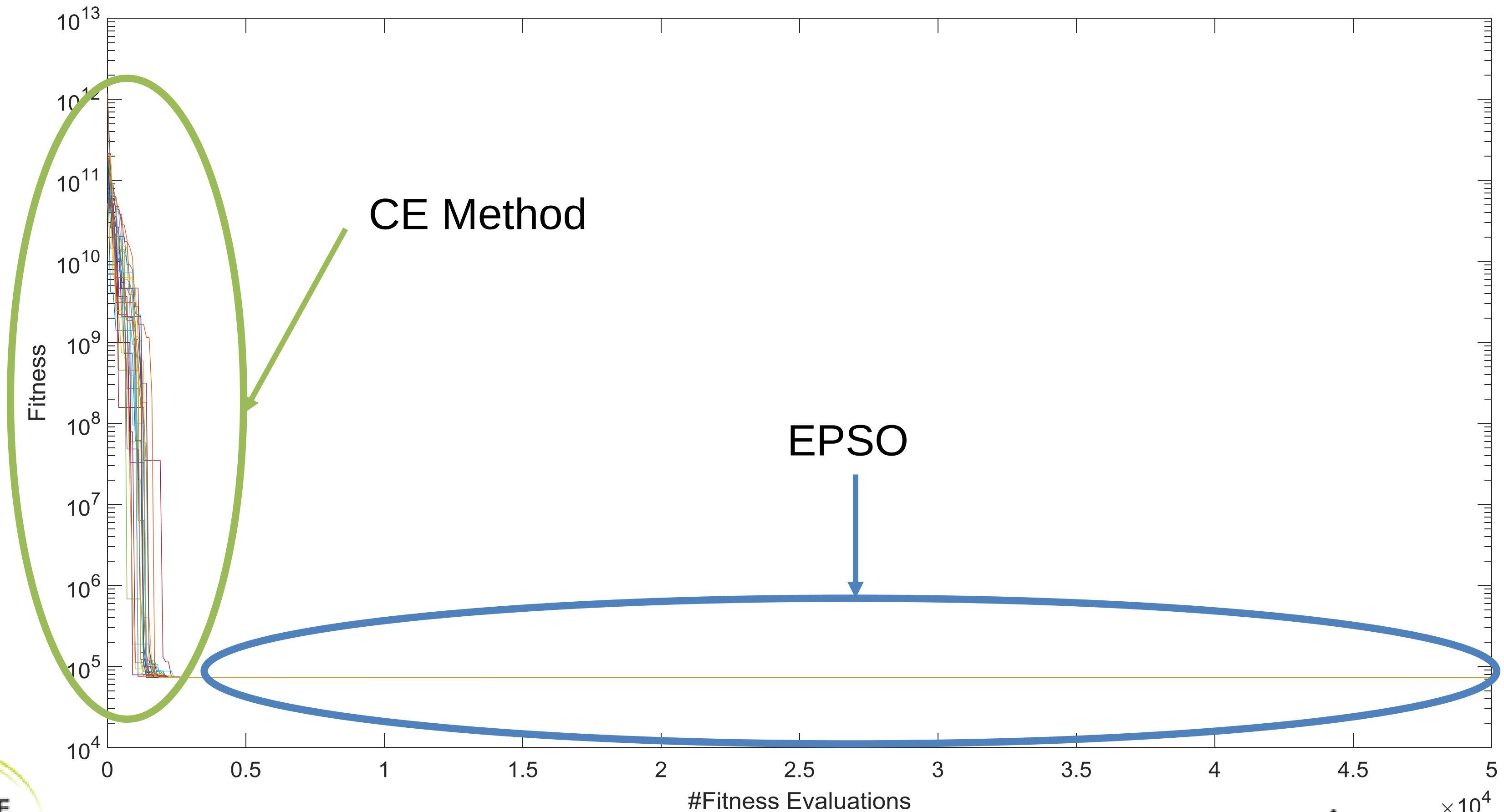
If $i \neq j$ then $P_{ij} = 0$

ElseIf $U(0,1) > p$ then $P_{ij} = 0$

Else $P_{ij} = 1$

Example

- Test Bed OPF: Case 1 – 31 independent runs



EPSO Parameter Tuning

- **Iterative method** based on **2^2 factorial design** and the **Two-way ANOVA** to guarantee the **best EPSO performance** in every problem

  Define maximum allowable interval for τ and P (e.g. [0.2, 0.8])

  Run a 2^2 factorial design (**4 experiments**)

    Perform 4 × 31 runs of EPSO for the 4 combinations of the limit values of τ and P

  Do

    Run Two-way ANOVA and **select** the variable with the highest F-test statistic

    Compute the main effect to determine which limit should be updated

    Update the limit to the central value of the interval (e.g. if the output decreases with the variable increase, then set the lower limit to the central value)

    Run a 2^2 factorial design with the updated limits (**+2 experiments**)

  While there is evidence that τ and P affect the output (check p-values) or if the difference between the limits is greater than a threshold (e.g. 0.1)

EPSO Parameter Tuning

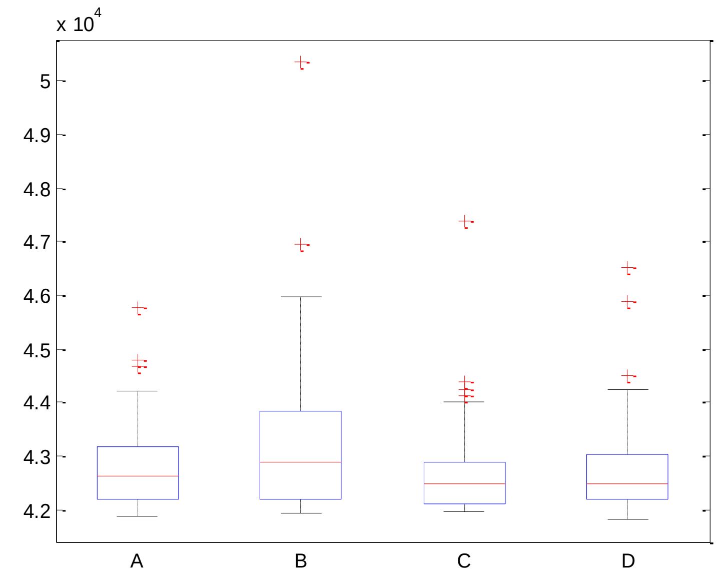
Two-way ANOVA results

Factor	F-test Statistic	P-value	Main Effect
	1st Iteration		
τ	3.474	0.064	-350.73
P	**6.675**	0.011	**486.14**
τ × P	0.225	0.636	89.17
	2nd Iteration		
τ	5.469	0.021	-548.35
P	**6.022**	0.015	**575.42**
τ × P	1.496	0.223	286.79
	3nd Iteration		
τ	**5.508**	0.020	**-448.7**
P	1.448	0.231	230.01
τ × P	0.958	0.329	187.14
	4th Iteration		
τ	2.849	0.093	-306.78
P	2.497	0.116	287.21
τ × P	1.807	0.181	244.34

Greater than the threshold level of 0.05

Setting values for *τ* and *P*

Factor	Iteration			
	1st	2nd	3rd	4th
τ (low)	0.2	0.2	0.2	**0.5**
τ (high)	0.8	0.8	0.8	0.8
P (low)	0.2	0.2	0.2	0.2
P (high)	0.8	**0.5**	**0.35**	0.35



Box-plots for the final range of *τ* and *P*

Results

- Test Bed OPF: 31 independent runs

Fitness	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Best	72682.29	72044.58	60282.36	71391.65	70566.15	60799.88
Mean	72686.53	72049.53	60286.37	71396.14	70572.44	60805.33
Standard Deviation	2.95	4.08	3.45	4.33	4.36	4.26

Results

- Test Bed OSDER: 31 independent runs

Fitness	Case 33	Case 180
Best	-5216.82	-2566.78
Mean	-5183.73	-2549.76
Standard Deviation	20.82	17.36

Final Remarks

- **Combination of methods** to address different stages of the search process can greatly **improve accuracy** and **robustness**
  - Need to establish intelligent mechanisms to switch from methods
- **Systematic parameter tuning** is essential to reduce the information required from the user

Thank you for your attention!

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