Fundamental Parameters and Figures-of-Merit of Antennas

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Antennas
Single Elements and Arrays

Antennas:
'Electronic Eyes and Ears of the World'
John D. Kraus

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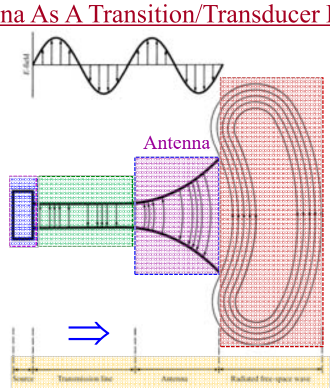
Antenna As A Transition/Transducer Device



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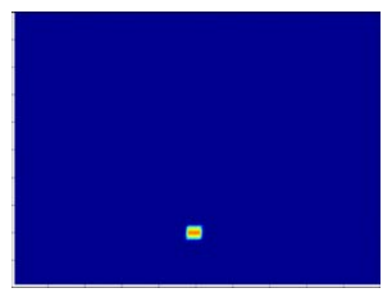
TE Horn Animation



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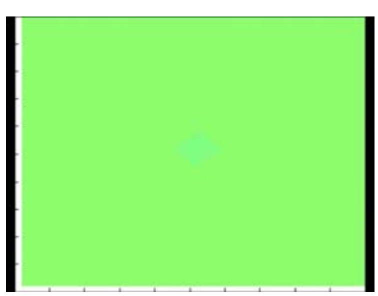
TM Open Animation



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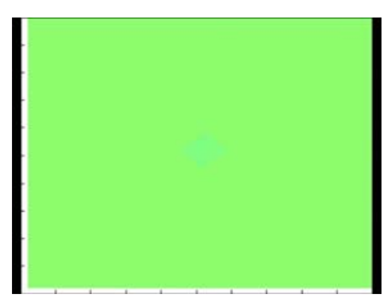
TM Box Animation



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Definition of Antenna

According to the *IEEE Standard Definitions of Terms for Antennas*:

"A means for radiating or receiving radio waves."

According to the *Webster's Dictionary*:

"A means for radiating or receiving radio waves."

In other words, the antenna is the *transitional structure / transducer* between free-space and a guiding device.

Definition of Antenna

The guiding device or transmission line may take the form of a coaxial line or a hollow pipe (waveguide), and it is used to transport EM energy from the transmitting source to the antenna, or from the antenna to the receiver.

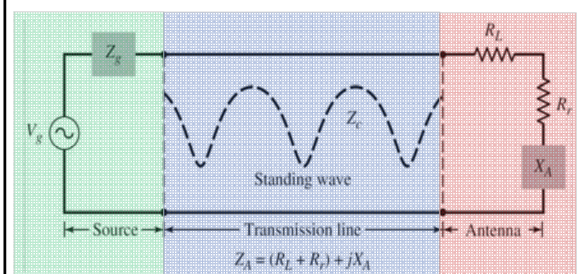
In the former case, we have a *transmitting* antenna and in the latter a *receiving* antenna.

Definition of Antenna

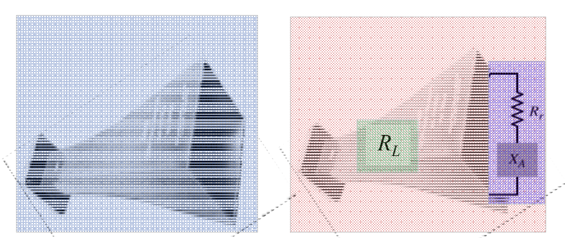
In addition to receiving or transmitting EM energy, an antenna in an advanced wireless communications system is usually required to *optimize or accentuate* the radiation energy in some directions and *suppress* it in others.

Thus the antenna must also serve as a directional device, in addition to a radiating device.

Thevenin Equivalent In Transmission Mode



Horn Antenna And Its Equivalent



Horn Equivalent

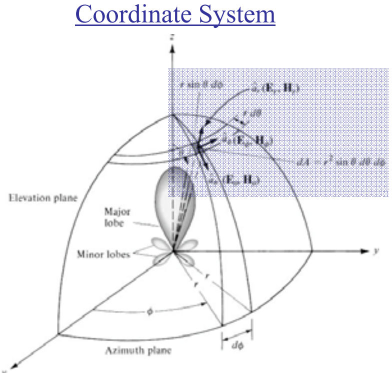
Types of Antennas

1. Wire antennas
2. Aperture antennas
3. Microstrip antennas
4. Array antennas
5. Reflector antennas
6. Lens antennas
7. Other Antennas for Mobile Units

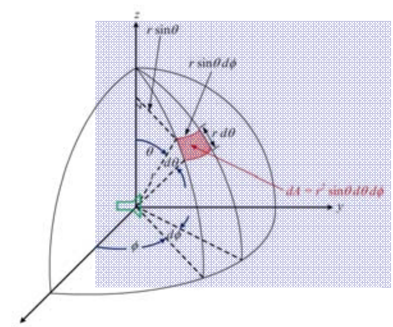
Fundamental Parameters

Coordinate System for Antennas

Coordinate System



Coordinate System



Spherical Angular Limits

$$\text{Area of Sphere} = \int_0^{\pi}\int_0^{2\pi} dA = \int_0^{\pi}\int_0^{2\pi} r^2 \sin\theta \, d\theta d\phi$$

$$\text{Area} = \int_0^{\pi}\left[\int_0^{2\pi} r^2 \sin\theta \, d\theta\right] d\phi$$

$$\text{Area} = (\pi)\left[2(1-1)\right] = 0 \ \ !!!$$

$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \pi$$

Spherical Angular Limits

$$\text{Area of Sphere} = \int_0^{2\pi}\int_0^{\pi} dA = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta d\phi$$

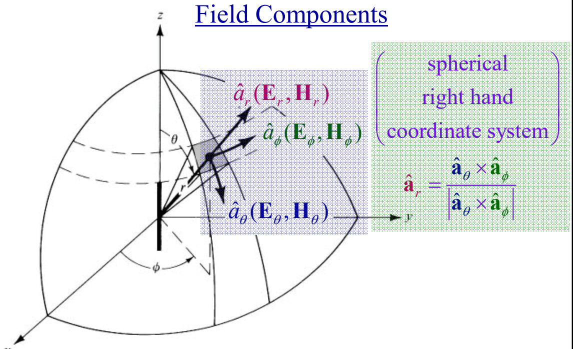
$$\text{Area} = \int_0^{2\pi}\left[\int_0^{\pi} r^2 \sin\theta \, d\theta\right] d\phi$$

$$\text{Area} = (2\pi)(2r^2) = 4\pi r^2$$

$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$

✓

Spherical Electric and Magnetic Field Components



Amplitude Radiation Pattern

- Field Pattern:

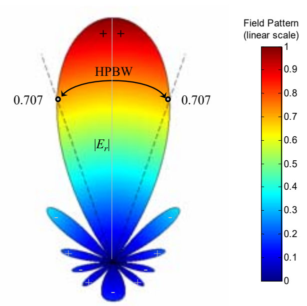
A plot of the field (either electric $|\underline{E}|$ or magnetic $|\underline{H}|$) on a *linear* scale

- Power Pattern:

A plot of the power (proportional to either the electric $|\underline{E}|^2$ or magnetic $|\underline{H}|^2$ fields) on a *linear* or *decibel* (dB) scale.

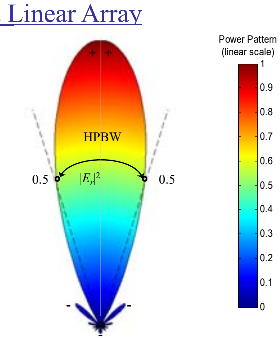
2-D Normalized *Field* $|\underline{E}_n|$ Pattern of a Linear Array

Linear Scale
N = 10 elements
$d = \lambda/4$ spacing
HPBW = 38.64°



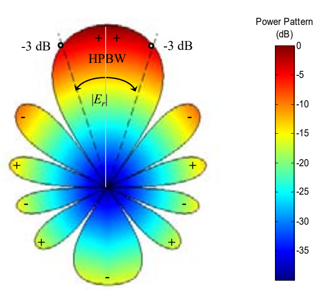
2-D Normalized *Power* $|\underline{E}_n|^2$ Pattern of a Linear Array

Linear Scale
N = 10 elements
$d = \lambda/4$ spacing
HPBW = 38.64°

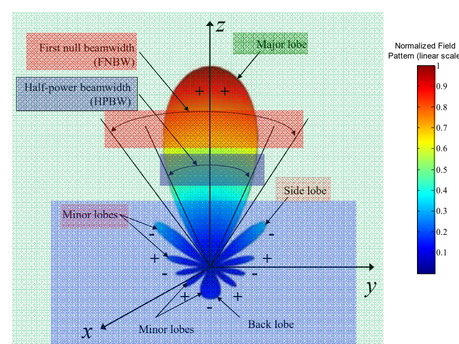


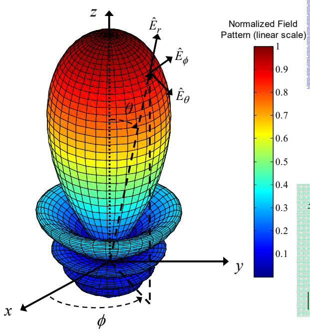
2-D Normalized *Power* $|\underline{E}_n|$ Pattern of a Linear Array in dB

dB Scale
N = 10 element
$d = \lambda/4$ spacing
HPBW = 38.64°



Polar Pattern





Normalized 3-D Amplitude *Field* Pattern of Linear Array

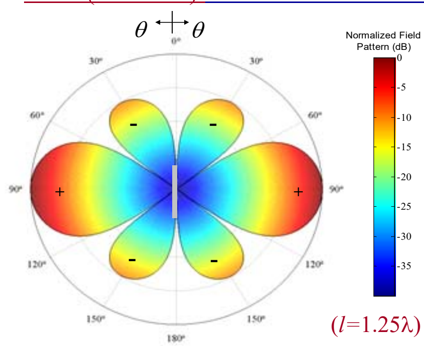
Linear Scale

$N = 10,\ d = \lambda/4$

$$\underline{E}(r,\theta,\phi)\big|_{r=r_c} = \hat{a}_r E_r(r_c,\theta,\phi) + \hat{a}_\theta E_\theta(r_c,\theta,\phi) + \hat{a}_\phi E_\phi(r_c,\theta,\phi)$$

$$|\underline{E}| = \sqrt{|E_r|^2 + |E_\theta|^2 + |E_\phi|^2}$$

Polar (dB Scale): Two-Dimensional Pattern



(l=1.25λ)

Directivity D

$$D = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}}$$

$$D_{\max} = D_0 = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

$$D(\text{dB}) = 10\log_{10}[D(\text{dimensionless})]$$

$$D_0(\text{dB}) = 10\log_{10}[D_0(\text{dimensionless})]$$

D = directivity (dimensionless)

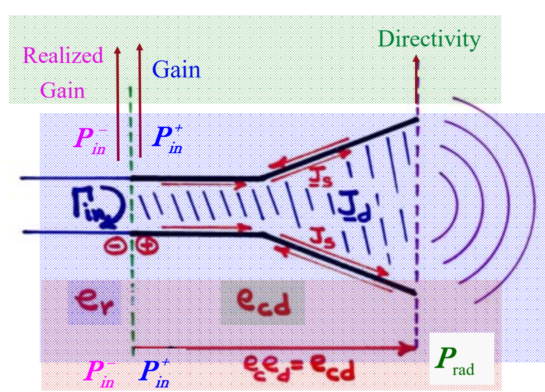
D_0 = maximum directivity (dimensionless)

U = radiation intensity (W/unit solid angle)

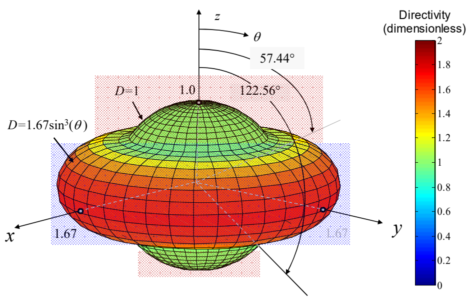
$U_{\max}$ = maximum radiation intensity

U_0 = radiation intensity of isotropic

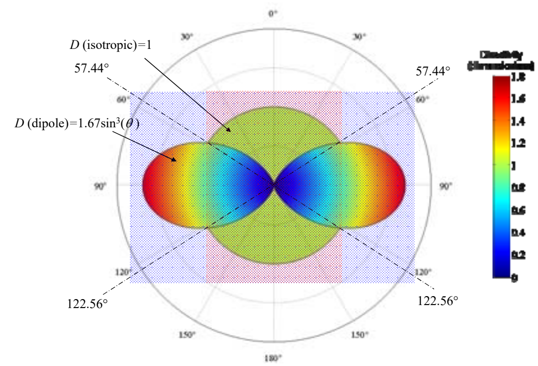
P_{rad} = radiated power (W)



Three-dimensional Directivity Pattern



Two-Dimensional Directivity Pattern



Example 1:

$$\underline{W}_{\text{rad}} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

Solution:

$$P_{\text{rad}} = \pi^2 A_0$$

$$U = r^2 W_{\text{rad}} = A_0 \sin\theta$$

$$U_{\max} = U\big|_{\max} = A_0 \sin\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)A_0}{\pi^2 A_0} = 1.27 \text{ dimensionless}$$

$$D_0 = 1.038 \text{ dB}$$

$$D = D_0 \sin\theta = 1.27\sin\theta$$

Example 2:

$$\underline{W}_{\text{rad}} = \hat{a}_r A_0 \frac{\sin^2\theta}{r^2}$$

Solution:

$$P_{\text{rad}} = \frac{8\pi}{3} A_0$$

$$U = r^2 W_{\text{rad}} = A_0 \sin^2\theta$$

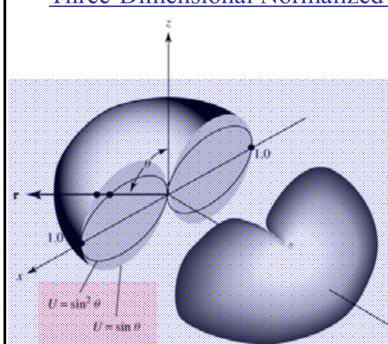
$$U_{\max} = U\big|_{\max} = A_0 \sin^2\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi A_0}{8\pi/3} = 1.5 \text{ dimensionless}$$

$$D_0 = 1.761 \text{ dB}$$

$$D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

Three-Dimensional Normalized Radiation Patterns

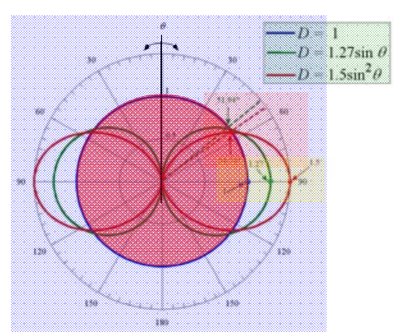


$$D = D_0 \sin\theta = 1.27\sin\theta$$

$$D_0 = 1.27 = 1.038 \text{ dB}$$

$$D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

$$D_0 = 1.5 = 1.761 \text{ dB}$$



$$1.27\sin\theta\big|_{\theta=\theta_0} = 1$$

$$\sin\theta_0 = \frac{1}{1.27} = 0.787$$

$$\theta_0 = \sin^{-1}(0.787) = 51.94^\circ$$

$$D \ge D(\text{isotropic}): \; 51.94^\circ \le \theta \le 128.06^\circ$$

$$1.5\sin^2\theta\big|_{\theta=\theta_0} = 1.5\sin^2\theta_0 = 1$$

$$\theta_0 = \sin^{-1}\left(\frac{1}{1.5}\right)^{1/2} = \sin^{-1}(0.667)^{1/2}$$

$$\theta_0 = \sin^{-1}(0.8165) = 54.74^\circ$$

$$D \ge D(\text{isotropic}): \; 54.74^\circ \le \theta \le 125.26^\circ$$

Approximate Formulas for Directivity

Kraus Formula

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\, d\theta\, d\phi \simeq \Theta_{1r}\Theta_{2r}$$

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}}$$

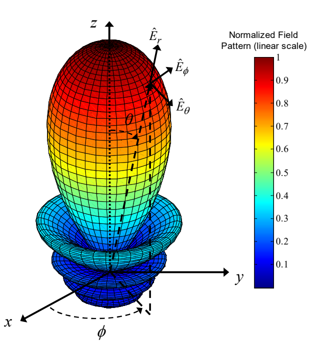
$$D_0 \approx \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}}$$

Tai & Pereira Formula

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1} + \frac{1}{D_2}\right) \quad \text{Arithmetic mean}$$

$$D_0 = \frac{4\pi}{\Omega_A} \approx \frac{32\ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2}$$

$$D_0 = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2}$$



N = 10 elements
$d = \lambda/4$ spacing
HPBW = 38.64º
FNBW = 73.8º

Kraus:

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}}$$

$$D_0 = \frac{41,253}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{(38.64)^2} = 27.63 = 14.51 \text{ dB}$$

Tai & Pereira:

$$D_0 = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{2(38.64)^2} = 24.38 = 13.87 \text{ dB}$$

Using the computer program Directivity,

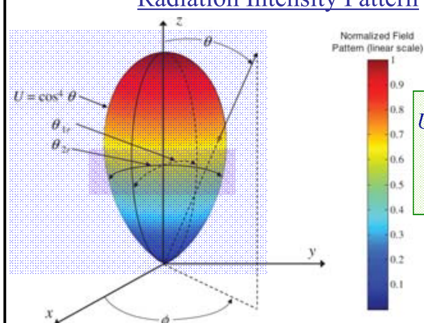
$$D_0 = 10.1158 = 10.05 \text{ dB}$$

Directional Patterns

$$U(\theta,\phi) = \begin{cases} B_o \cos^n(\theta) & \begin{cases} 0 \le \theta \le \pi/2 \\ 0 \le \phi \le 2\pi \end{cases} \\ 0 & \text{Elsewhere} \end{cases}$$

$$n = 1, 2, 3 \ldots 10, 15, 20$$

Radiation Intensity Pattern



$U(\theta,\phi) = \cos^4\theta$

$0 \le \theta \le 90^o$

$0 \le \phi \le 360^o$

Exact & Approximate Directivities

Table 2.1

n	Exact Equation (2-22)	Kraus Equation (2-26)	Kraus % Error	Tai and Pereira Equation (2-30a)	Tai and Pereira % Error
1	4	2.86	−28.50	2.53	−36.75
2	6	5.09	−15.27	4.49	−25.17
3	8	7.35	−8.12	6.48	−19.00
4	10	9.61	−3.90	8.48	−15.20
5	12	11.87	−1.08	10.47	−12.75
6	14	14.13	+0.93	12.46	−11.00
7	16	16.39	+2.48	14.47	−9.56
8	18	18.66	+3.68	16.47	−8.50
9	20	20.93	+4.64	18.47	−7.65
10	22	23.19	+5.41	20.47	−6.96
11.28	24.56	26.08	+6.24	23.02	−6.24
15	32	34.52	+7.88	30.46	−4.81
20	42	45.89	+9.26	40.46	−3.67

$n \approx 5.5$

Kraus' Error = 0

$n = 5.497 \approx 5.5$

HPBW = 56.35°

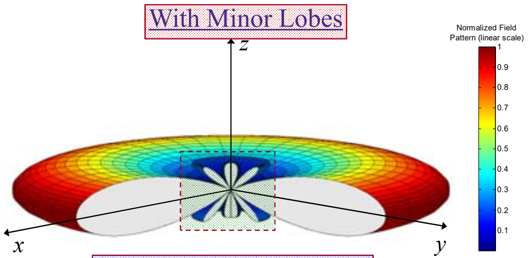
|Kraus' Error|=|T & P' Error|=6.24%

$n = 11.28$

HPBW = 39.77°

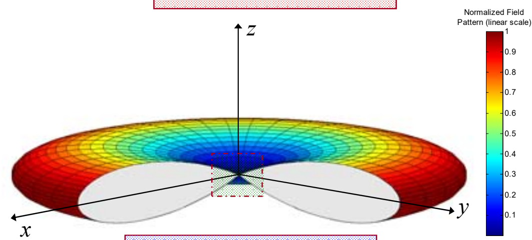
Omindirectional Patterns

Omnidirectional Pattern
With Minor Lobes



$$U \approx \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$

Omnidirectional Pattern
Without Minor Lobes



$$U \approx \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$

Omnidirectional Patterns

$$U = \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$

Directivity:

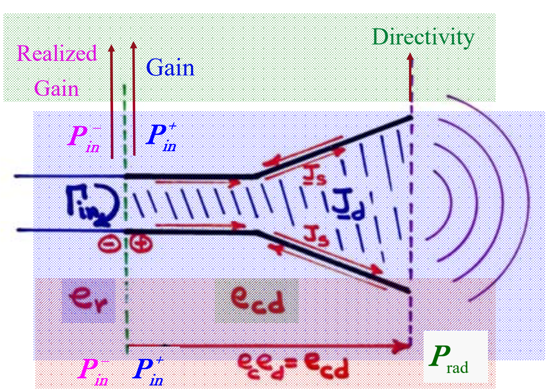
McDonald

$$D_0 = \frac{101}{\text{HPBW(degrees)} - 0.0027[\text{HPBW(degrees)}]^2}$$

Pozar

$$D_0 = -172.4 + 191\sqrt{0.818 + \frac{1}{\text{HPBW(degrees)}}}$$

Gain



$$G_0 = e_{cd} D_0 \quad \text{(2-49a)}$$

$e_{cd} = e_c e_d$ = Radiation efficiency

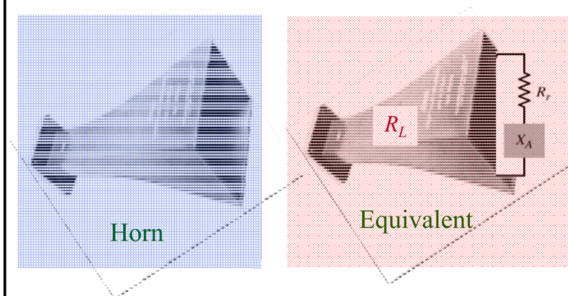
e_c = Conduction efficiency

e_d = Dielectric efficiency

Antenna Radiation Efficiency

e_{cd}

Horn Antenna And Its Circuit Equivalent



e_{cd} = Radiation Efficiency

$$e_{cd} = \frac{\text{Power Radiated by Antenna } (P_r)}{\text{Power Delivered to Antenna } (P_r + P_L)}$$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

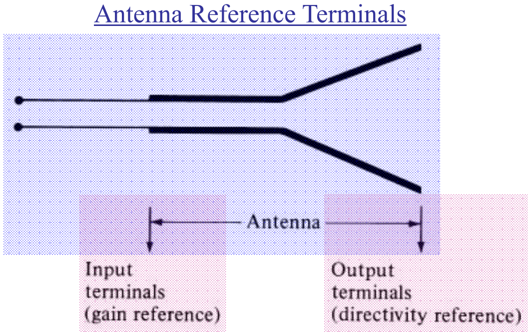
$$G = e_{cd} D$$

$$G_0 = e_{cd} D_0$$

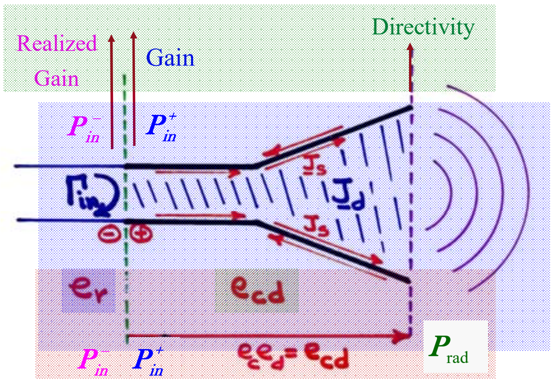
$$G_0(\text{dB}) = 10\log_{10}\left[e_{cd} D_o\right]$$

$$G_0(\text{dB}) = 10\log_{10}(e_{cd}) + 10\log_{10}(D_o)$$

Antenna Reference Terminals



Realized Gain



Realized Gain G_{re}

$$G_{re}(\theta,\phi) = e_o D(\theta,\phi) = e_r e_{cd} D(\theta,\phi)$$

$$G_{re}(\theta,\phi) = (1-|\Gamma_{in}|^2) e_{cd} D(\theta,\phi)$$

e_o = antenna total efficiency

$e_r = (1-|\Gamma_{in}|^2)$ = Reflection efficiency

$e_{cd} = e_c e_d$ = Radiation efficiency

e_c = Conduction efficiency

e_d = Dielectric efficiency

Today antenna technology
is
Science
Not
Art

Very bright future and many challenges ahead.
We just need exercise creativity,
imagination and science
for its advancement.