Estimating the Effects of Small Voltage and Frequency Changes on Industrial Induction Motor Loading

Areeg Ahmed, Anas Ali, Abdelrahman Karrar
University of Tennessee at Chattanooga
Chattanooga, USA
email: abdelrahman-karrar@utc.edu

Mark Bowman, Preston Cooper
Tennessee Valley Authority
Chattanooga, USA
email: mdbowman@tva.gov

***Abstract*—This work develops expressions for estimating the change in active and reactive power for industrial motors in response to voltage and frequency variations around the operating point. Magnetizing circuit saturation is accounted for by using incremental reactance. The expressions are simplified for a quick estimate using stator measurements and parameters. The expressions are validated using simulation, and further, a test was carried out in a nuclear power plant to determine the voltage sensitivities experimentally. Both simulation and test show that the expressions are capable of yielding a fair estimation of the required sensitivities.**

***Index Terms*—Induction motors, voltage sensitivity, frequency sensitivity, nuclear power plants.**

I. Introduction

There has been some previous interest in the behavior of loads in response to small changes in voltage around the operating point, primarily for purposes of efficiency calculations [1]. An EPRI commissioned report in 1981 [2], and a subsequent study [3] provided calculations for energy reduction for micro (equipment-level) and macro (feeder-level) responses to a conservation voltage reduction (CVR) exercise. The studies show that a voltage reduction within allowable ANSI standards on motor loads in particular would result in better efficiency and lower active and reactive power consumption. Other studies have attempted to calculate load sensitivities (both individual and composite) to voltage and frequency changes for the benefit of stability and transient analysis [4], [5].

This study attempts to take a closer look at the steady-state load voltage and frequency sensitivity characteristics around the operating point in the context of industrial plants, in particular nuclear power plant auxiliaries. The incentive for this work was to explore the effects of voltage and frequency tolerances on the loading of diesel generators (DGs) providing power to safety-related equipment in nuclear power plants upon loss of offsite power. Sample calculations on active power of motors are provided in a Westinghouse topical report [6], but this work probes deeper into the problem through a more rigorous approach, and further adding kVAr effects of voltage and frequency to arrive at total kVA of diesel generator loading. Typical nuclear power plant loads are largely motor loads, sometimes more than 90% so. The large motors include reactor coolant pumps, safety-injection pumps, auxiliary feed-water pumps, and residual-heat removal pumps.

Typical plant technical specification tolerances for the steady voltage and frequency are ±10% and ±2%, and Westinghouse [6] provides guidance to validate these or tighter specifications. This work, in accordance with TVA plant speciation considers a voltage change of 5% around nominal and frequency of 0.33% around 60 Hz. The model used for the induction motor is the steady-state equivalent circuit with parameters taken from [7]. Most of the motors loads are variable-torque loads ($T \propto \omega^2$) typical of centrifugal pumps, fans and blowers, but a small percentage associated mostly with the cooling loads are constant-torque.

What the study does not pertain to do is to develop a small signal model suitable for dynamic analysis, such as typical in stability or speed control studies. Rather the aim is to provide a qualitative picture surrounding the steady-state effects of small changes in voltage and frequency on the motor power consumption. To offer further physical insight, preference is given to expressing the performance relations in terms of measurable quantities such as stator current and voltage, rather than using more elusive concepts such as flux linkage, etc.

II. Voltage And Frequency Sensitivities Based On incremental Steady State Motor Model

A. Steady-state motor models

Since interest is only near the steady-state operating region, the standard single cage model is used, even for deep bar and double cage motors. A double cage model would be required for dynamic and starting studies [8]. Parameters used are relevant to full load rated slip operation. It is recognized that the magnetizing reactance of an induction motor under normal operating conditions is in the saturation region, while leakage reactance saturation may be neglected for normal operating currents [9]. Further, for small signal expressions, the incremental values of the p.u. magnetizing inductance $X_m = \frac{\partial v_m}{\partial i_m}$ at the operating point are used [10].

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B. Development of expressions; Voltage sensitivity.

Constant torque motor loads are considered first, since the expressions are somewhat simpler than the variable torque loads. For both, Fig 1.0 shows the Thévenin equivalent the standard motor circuit model, and is the basis for the expressions developed in the paper, where all the parameters used approximate the motor's performance in the steady-state normal operation zone. The main simplification is the assumption of linear torque-slip characteristics, namely (using the p.u. formulation):

$$T = \frac{V_{th}{}^2 s}{R_r} \tag{1}$$

And, as a consequence of (1)

$$I_r = \frac{V_{th} s}{R_r} \tag{2}$$

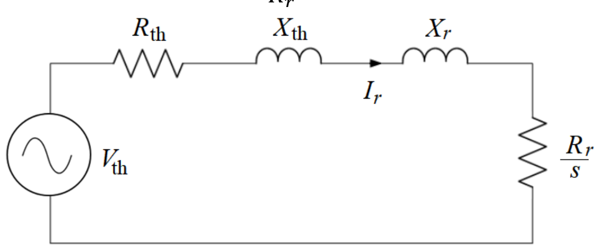


Fig. 1. Thevenin Equivalent Circuit for an Induction Motor

For constant torque: $\Delta T = 0$

$$\Delta T = \frac{2V_{th} s}{R_r}\Delta V_{th} + \frac{V_{th}{}^2}{R_r}\Delta s = 0 \tag{3}$$

$$\Delta s = -\frac{2s}{V_{th}}\Delta V_{th} \tag{4}$$

From (2)

$$\Delta I_r = \frac{s}{R_r}\Delta V_{th} + \frac{V_{th}}{R_r}\Delta s \tag{5}$$

Substituting Δs from (4)

$$\Delta I_r = -\frac{s}{R_r}\Delta V_{th} \tag{6}$$

Rotor power,

$$P_r = I_r{}^2 \frac{R_r}{s} \tag{7}$$

Then,

$$\Delta P_r = \frac{2I_r R_r}{s}\Delta I_r - \frac{I_r{}^2 R_r}{s^2}\Delta s \tag{8}$$

Substituting (4), (6), and using (2) leads to

$$\Delta P_r = 0 \tag{9}$$

The stator current is found as

$$\bar{I}_s = \bar{I}_r + \bar{I}_r \frac{jX_r + \frac{R_r}{s}}{jX_m} \tag{10}$$

Finding the magnitude of (10) results in,

$$I_s{}^2 = \frac{I_r{}^2}{X_m{}^2}\left[(X_m + X_r)^2 + \frac{R_r{}^2}{s^2}\right] \tag{11}$$

Since $X_r \ll X_m$,

$$I_s{}^2 \approx I_r{}^2\left(1 + \frac{R_r{}^2}{X_m{}^2 s^2}\right) = I_r{}^2 + \frac{V_{th}{}^2}{X_m{}^2} = I_r{}^2 + I_m{}^2 \tag{12}$$

The change in stator power loss, $\Delta(I_s{}^2 R_s)$, is then

$$\Delta P_s = \Delta(I_s{}^2 R_s) = 2R_s\left(I_r \Delta I_r + \frac{V_{th}}{X_m}\frac{\Delta V_{th}}{X_{inc}}\right) \tag{13}$$

$I_r \Delta I_r$ is substituted from (2) and (6); further define X_{inc} as the incremental magnetizing reactance, given by $\Delta V_m/\Delta I_m$ (I_m is the magnetizing current and V_m is approximated here by V_{th}).

$$\Delta P_s = \Delta P = \frac{2R_s}{V_{th}}\left(-I_r{}^2 + \frac{V_{th}{}^2}{X_m}\frac{1}{X_{inc}}\right)\Delta V_{th} \tag{14}$$

With further manipulations involving (12), (14) may be written as,

$$\Delta P = \frac{2R_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right) - I_s{}^2\right]\Delta V_{th} \tag{15}$$

Equation (15) is more insightful because the values of I_s and R_s are easily measurable, while V_{th} is typically in the range of 0.95 – 0.98 of supply voltage, further simplified by assuming it equal to supply voltage. A knowledge of magnetizing characteristics gives X_m and X_{inc}. Typical values for industrial motors are found in [7]. Further, (15) tells us (a) the incremental power ΔP is small (multiplied by R_s), and (b) it could become negative for high values of load, where I_s goes up.

The reactive power change is more tangible, since it involves rotor, stator, and magnetizing components. The latter, being somewhere into saturation contributes generously to the total value of ΔQ. Starting with rotor reactive power loss,

$$Q_r = I_r{}^2 X_r \tag{16}$$

Then, using (2) and (6)

$$\Delta Q_r = 2I_r X_r \Delta I_r = -\frac{2I_r{}^2 X_r}{V_{th}}\Delta V_{th} \tag{17}$$

For the stator reactive power loss, we may write a similar expression to (14), replacing R_s with X_s,

$$\Delta Q_s = \frac{2X_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right) - I_s{}^2\right]\Delta V_{th} \tag{18}$$

Finally, magnetizing branch reactive power is,

$$Q_m \approx V_{th} I_m \tag{19}$$

Thus

$$\begin{aligned}\Delta Q_m &= V_{th}\Delta I_m + I_m \Delta V_{th} \\ &= V_{th}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right)\Delta V_{th}\end{aligned} \tag{20}$$

Adding everything together, $\Delta Q = \Delta Q_r + \Delta Q_s + \Delta Q_m$,

$$\begin{aligned}\Delta Q = \frac{2X_s}{V_{th}}\Bigg[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right) \\ - I_s{}^2\left(1 + \frac{I_r{}^2 X_r}{I_s{}^2 X_s}\right)\Bigg]\Delta V_{th} + V_{th}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right)\Delta V_{th}\end{aligned} \tag{21}$$

The reactive power term is dominated by the last term, which represents the reactive power change attributed to the magnetizing circuit. The first term (between square brackets) is smaller, being multiplied by X_s and adds a fraction in either direction, depending on the motor loading similar to the discussion for active power. If $\frac{I_r{}^2 X_r}{I_s{}^2 X_s}$ is taken as ≈ 1.0, (for the motors we simulated it was in the range 0.5 to 1.5) , we may reduce (21) to,

$$\Delta Q = \frac{2X_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_{inc}}+\frac{1}{X_m}\right)-2I_s{}^2\right]\Delta V_{th} + V_{th}\left(\frac{1}{X_m}+\frac{1}{X_{inc}}\right)\Delta V_{th} \tag{22}$$

For variable-torque motor loads, the fundamental equations are different. The torque equation (1) is modified to represent speed dependency as follows,

$$T = k\omega^2 = k(1-s)^2 = \frac{V_{th}{}^2 s}{R_r} \tag{23}$$

ω is the p.u. speed, and k is some constant.

For small increments around the operation point, then,

$$\Delta s = -\frac{2s(1-s)}{V_{th}\,(1+s)}\,\Delta V_{th} \tag{24}$$

And, using (5)

$$\Delta I_r = -\frac{s}{R_r}\left[\frac{1-3s}{(1+s)}\right]\Delta V_{th} \tag{25}$$

Equations (24) and (25) may then be used as the basis for obtaining active and reactive power changes similar to the derivations carried out for constant torque motors. The manipulations are not outlined here for reasons of space. Two expressions for active power change are presented; an elaborate (26) and a simplified (27) which is possible if the slip is reasonably small, and $\frac{I_r{}^2 R_r}{I_s{}^2 R_s} \approx 1.0$ (again this can be anywhere between 0.5 and 1.5).

$$\Delta P = \frac{2R_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_{inc}}+\frac{(1-3s)}{(1+s)X_m}\right) - \frac{I_s{}^2}{(1+s)}\left((1-3s)-\frac{2I_r{}^2 R_r}{I_s{}^2 R_s}\right)\right]\Delta V_{th} \tag{26}$$

$$\Delta P \approx \frac{2R_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_{inc}}+\frac{1}{X_m}\right)+I_s{}^2\right]\Delta V_{th} \tag{27}$$

The simplified expressions have the advantage of using stator quantities only, in addition to the slip. For reactive power changes we have,

$$\Delta Q = \frac{2X_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_{inc}}+\frac{(1-3s)}{(1+s)X_m}\right) - \frac{(1-3s)I_s{}^2}{(1+s)}\left(1+\frac{I_r{}^2 X_r}{I_s{}^2 X_s}\right)\right]\Delta V_{th} + V_{th}\left(\frac{1}{X_m}+\frac{1}{X_{inc}}\right)\Delta V_{th} \tag{28}$$

Further reduced by ignoring the slip and taking $I_r{}^2 X_r / I_s{}^2 X_s$ as 1.0.

$$\Delta Q \approx \frac{2X_s}{V_{th}}\left[\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_{inc}}+\frac{1}{X_m}\right)-2I_s{}^2\right]\Delta V_{th} + V_{th}\left(\frac{1}{X_m}+\frac{1}{X_{inc}}\right)\Delta V_{th} \tag{29}$$

Within an approximation, the reactive power change in variable-torque motors is the same as that of constant torque. The simplifications in (27) and (29) lead to a slight overestimation of power change, which is favorable for conservatively determining DG loading in power plants.

C. Development of expressions; Frequency sensitivity

Again, we start with constant torque motor loads. Since the synchronous frequency ω_s has changed from the nominal, there is a need to reflect this explicitly in the p.u. torque equation. The torque is then written as,

$$T = \frac{1}{\omega_s}\frac{V_{th}{}^2 s}{R_r} = \frac{V_{th}{}^2}{R_r}\left(\frac{\omega_s-\omega}{\omega_s{}^2}\right) \tag{30}$$

Then,

$$\Delta T = 0 = \frac{V_{th}{}^2}{R_r}\left[\frac{(2\omega-\omega_s)\Delta\omega_s}{\omega_s{}^3}-\frac{\Delta\omega}{\omega_s{}^2}\right] \tag{31}$$

Giving,

$$\Delta\omega = \frac{(2\omega-\omega_s)}{\omega_s}\Delta\omega_s = (1-2s)\Delta\omega_s \tag{32}$$

Now, since

$$s = \frac{\omega_s-\omega}{\omega_s} \tag{33}$$

Then

$$\Delta\boldsymbol{s} = \frac{\omega\Delta\omega_s-\omega_s\Delta\omega}{\omega_s{}^2} \tag{34}$$

Solving (32) with (34) to eliminate $\Delta\omega$ gives

$$\Delta s = \frac{s\Delta\omega_s}{\omega_s} \tag{35}$$

Constant V_{th} in (2) leads to

$$\Delta I_r = \frac{V_{th}}{R_r}\Delta\boldsymbol{s} \tag{36}$$

Equations (35) and (36) are the basic equations needed to determine active and reactive power changes for constant torque motor in response to a frequency change $\Delta\omega_s$. The mathematical development is omitted for reasons of space, and the final expressions are presented in (37), (38) for original and simplified active power change, and (39), (40) for corresponding reactive power change.

$$\Delta P = 2R_s\left[I_s{}^2-\frac{V_{th}{}^2}{X_m}\left(\frac{1}{X_m}+\frac{1}{X_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} + \frac{I_r{}^2 R_r}{s}\frac{\Delta\omega_s}{\omega_s} \tag{37}$$

$$\Delta P \approx \left[P + I_s{}^2 R_s - 2\frac{V_{th}{}^2 R_s}{X_m}\left(\frac{1}{X_m}+\frac{1}{X_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} \tag{38}$$

Note that if all the terms multiplied by R_s are ignored in (38), the percentage active power change reverts to original power multiplied by the percentage frequency change.

$$\Delta Q = \left[3I_r^2X_r + 3I_s^2X_s - \frac{V_{th}^2}{X_{inc}} - \frac{2V_{th}^2X_s}{X_m}\left(\frac{1}{X_m} + \frac{1}{X_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} \quad (39)$$

With $I_r^2X_r \approx I_s^2X_s$

$$\Delta Q \approx \left[6I_s^2X_s - V_{th}^2\left(\frac{1}{X_{inc}} + \frac{2X_s}{X_m^2} + \frac{2X_s}{X_mX_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} \quad (40)$$

For variable torque expressions, we need to take the derivative of (23), which gives

$$\Delta T = 2k\omega\Delta\omega = \frac{V_{th}^2}{R_r}\left[\frac{(2\omega - \omega_s)\Delta\omega_s}{\omega_s^3} - \frac{\Delta\omega}{\omega_s^2}\right] \quad (41)$$

Substituting for k will eventually lead to

$$\Delta\omega = \frac{(1-2s)(1-s)}{(1+s)}\Delta\omega_s \quad (42)$$

Solving (42) with (34) to eliminate $\Delta\omega$, we get,

$$\Delta s = 3s\left(\frac{1-s}{1+s}\right)\frac{\Delta\omega_s}{\omega_s} \quad (43)$$

The slip changes almost three times as the frequency, compared to changing as the frequency for constant torque. The original and simplified active and reactive power changes are given in (44) through (47).

$$\Delta P = 2R_s\left[3I_s^2 - \frac{V_{th}^2}{X_m}\left(\frac{3}{X_m} + \frac{1}{X_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} + \frac{3(1-s)}{(1+s)}\frac{I_r^2R_r}{s}\frac{\Delta\omega_s}{\omega_s} \quad (44)$$

$$\Delta P \approx \left[3P + 3I_s^2R_s - 2\frac{V_{th}^2R_s}{X_m}\left(\frac{3}{X_m} + \frac{1}{X_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} \quad (45)$$

Note that if we ignore terms multiplied by R_s, the percentage active power change is three multiples of the percentage frequency change.

$$\Delta Q = 7I_r^2X_r\frac{\Delta\omega_s}{\omega_s} + 7I_s^2X_s\frac{\Delta\omega_s}{\omega_s} - \frac{V_{th}^2}{X_{inc}}\frac{\Delta\omega_s}{\omega_s} - \frac{2V_{th}^2X_s}{X_m}\left(\frac{3}{X_m} + \frac{1}{X_{inc}}\right)\frac{\Delta\omega_s}{\omega_s} \quad (46)$$

And, with $I_r^2X_r/I_s^2X_s \approx 1.0$

$$\Delta Q \approx \left[14I_s^2X_s - V_{th}^2\left(\frac{1}{X_{inc}} + \frac{6X_s}{X_m^2} \frac{2X_s}{X_mX_{inc}}\right)\right]\frac{\Delta\omega_s}{\omega_s} \quad (47)$$

III. Comparisons of Approximate Formulae with Simulations

Five types of motors are studied, corresponding to the following categories; (1) Small industrial, (2) Large industrial, (3) Power plant auxiliary, (4) Weighted aggregate of residential and industrial motors, (5) Weighted aggregate of motors dominated by air conditioning. The parameters for each type were taken from the IEEE Task Force on Load Representation for Dynamic Performance [7]. Saturation modeling follows the guidelines mentioned in the same reference.

Table I shows a comparison between simulation and calculations for voltage changes. The calculations refer to (26) - (29) for variable torque motors, which define the first four categories. The fifth category is a constant torque motor, so comparisons were made against (14), (15), (21) and (22). Simulation was performed in MATLAB, using the asynchronous machine block (care must be exercised to set the solver for high accuracy). Active and reactive power sensitivities are calculated for a voltage increase of 5% above nominal.

Table II shows the same comparison for frequency changes where the frequency is changed from 60 Hz to 60.2, a change of 0.33%. The calculations refer to (44) - (47) for variable torque motors, and (37) - (40) for the constant torque motor.

Table I
SIMULATED AND CALCULATED ACTIVE & REACTIVE POWER VOLTAGE SENSITIVITIES (ORIGINAL & SIMPLIED EXPRESSIONS)

Motor Category	$(\Delta P/P)/(\Delta V/V)$			$(\Delta Q/Q)/(\Delta V/V)$		
	Sim.	Calc. Orig.	Calc. Simple	Sim.	Calc. Orig.	Calc. Simple
(1)	0.040	0.032	0.092	2.797	2.508	2.690
(2)	0.017	0.013	0.035	2.034	1.612	1.947
(3)	0.028	0.025	0.054	2.847	2.636	2.697
(4)	0.122	0.119	0.121	3.171	2.912	3.037
(5)	-0.042	0.073	0.088	2.899	3.149	3.192

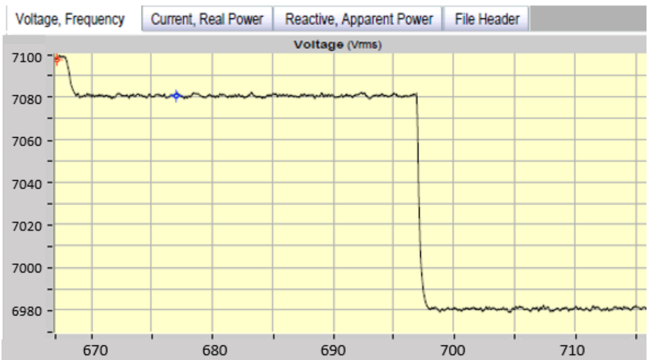
Table II
SIMULATED AND CALCULATED ACTIVE & REACTIVE POWER FREQUENCY SENSITIVITIES (ORIGINAL & SIMPLIED EXPRESSIONS)

Motor Category	$(\Delta P/P)/(\Delta f/f)$			$(\Delta Q/Q)/(\Delta f/f)$		
	Sim.	Calc. Orig.	Calc. Simple	Sim.	Calc. Orig.	Calc. Simple
(1)	2.980	2.970	3.041	1.174	1.189	0.852
(2)	2.992	2.984	3.031	2.226	2.141	1.106
(3)	2.984	2.976	3.018	1.244	1.283	1.711
(4)	2.851	2.853	3.032	0.651	0.698	0.539
(5)	1.039	1.020	1.015	-0.043	-0.071	0.057

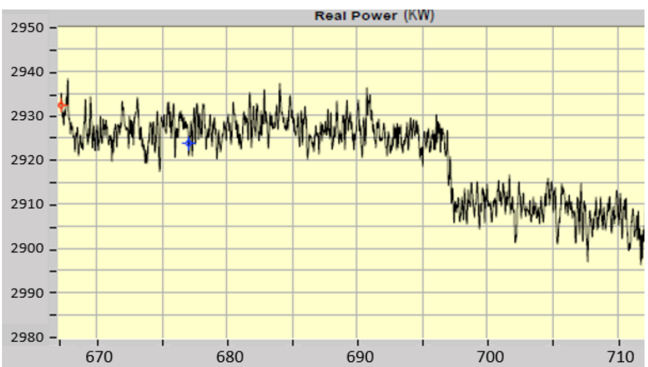
It is generally observed that active power response for voltage changes was negligible for the motors investigated, while the reactive power response was strong and in the positive direction. The reactive power voltage sensitivity tends to be greater than that of theoretical static reactive loads, and approaches 2.0 - 3.0 principally due to saturation. Equations (22) and (29) tell us that lower loads would increase this sensitivity factor further. Conversely, the active power sensitivity to frequency is stronger than that of reactive power, at around 3.0 for variable torque motors, and 1.0 for constant torque. Reactive power sensitivities to frequency are smaller, and can go negative on reduced loading, as (40) and (47) imply.

IV. A VOLTAGE VARIATION EXPERIMENT

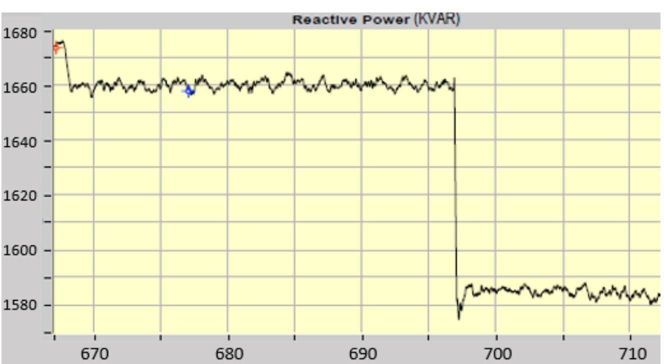
In Oct 2018 TVA took measurements in a LOCA (loss-of-coolant accident) test involving reducing the voltage of the diesel generator by 1.4%. Total load was 95% motor. Diesel gen. was in isochronous mode (isolated) and most of the accident loads were carried. Frequency was steady. Voltage, active and reactive power waveforms are shown against time in Fig 2.0.



(a)



(b)



(c)

Fig. 2.0 Diesel generator Voltage (a), Active power (b), and Reactive power (c) for a change in voltage by -1.4% at a TVA Nuclear Power Plant.

The calculated sensitivities were $(\Delta P/P)/(\Delta V/V)$ = 0.6, and $(\Delta Q/Q)/(\Delta V/V)$ = 3.4. Proceeding from active and reactive power sensitivities, it is an easy matter to calculate apparent power (kVA) sensitivities using $S^2 = P^2 + Q^2$, which lead to,

$$\Delta S = \frac{\mathrm{P}}{\mathrm{S}}\Delta P + \frac{\mathrm{Q}}{\mathrm{S}}\Delta Q \qquad (48)$$

Hence

$$\frac{\Delta S}{S} / \frac{\Delta V}{V} = \frac{\mathrm{P}^2}{\mathrm{S}^2}\left(\frac{\Delta P}{P} / \frac{\Delta V}{V}\right) + \frac{\mathrm{Q}^2}{\mathrm{S}^2}\left(\frac{\Delta Q}{Q} / \frac{\Delta V}{V}\right) \qquad (49)$$

The sensitivities are weighted by the square of the active and reactive power factors to give overall kVA sensitivity. In the case of the above described test, a 1% change in voltage leads to a 1.03% change in total generator kVA.

The test supports the general premise that power/voltage sensitivity is small and reactive power sensitivity to voltage is greater, 2.0 – 3.0 or more. One explanation for why the observed sensitivities are slightly higher than those in the Table I is that the actual loading for many large motors was lower than the typical loadings taken from [7].

V. CONCLUSIONS

Expressions that estimate the change in active and reactive power for industrial motors in response to voltage and frequency variations around the operating point have been developed. Simplification of the expressions allowing use of stator quantities and measurements were attempted. Saturation was accounted for by using incremental magnetizing reactances. The expressions were validated using simulation which shows that they provide fair approximations.

The expressions may be useful in the context of the power plant industry for determining compliance of emergency diesel generator tolerance to voltage and frequency changes. A test to validate the theoretical derivations was done in a TVA nuclear power plant. The test shows fair confirmation of theory.

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