



# Exploring Power System State Estimation Divergence via Synthetic Measurement Data Creation

- Efforts Led by IEEE Task Force on Standard Test Cases for Power System State Estimation

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# Collaborators and Sponsors of Presented Work

## Collaborators (sorted by their last names)

- Shahrokh Akhlaghi (Ulteig Engineers Inc.)
- Renke Huang, Zhenyu Huang, Shaobu Wang, et al. (PNNL)
- Yuzhang Lin (New York University)
- Da Meng, Shuai Lu (EnerMod)
- Greg Welch (University of Central Florida)
- Junbo Zhao (Mississippi State University)

## Sponsors (sorted by their names)

- DOE Advanced Grid Modeling program
- NSF CAREER grant no. #1845523

# Outlines

- **Background**
- Creation of Challenging Cases to SSE
- Challenges from Time-Skew Problems
- Summary of Findings and Future Work

# SCADA/EMS Systems

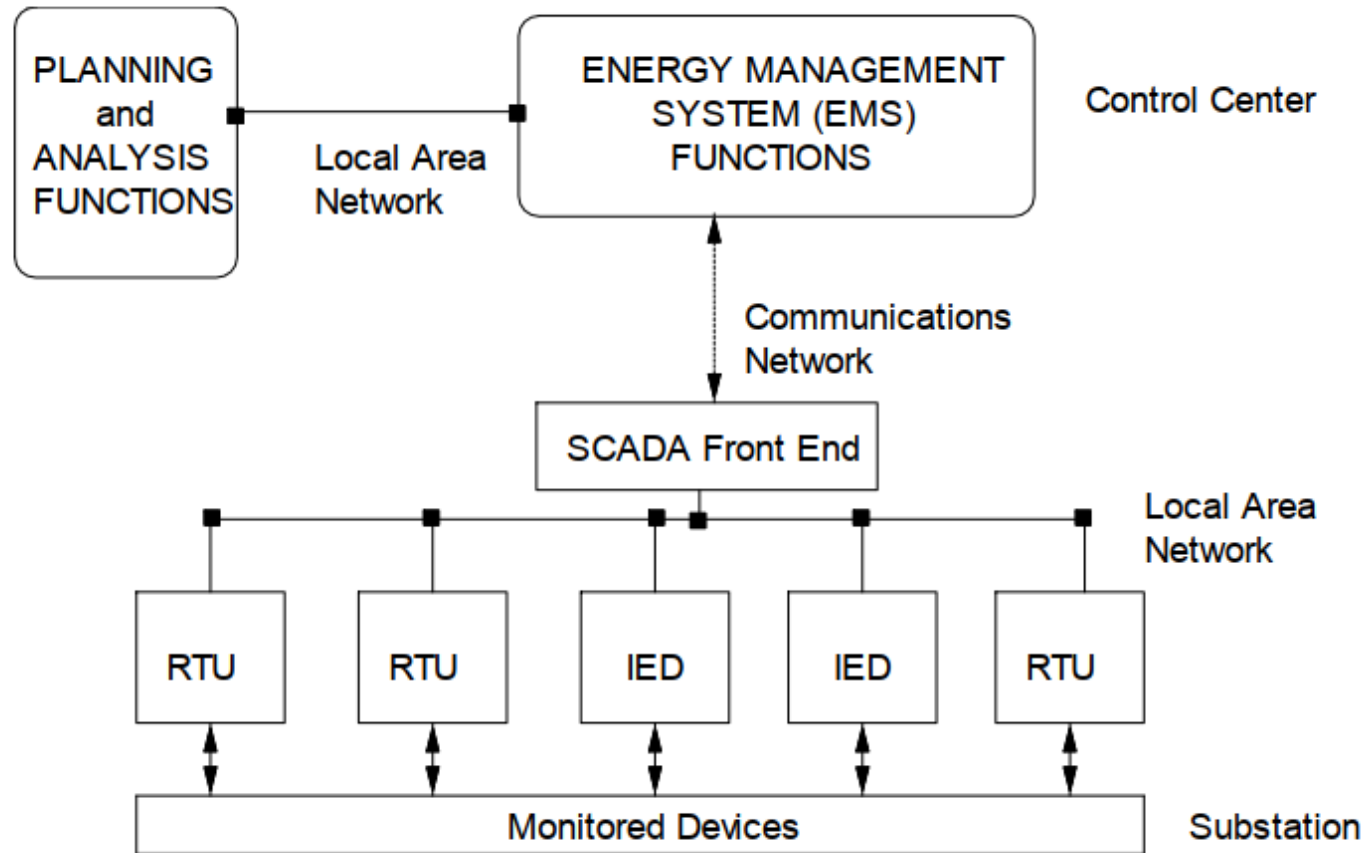
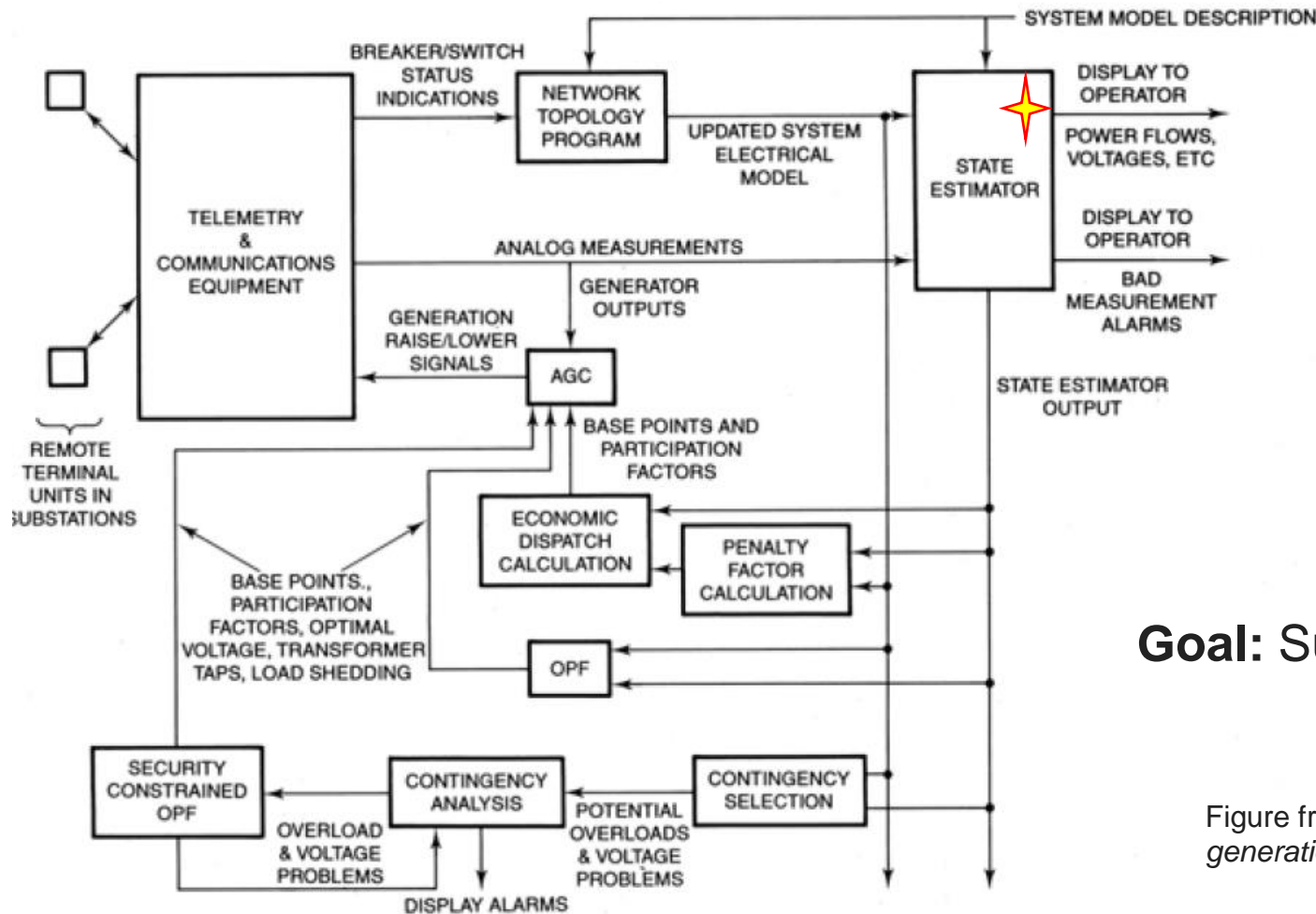


Figure from Abur, Ali, and Antonio Gomez Exposito. *Power system state estimation: theory and implementation*. CRC press, 2004.



# Critical Role of State Estimation in Power System Operations



## Inputs:

- Measurement Data
- Power Flow Models

## Outputs:

- Estimated States
- Improved Models

**Goal:** Support well-informed decision making.

Figure from Wood, Allen J., Bruce F. Wollenberg, and Gerald B. Sheblé. *Power generation, operation, and control*. John Wiley & Sons, 2013.

# Problem Statement

## Functions of a State Estimator:

- Monitors operating conditions (variables) of a power grid in a control center
- Supports real-time operations (e.g., ED, OPF)

## Challenges:

- Noise and even bad data in measurements (Inaccuracy)
- Limited number of direct measurements (limited scope)

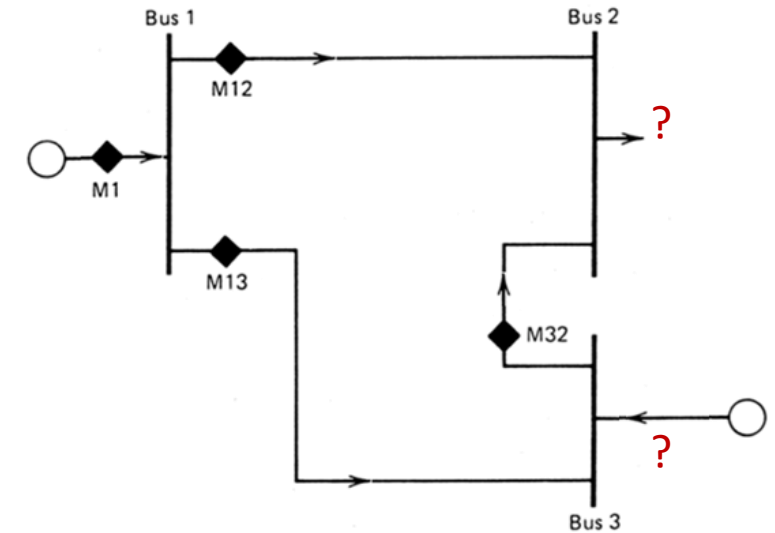


Figure from Wood, Allen J., Bruce F. Wollenberg, and Gerald B. Sheblé. *Power generation, operation, and control*. John Wiley & Sons, 2013.

# Classical Solutions: SSE

## Static State Estimation (SSE)

- Estimate  $\mathbf{x}$ , bus voltage phasors
- By integrating
  - SCADA/PMU measurements:  $z$
  - Power flow models  $z = h(\mathbf{x}) + r$
- To
  - Filter out noise using spatial redundancy
  - Estimate variables that are not measured

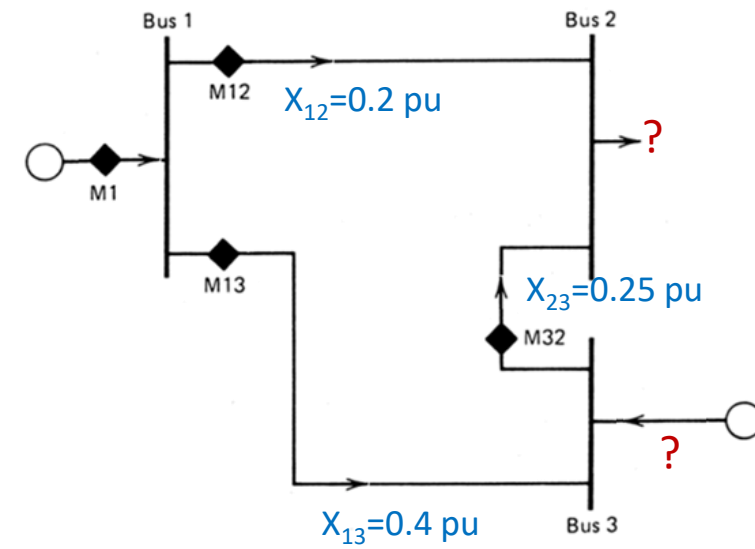


Figure from Wood, Allen J., Bruce F. Wollenberg, and Gerald B. Sheblé. *Power generation, operation, and control*. John Wiley & Sons, 2013.

## Example Algorithms

- Weighted least squares (WLS)
- Least absolute values (LAV)

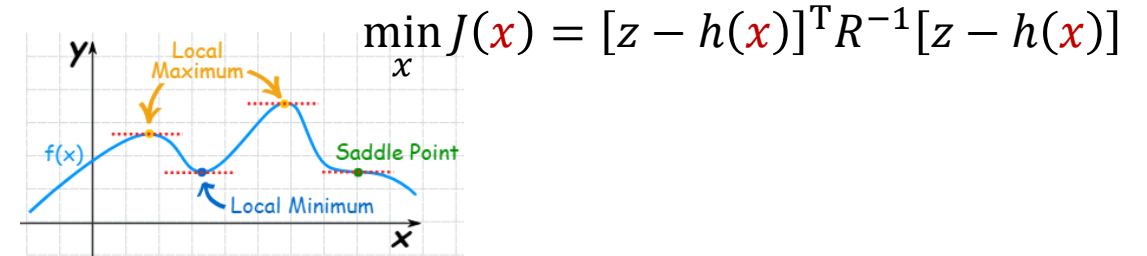
$$\min_x J(\mathbf{x}) = [z - h(\mathbf{x})]^T R^{-1} [z - h(\mathbf{x})]$$

$$\min_x J(\mathbf{x}) = \sum_{i=1}^m \frac{1}{\sqrt{R_{ii}}} |z_i - h_i(\mathbf{x})|$$

# Challenges to Conventional SSE

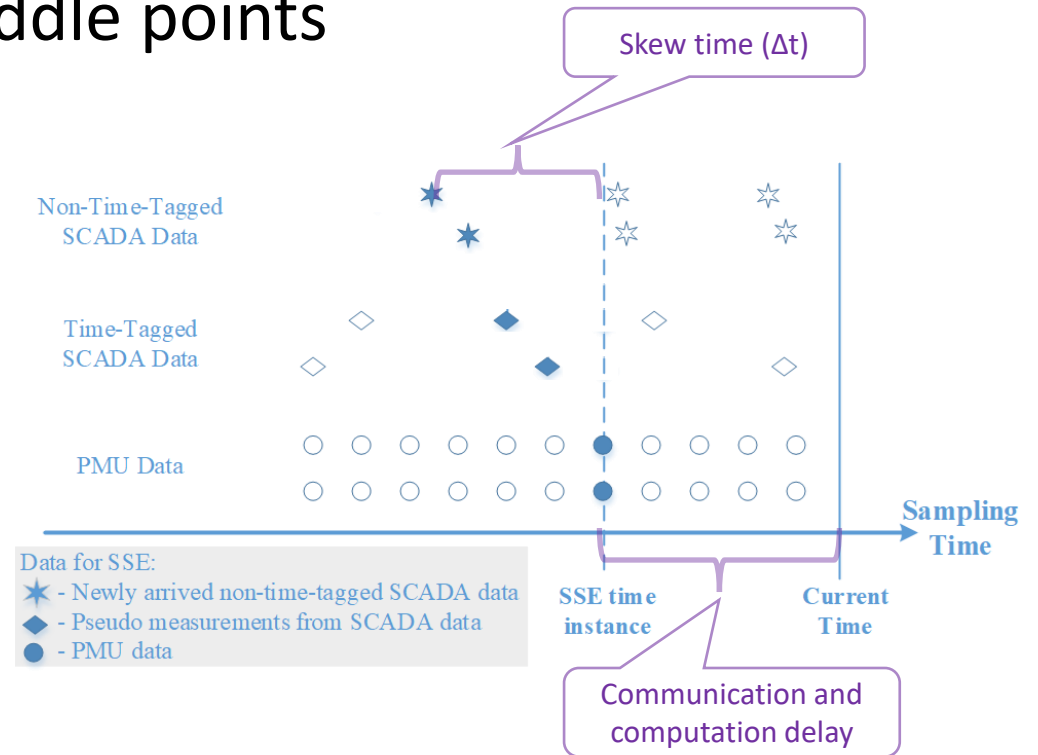
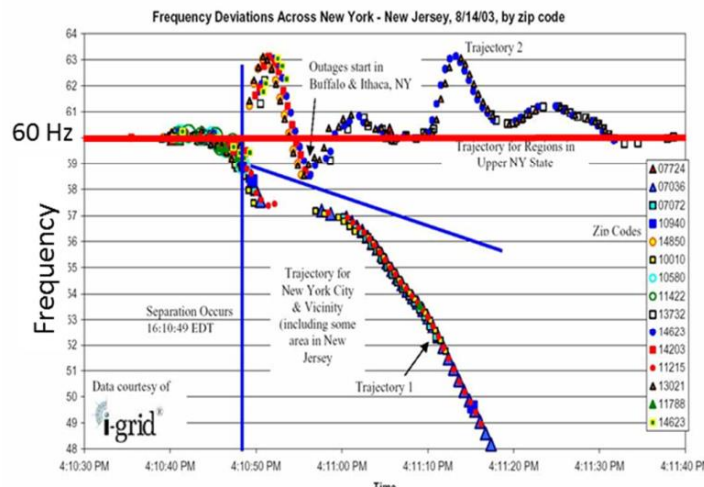
Divergence caused by

- time-skewed measurements,
- local minimums, maximums, and saddle points



$$\min J(x) = [z - h(x)]^T R^{-1} [z - h(x)]$$

Significant frequency deviations from the nominal 60 Hz during the August 14, 2003 Northeast Blackout [1]



[1] Z. Huang, N. Zhou, R. Diao, S. Wang, S. Elbert, D. Meng and S. Lu, "Capturing real-time power system dynamics: Opportunities and challenges," in 2015 IEEE Power & Energy Society General Meeting, Denver, CO, USA, 2015.



# Normal Equations Solutions

## Objective function

$$x_{MLE} = \arg \min_x \{J(x) \triangleq [z - h(\mathbf{x})]^T R^{-1} [z - h(\mathbf{x})]\}$$

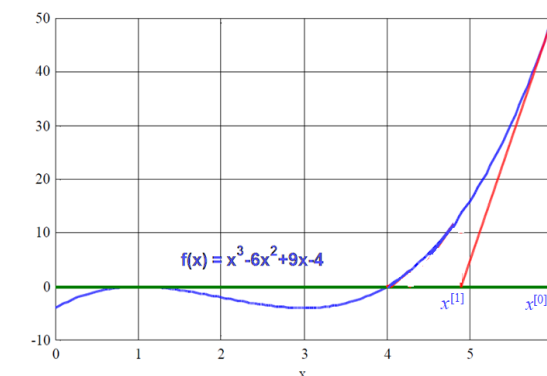
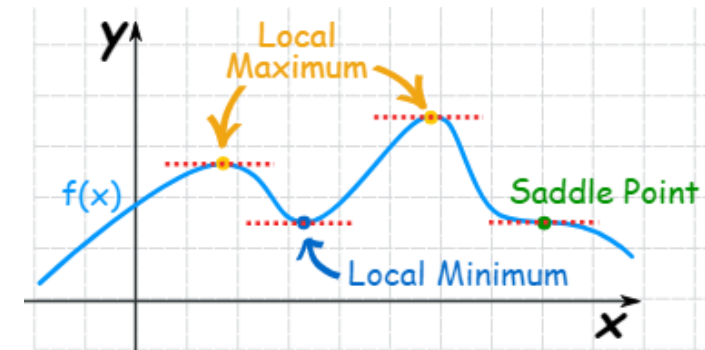
## Necessary conditions

$$g(x) = \frac{\partial J(x)}{\partial x} \rightarrow 0$$

## Normal Equation from Newton Method

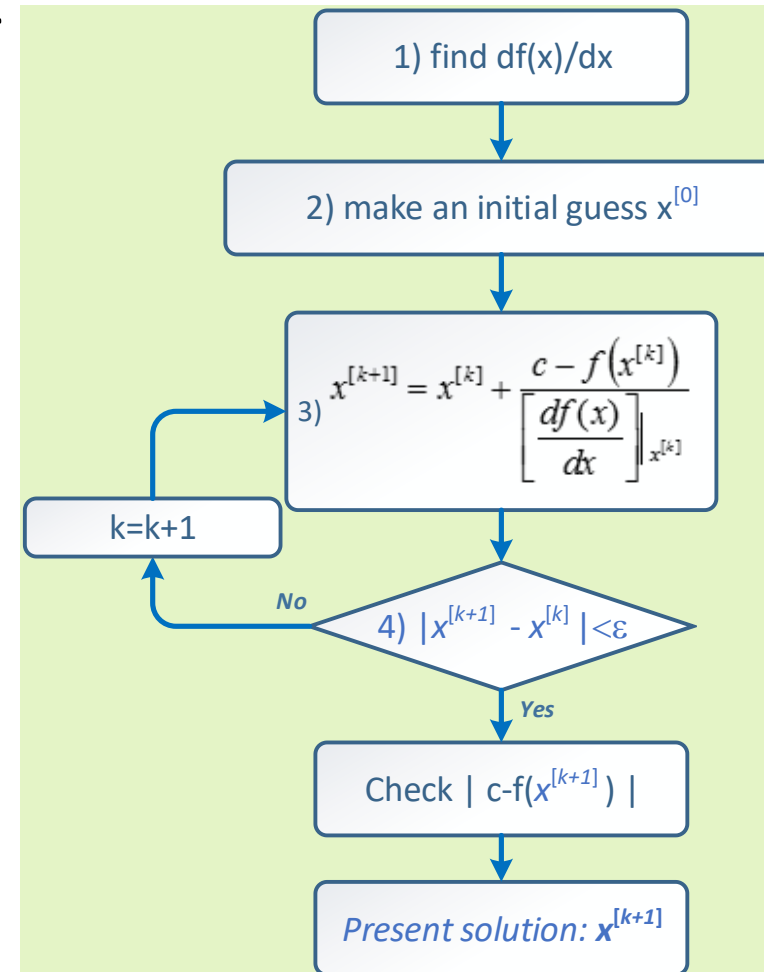
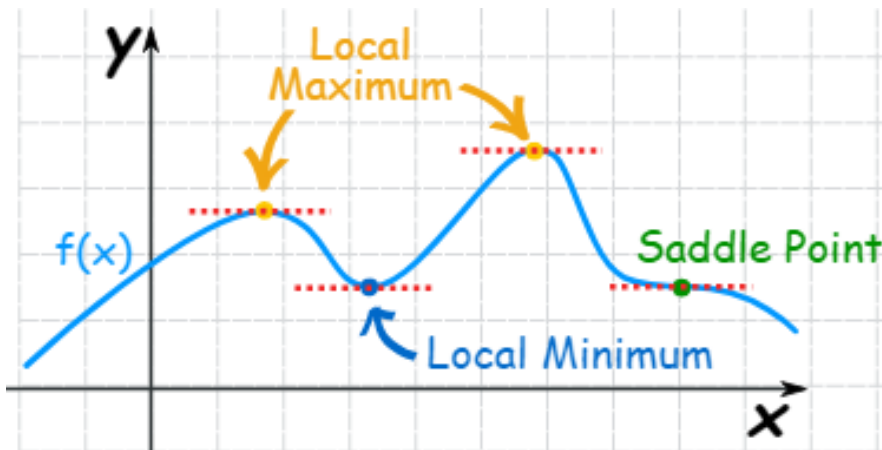
$$\begin{aligned} \Delta x^k &= \left[ \frac{\partial g(x)}{\partial x} \right]^{-1} [0 - g(x)] \\ &\approx [H^T(x^k) R^{-1} H(x^k)]^{-1} H^T(x^k) R^{-1} (z - h(x^k)) \end{aligned}$$

$$\frac{\partial g(x)}{\partial x} = H^T(x^k) R^{-1} H(x^k) + 2^{nd} \text{ order derivative}$$



# Divergence

Converge to local instead of global min  
Increased  $J(x)$

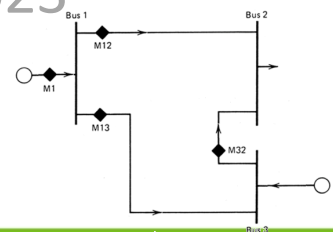


# Outlines

- Background
- **Creation of Challenging Cases to SSE**
- Challenges from Time-Skew Problems
- Summary of Findings and Future Work

# Factors that may Trigger Divergence of SSE

- (1) **Initial values of the states** far away from the true value [Weng 2012][Gou 2023]
- (2) Presence of numerous **bus power injection measurements** tends to make the normal equations method increasingly unstable. [Gu 1983]
- (3) **An injection flow measurement** associated with a large number of branches [Gou 2023]
- (4) **Very heavy weights** for modeling very accurate virtual measurements (such as zero injections) [Abur 2004 ][Gou 2023]
- (5) **A long line and a short line** at the same bus, i.e., lines with significantly different impedances [Abood 2017] [Tripathy 1982][Gou 2023]
- (6) **Bad data** that cannot be effectively rejected by the WLS algorithm [Gou 2023]



# A. Divergent Case through Changing Operational Points

## Base case:

- ❑ IEEE 57-bus model with low redundancy measurements

## Approach:

- ❑ Change the true voltage magnitudes of bus 36:38 by increasing them to 1.3\*

## Divergent cases:

- ❑ B0057s\_033.mat
- ❑ B0057s\_data\_033.csv
- ❑ B0057s\_header\_033.csv
- ❑ state\_zn33\_divergent.mat (stores true states)

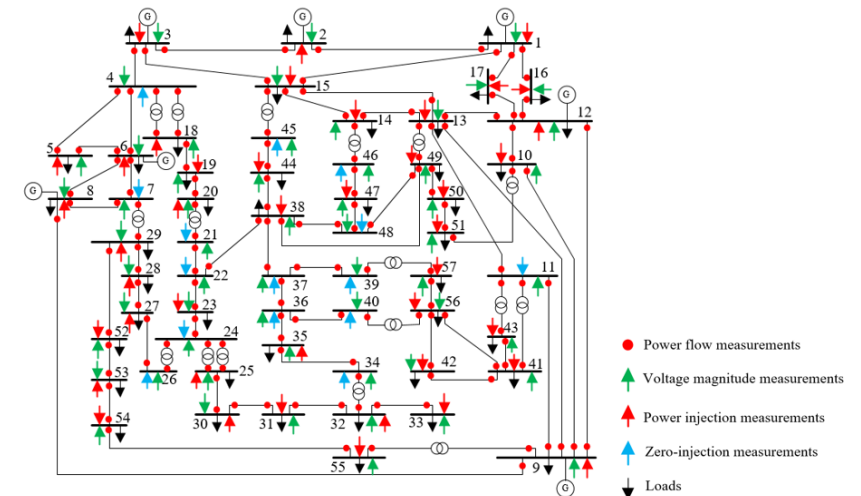
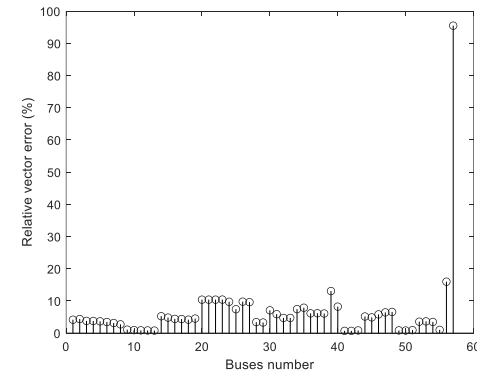
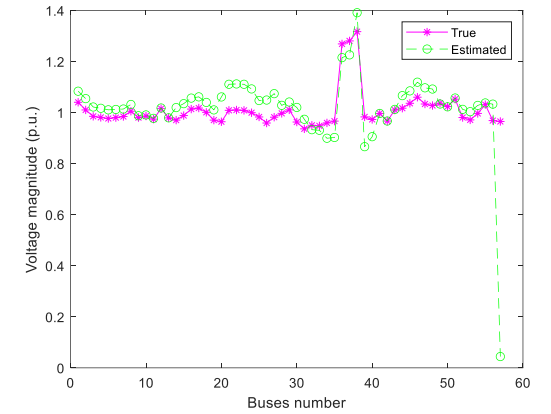
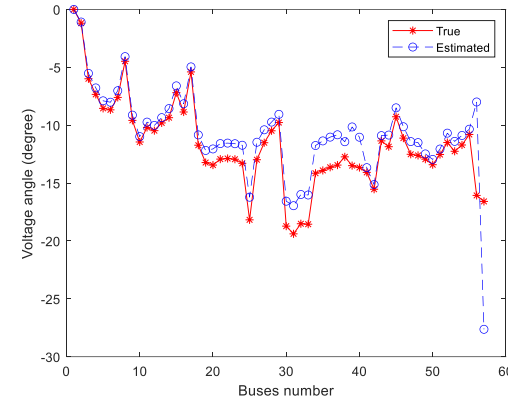
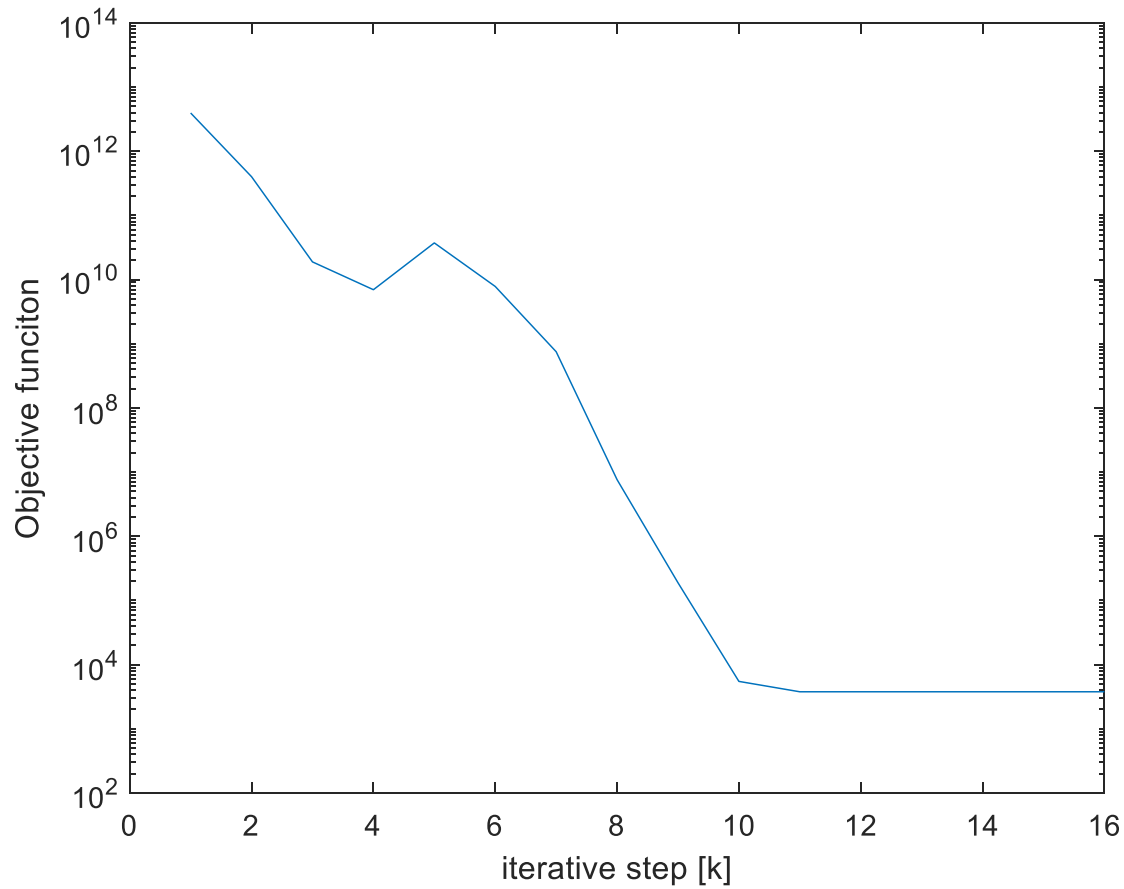


Fig. 1. IEEE 57-bus test system measured by pure SCADA measurements.



# Case A. WLS State Estimation Results



- Converged to a local minimum point

# B. Divergent Case through Changing Operational Points and Using Only Bus Injection Measurements

## Base case:

- IEEE 57-bus model with high redundancy measurements

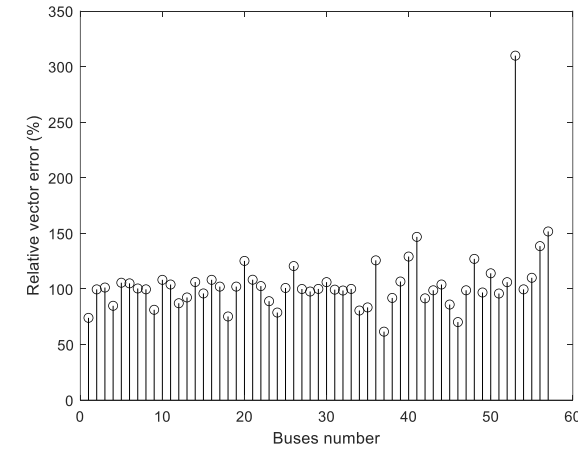
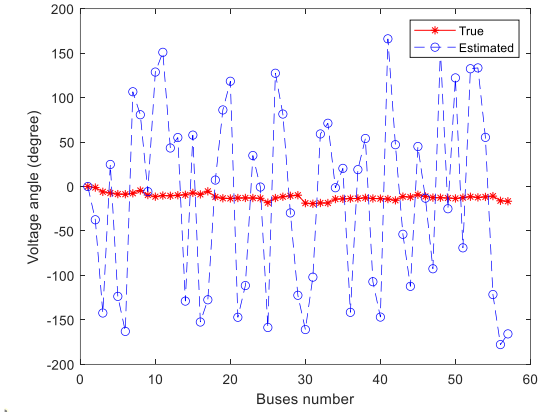
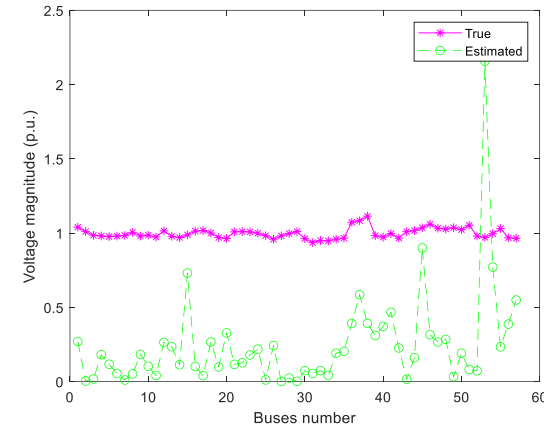
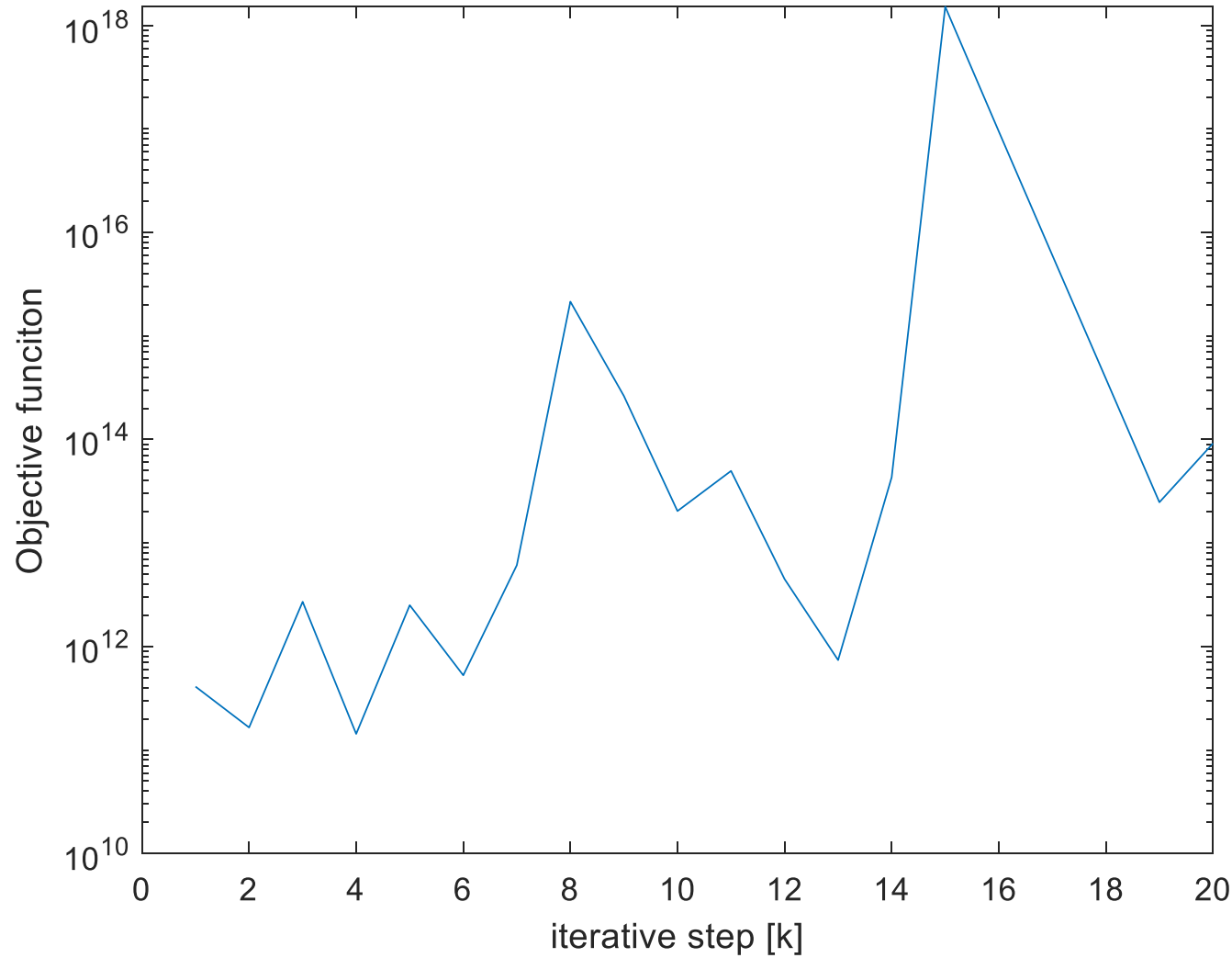
## Approach:

- Change the true voltage magnitudes of bus 36:38 by increasing them to 1.1\*
- Change the measurements by including only bus power injection.

## Divergent cases:

- B0057s\_022.mat
- B0057s\_data\_022.csv
- B0057s\_header\_022.csv
- state\_zn22\_divergent.mat (stores true states)

# Case B. WLS State Estimation Results



• Diverged

# Factors that may trigger divergence of SSE

- (1) **Initial values of the states** far away from the true value [Weng 2012][Gou 2023]
- (2) Presence of numerous **bus power injection measurements** tends to make the normal equations method increasingly unstable. [Gu 1983]
- (3) **An injection flow measurement** associated with a large number of branches [Gou 2023]
- (4) **Very heavy weights** for modeling very accurate virtual measurements (such as zero injections) [Abur 2004 ][Gou 2023]
- (5) **A long line and a short line** at the same bus, i.e., lines with significantly different impedances [Abood 2017] [Tripathy 1982][Gou 2023]
- (6) **Bad data** that cannot be effectively rejected by the WLS algorithm [Gou 2023]

# C. Divergent Cases by Putting Very Heavy Weights on the Virtual Measurement of Net Zero Injection

## Base case:

- IEEE 57-bus model with low redundancy measurements

## Approach:

- Change the weights on the **15\*2 net zero injections** ( $P_i, Q_i$ ) by decreasing their standard deviations  $m$  from  $1e-5$  to  **$1e-11$**

## Setup: 491 measurements

- 57 Vmag
- 57  $P_i$ , 57  $Q_i$  160  $P_{ij}$ , 160  $Q_{ij}$

## Divergent cases:

- B0057s\_044.mat
- B0057s\_data\_044.csv
- B0057s\_header\_044.csv
- state\_zn44\_divergent.mat (stores true states)

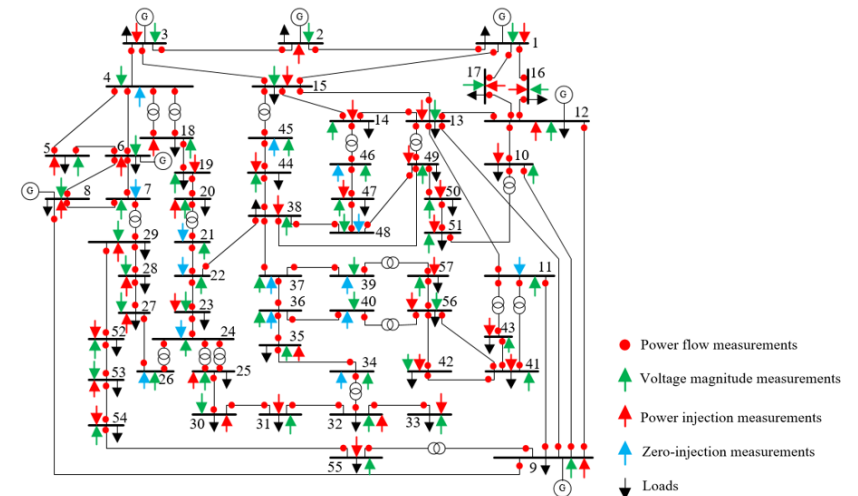
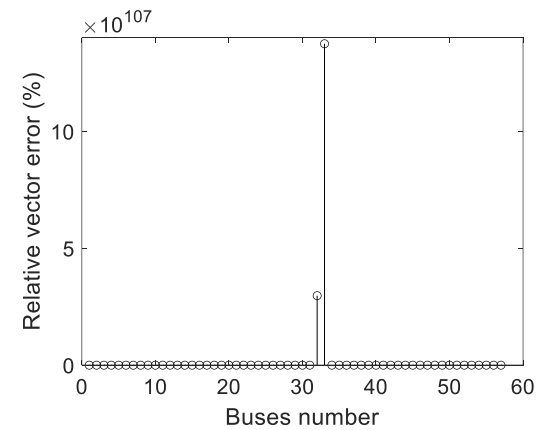
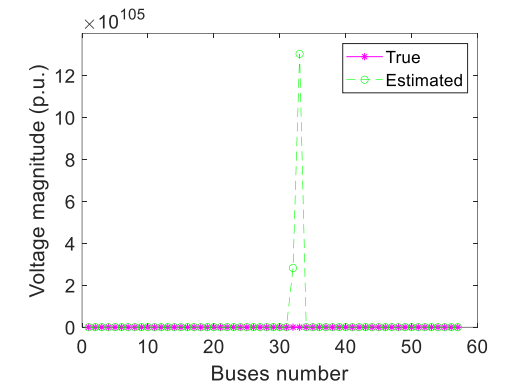
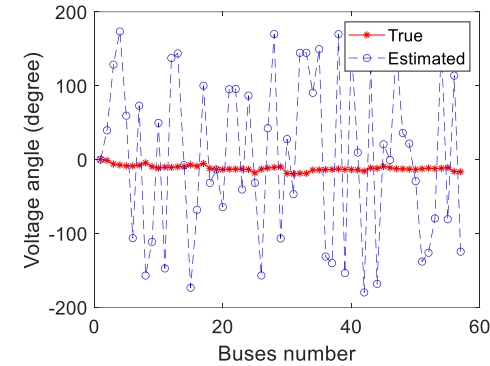
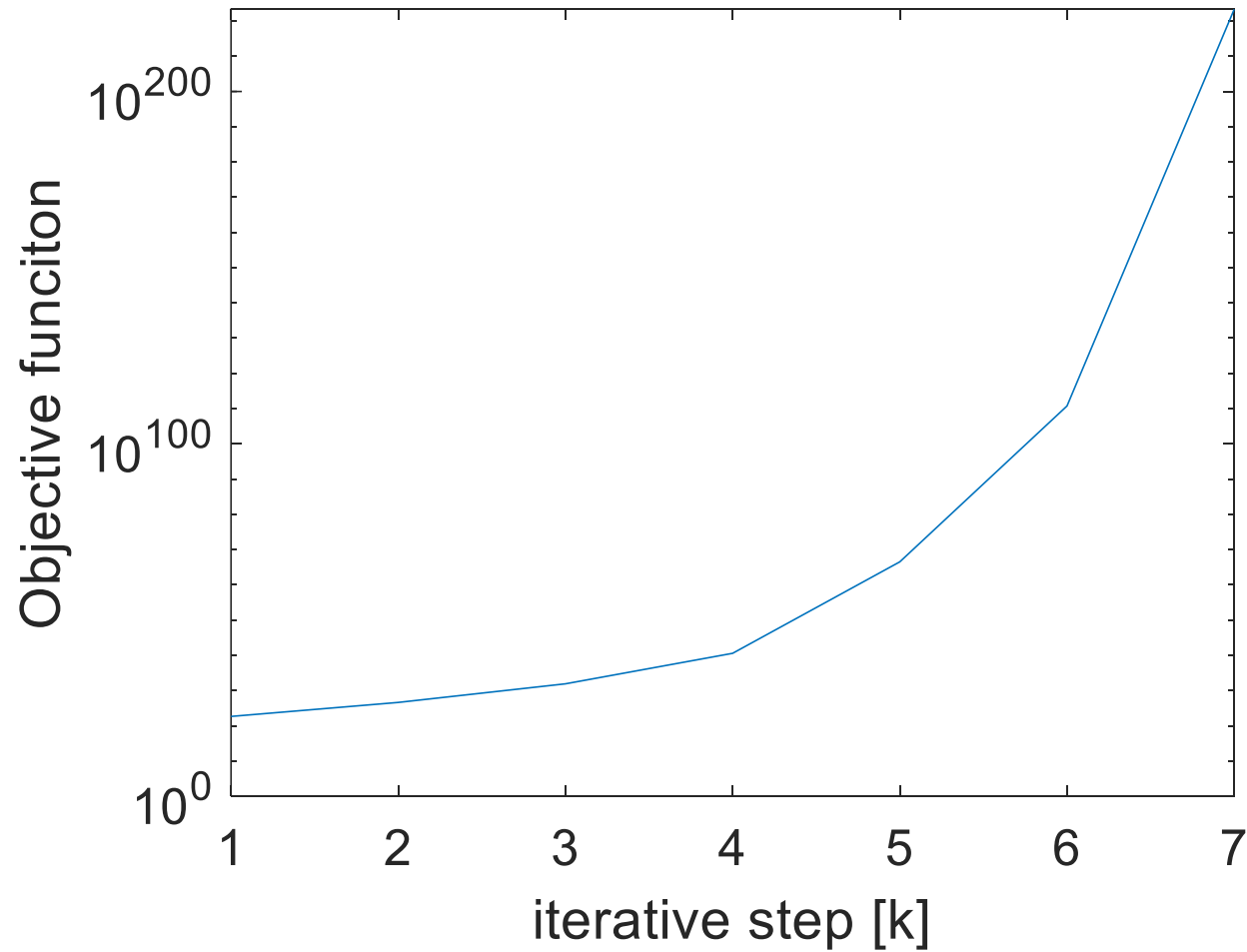


Fig. 1. IEEE 57-bus test system measured by pure SCADA measurements.



# Case C. WLS State Estimation Results



- Diverged

# D. Divergent Case through Shortening a Transmission Line

## Base case:

- ❑ IEEE 57-bus model with low redundancy measurements

## Approach:

- ❑ Make the shortest line (line 33) shorter by changing its impedance to  $1e-16$ \*original impedance.

## Setup: 171 measurements

- ❑ 57 Vmag
- ❑ 57  $P_i$ , 57  $Q_i$

## Divergent cases:

- ❑ B0057s\_055.mat
- ❑ B0057s\_data\_055.csv
- ❑ B0057s\_header\_055.csv
- ❑ state\_zn55\_divergent.mat (stores true states)

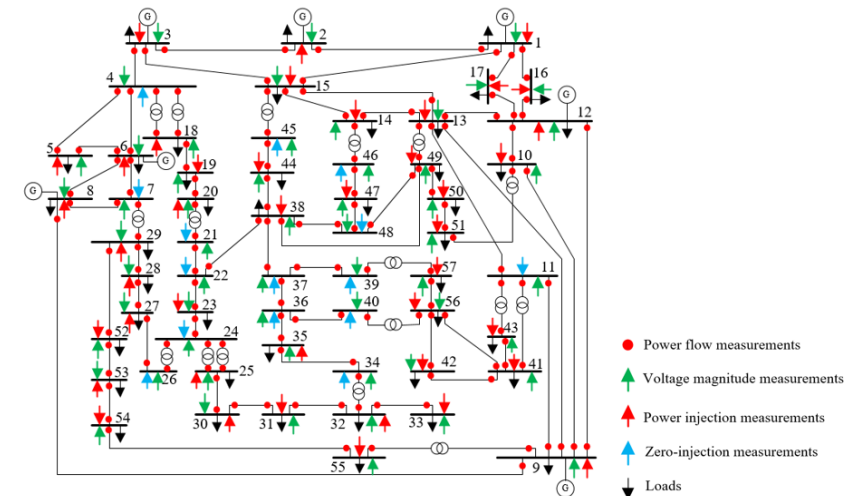
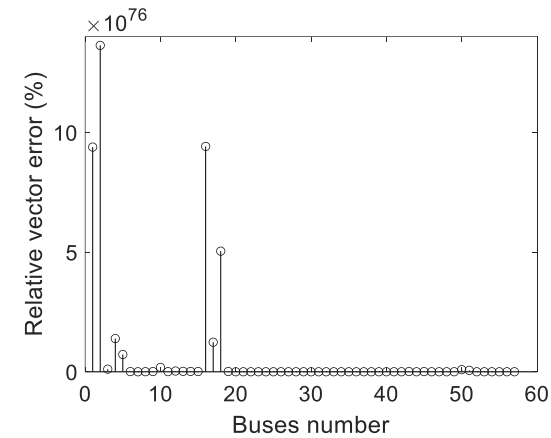
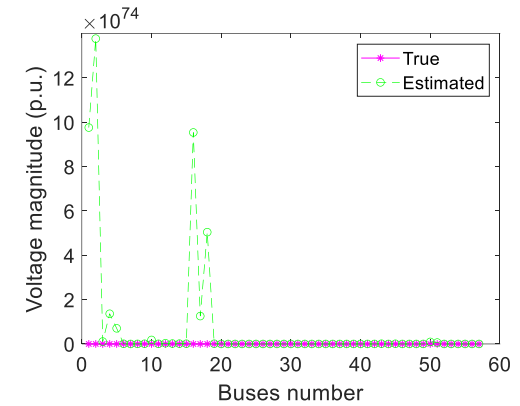
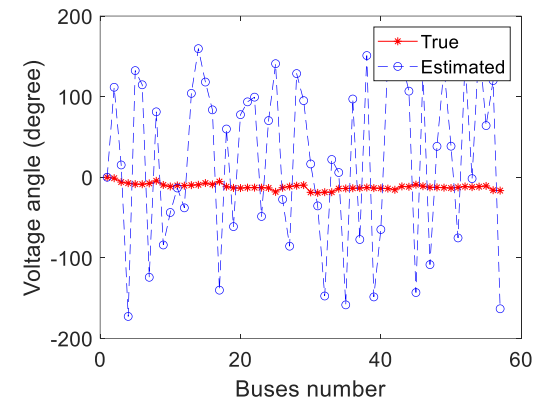
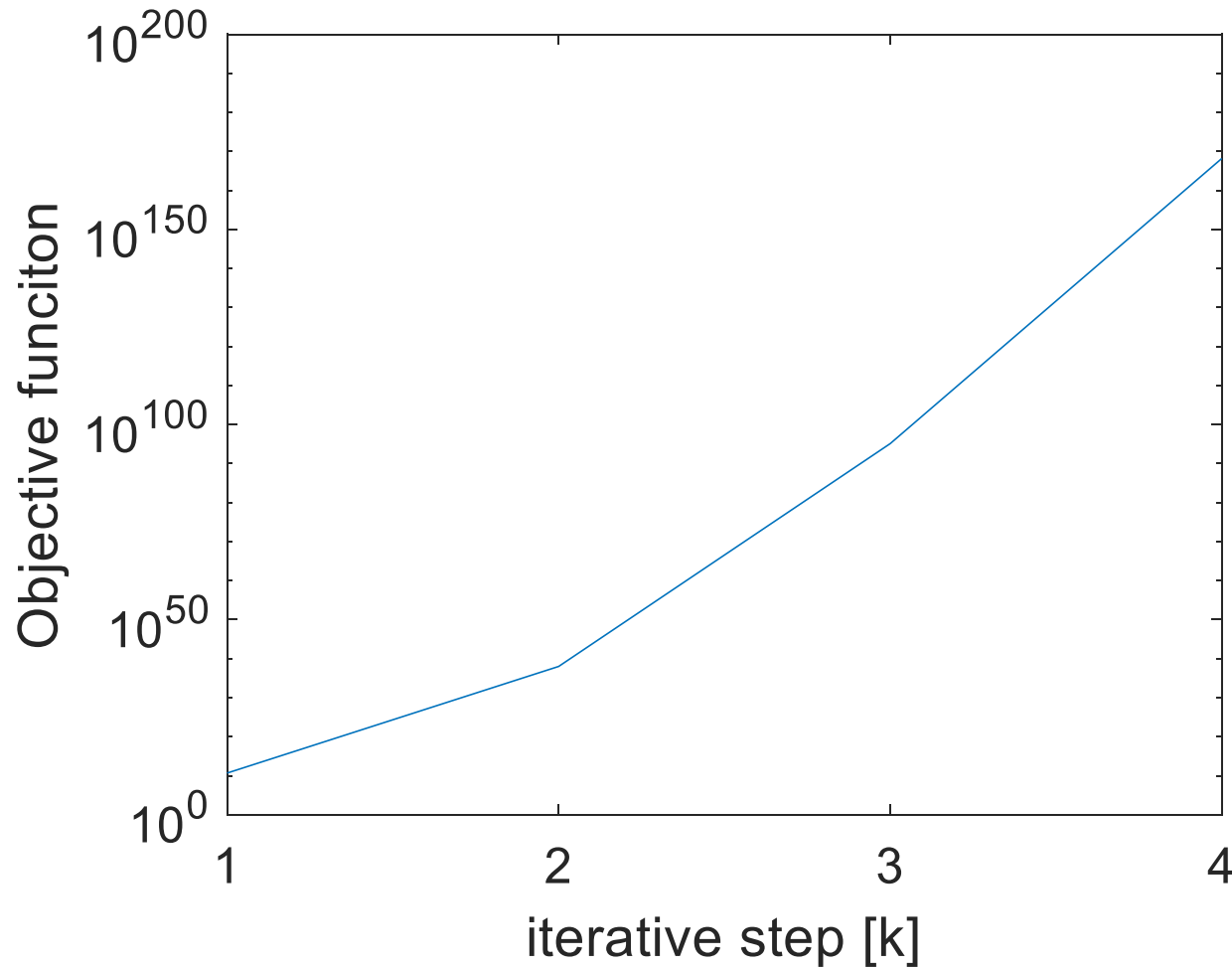


Fig. 1. IEEE 57-bus test system measured by pure SCADA measurements.

# Case D. WLS State Estimation Results



- Diverged

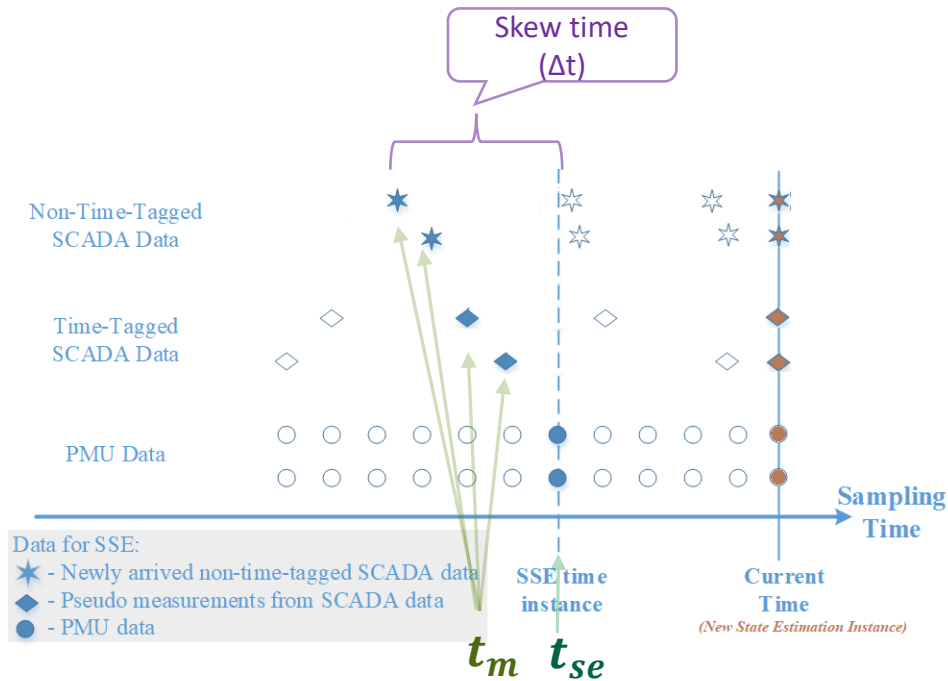
# Outlines

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# Problem Definition of SSE and its Time-Skew Problem

- **Ideal SSE:** Assume a snapshot of measurements at  $t_{se}$

$$\hat{x}_{t_{se}} = \mathop{\text{arg min}}_{x_{t_{se}}} \left\{ \sum_{m=1}^M \frac{(z_{m,t_{se}} - h_{m,t_{se}}(x_{t_{se}}))^2}{R_{mm}} \right\}$$



- **Actual SSE:**  $\hat{z}_{m,t_{se}} = z_{m,t_m}$

$$\hat{x}_{t_{se},\Delta t} = \mathop{\text{arg min}}_{x_{t_{se}}} \left\{ \sum_{m=1}^M \frac{(z_{m,t_m} - h_{m,t_{se}}(x_{t_{se}}))^2}{R_{mm}} \right\}$$

$$t_m \in [\min(t_m), \min(t_m) + \Delta t]$$

- Skew time:

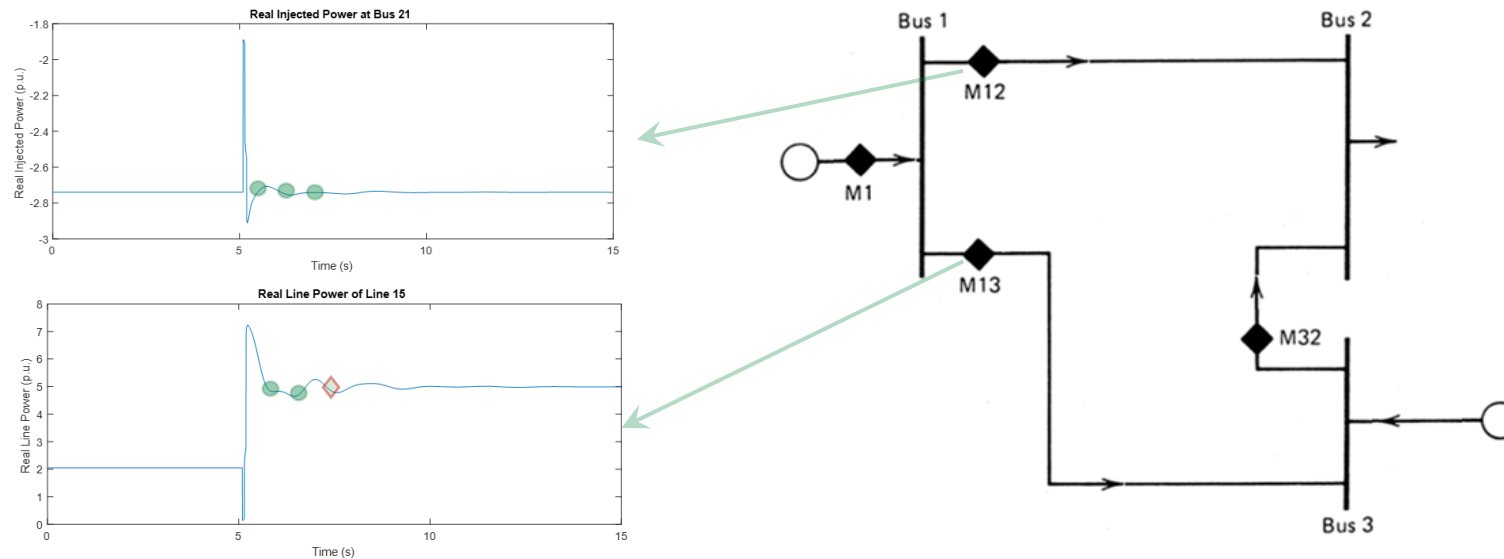
$$\Delta t = t_{se} - \min_{m \in [1,M]}(t_m)$$

- Time skew errors:

$$\Delta z_{m,\Delta t} = z_{m,t_m} - z_{m,t_{se}}$$



# Constructing Forecasting Models based on Spatial and Temporal Correlation



$$\hat{z}_{m,t_{se}} = \sum_{i=1}^M \sum_{k_i=0}^{K_i} z_{i,t_i-k_i} \beta_{i,t_i-k_i} + \beta_{m,0}$$

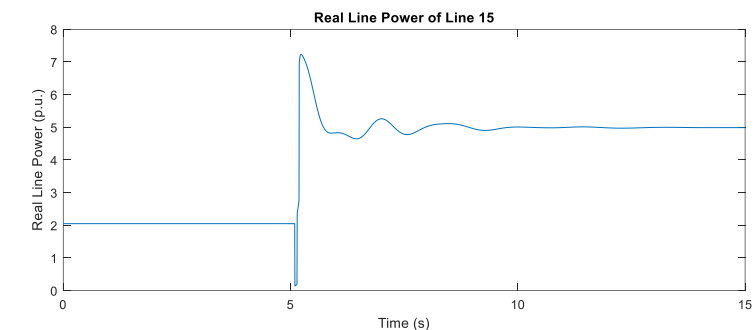
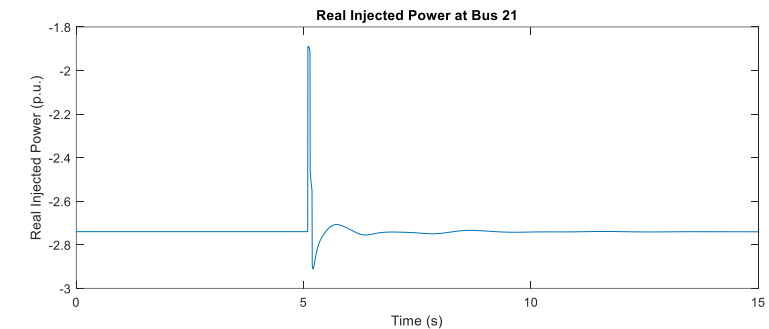
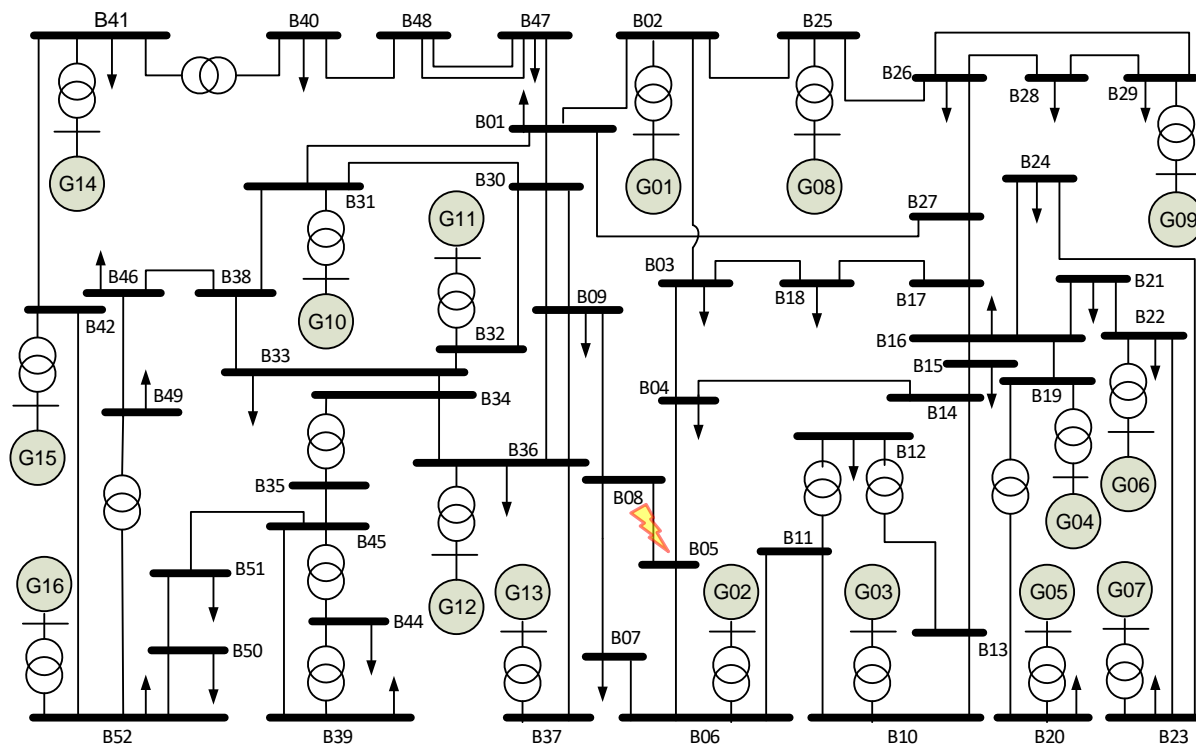
$$\hat{z}_{m,t_{se}} = F_m(z_{i,t_i-k_i} | i = 1, 2, \dots, M; k_i = 0, 1, \dots, K_i)$$

# Simulation Setup

16-machine 68-bus system with PST

Fault between bus 5 and bus 8 at 5.1<sup>th</sup> second

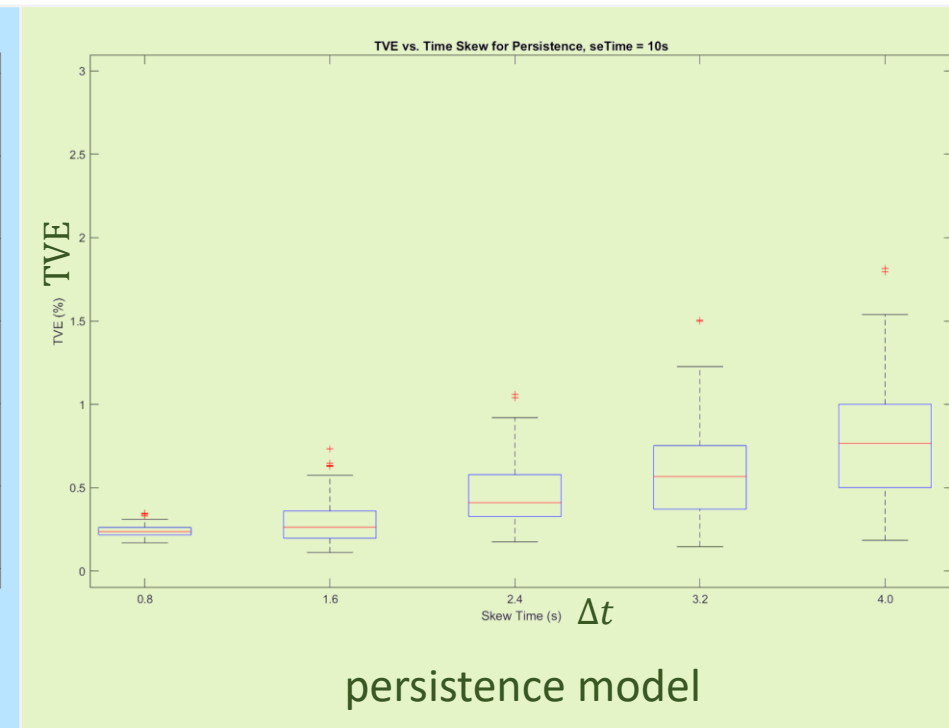
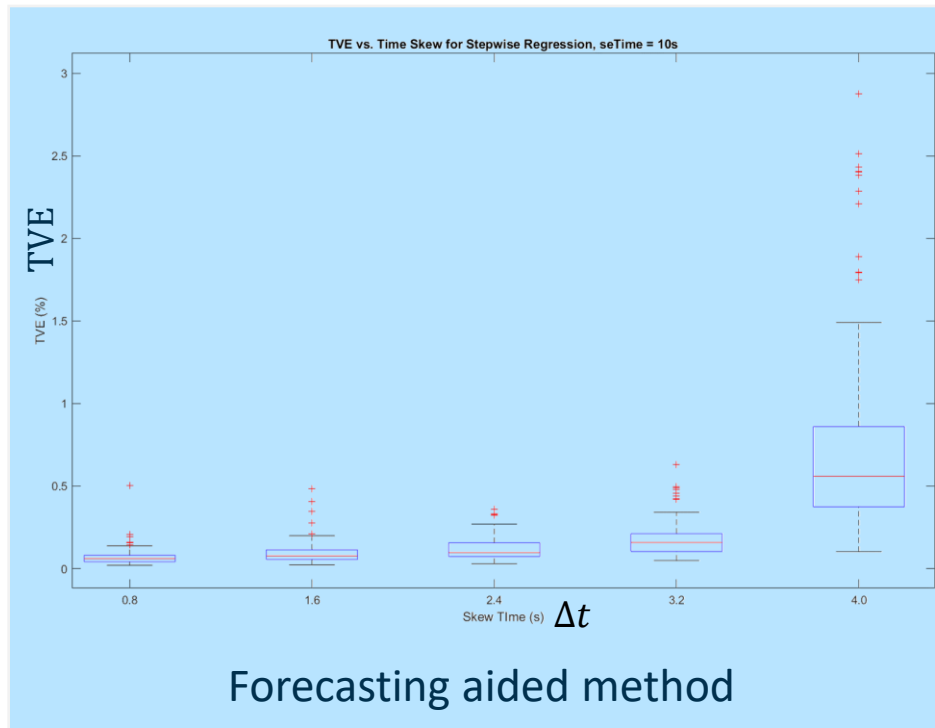
Gaussian white noise:  $\sigma = 3 \times 10^{-2}$  (100 Instances of MC)



# Estimation Accuracy w.r.t. to $\Delta t$

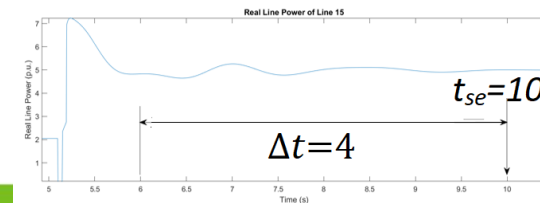
Estimation errors increase with  $\Delta t$

Forecasting-Aided method is more accurate than persistence model



$$TVE_{t_{se}} = \frac{\sum_{i=1}^n |\hat{V}_{i,t_{se}} \angle \hat{\theta}_{i,t_{se}} - V_{i,t_{se}} \angle \theta_{i,t_{se}}|}{\sum_{i=1}^n |V_{i,t_{se}} \angle \theta_{i,t_{se}}|}$$

$t_{se} = 10s$



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## 6. Summary of Findings

- SSE formulated as an **optimization problem**, solved iteratively using numerical methods
- **Ill-conditioning of the gain matrix  $\mathbf{G}$**  contributes to numerical instability and potential divergence.
- **Contributing factors:**
  - Heavy weights on virtual measurements.
  - Presence of both very short and long lines at the same bus.
  - Measurement sets dominated by bus injection measurements.
  - Initial state values significantly deviating from true values.
  - Ineffective bad data rejection.
  - Large systems with many state variables and low redundancy.
  - Time-skew errors during large variations and topology changes

# On-going Efforts and Works

## Comprehensive Report:

- Title: "Standard Test Cases for Static and Dynamic State Estimation in Power Systems."
- Published by **IEEE Task Force on Standard Test Cases for Power System State Estimation.**

## Data Accessibility:

- Test case data will be made publicly available.
- Data formatted using IEEE standard format and other convenient format

# Questions or Comments?

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