



The Functional Basis Analysis for the Parametric Reconstruction of Synchronised Waveforms in Power Systems

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Contributors

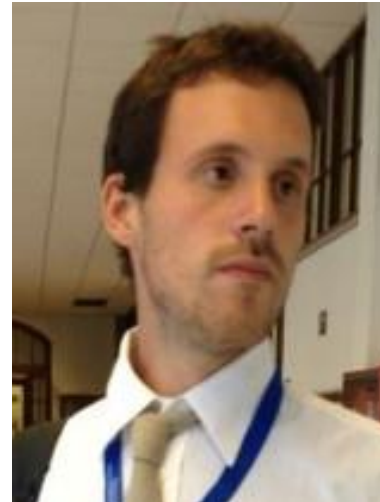
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Context

Event of September 28, 2016, Australia

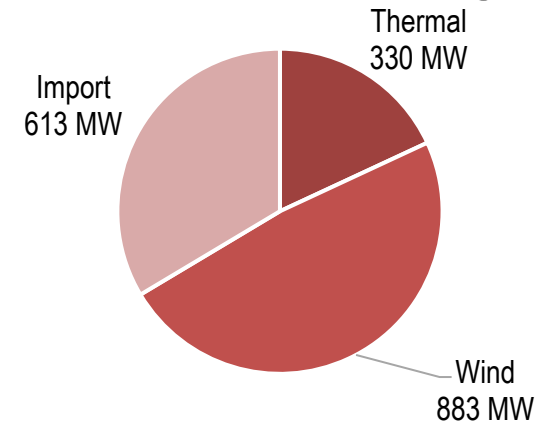
Multiple **tornadoes** in South Australia (SA) tripped multiple **275 kV transmission circuits** and resulted in multiple **faults in quick succession**.

The series of voltage dips from the faults **triggered protection on several wind farms to runback about 456 MW** of wind generation.

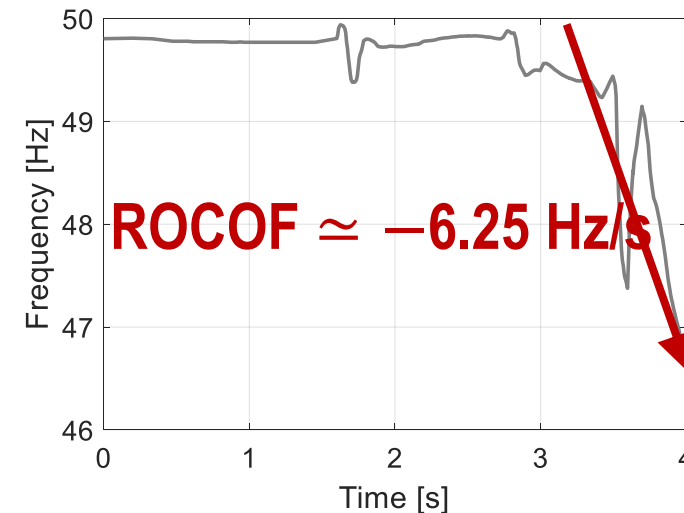
The reduction in wind farm output was **compensated by an increase in power imported** from Victoria. However, the **import reached a level that tripped the interconnector** on loss of synchronism protection.

The loss of power infeed from the wind farms and import from Victoria resulted in the **frequency falling so fast that load shedding schemes were unable to stop the fall**, resulting in a blackout.

SA pre-black-out generation mix



System frequency

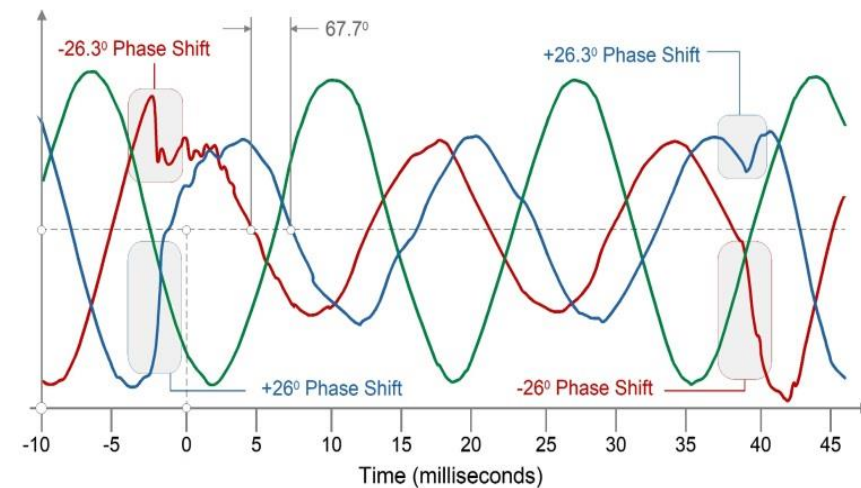
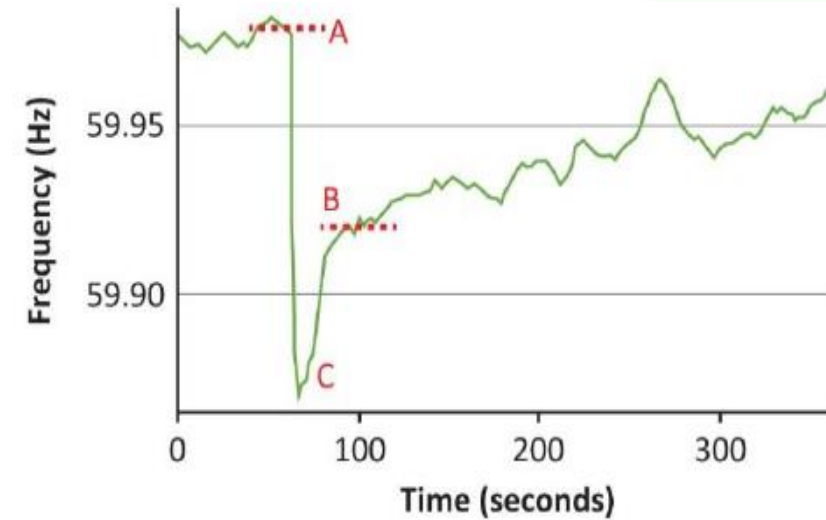


Context

Event of August 16, 2016, California

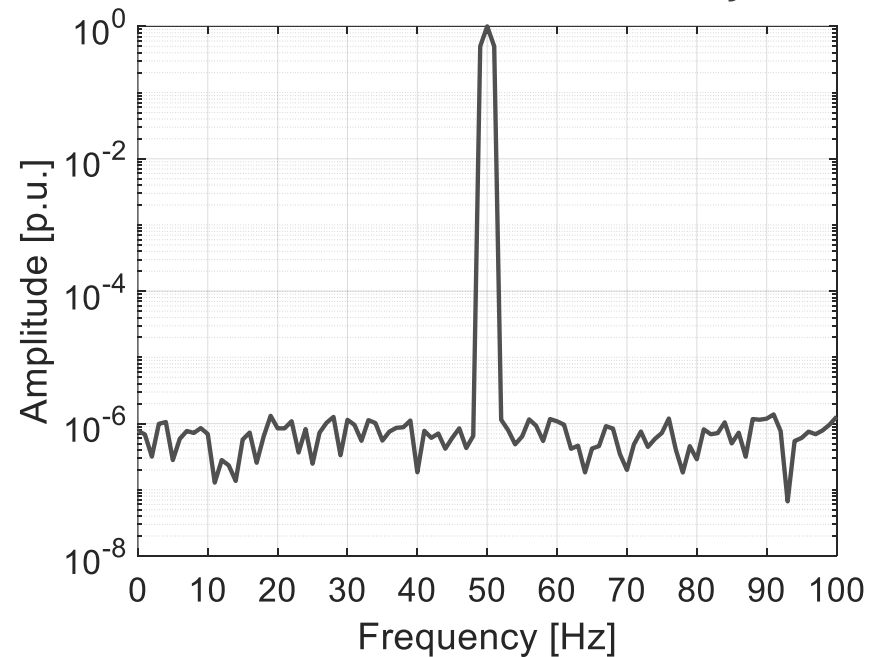
A fault induced a total of 1'178 MW of PV power interruption.

- The majority of inverters trip instantaneously based on frequency measurements obtained from PLLs; it was determined that these inverters are susceptible to erroneous tripping due to frequency changes; this response accounted for **700 MW of the lost generation**.
- A number of inverters that tripped were configured to **cease current injection if the voltage goes above 1.1 pu or below 0.9 pu**; the inverters were returned to pre-disturbance level at a slow ramp rate; this response accounted for about **450 MW of lost generation**.



Adequacy of signal processing techniques

$$v(t) = A \cdot \cos(2\pi ft)$$

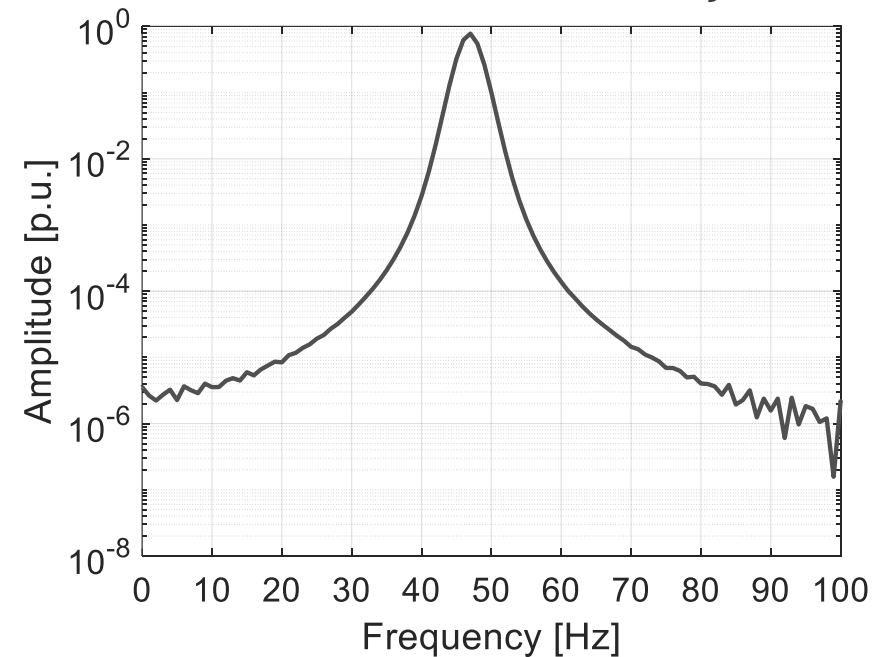


Energy of the signal in [48, 52] Hz 99.X%

→ **Narrow-band signal**

Transient with ROCOF $R=-6.25$ Hz/s

$$v(t) = A \cdot \cos(2\pi(f + Rt)t)$$



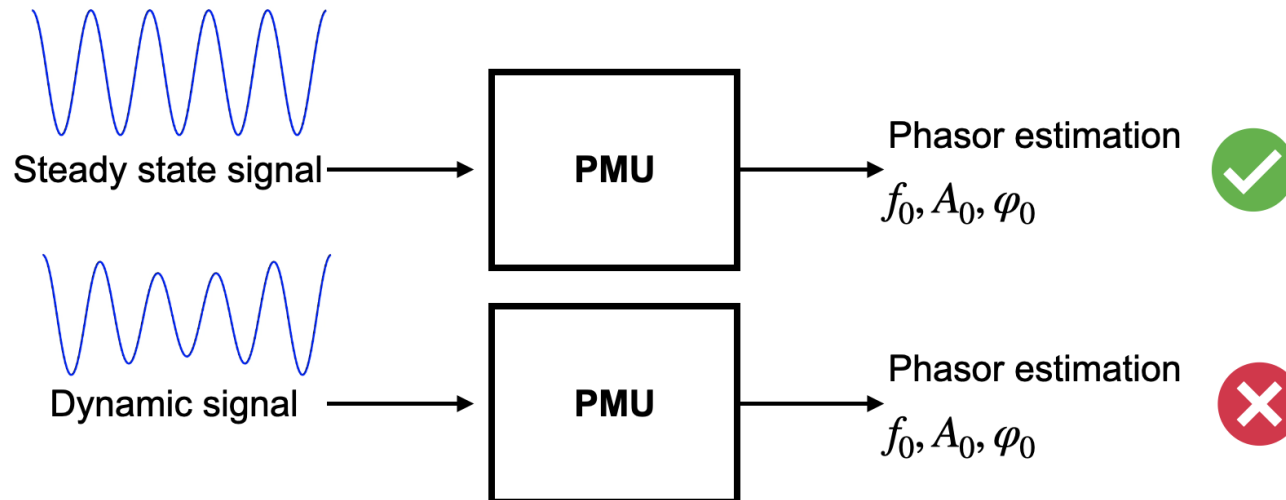
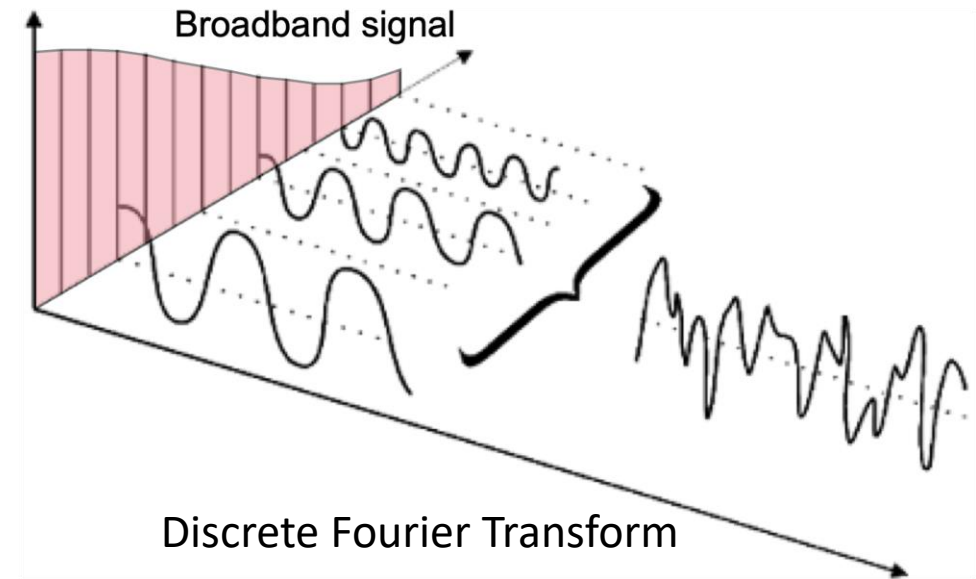
Energy of the signal in [48, 52] Hz 32%

→ **Broad-band signal**

Broadband Signal Analysis

Phasor limitations

- Real-world signal dynamics exhibit continuous and broadband spectra.
- Narrowband phasor estimation can yield high errors.



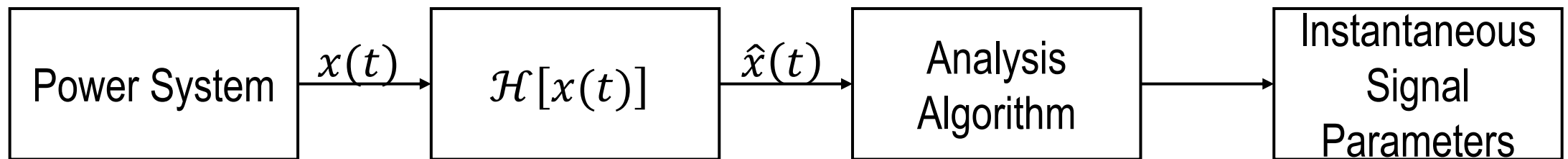
Incorrect model
 ↓
 Incorrect parameters
 ↓
 Inappropriate control actions!

Analysis of Broad-Band Signals in Reduced-Inertia Power Systems Using the Hilbert Transform (HT)

As seen, the exact modeling of voltage/current signals in inertia-less power systems may require the use of more sophisticated representations other than phasors.

Proposal: Can we project on a **different functional basis** not based on sinusoids that enables us to reconstruct the whole spectrum and, therefore, capable to **model the power transfer on the whole spectrum?**

The **Hilbert Transform (HT) and analytic signals** may be the appropriate tools since allows for a seamless parametrization of the generic transients of voltage/current signals.



A. Derviškić, G. Frigo and M. Paolone, "Beyond Phasors: Modeling of Power System Signals Using the Hilbert Transform," in IEEE Transactions on Power Systems, vol. 35, no. 4, pp. 2971-2980, July 2020.

A. Karpilow, A. Derviškić, G. Frigo and M. Paolone, "Characterization of Non-Stationary Signals in Electric Grids: A Functional Dictionary Approach," in IEEE Transactions on Power Systems, vol. 37, no. 2, pp. 1126-1138, March 2022.

Broadband Signal Analysis

Hilbert Transform

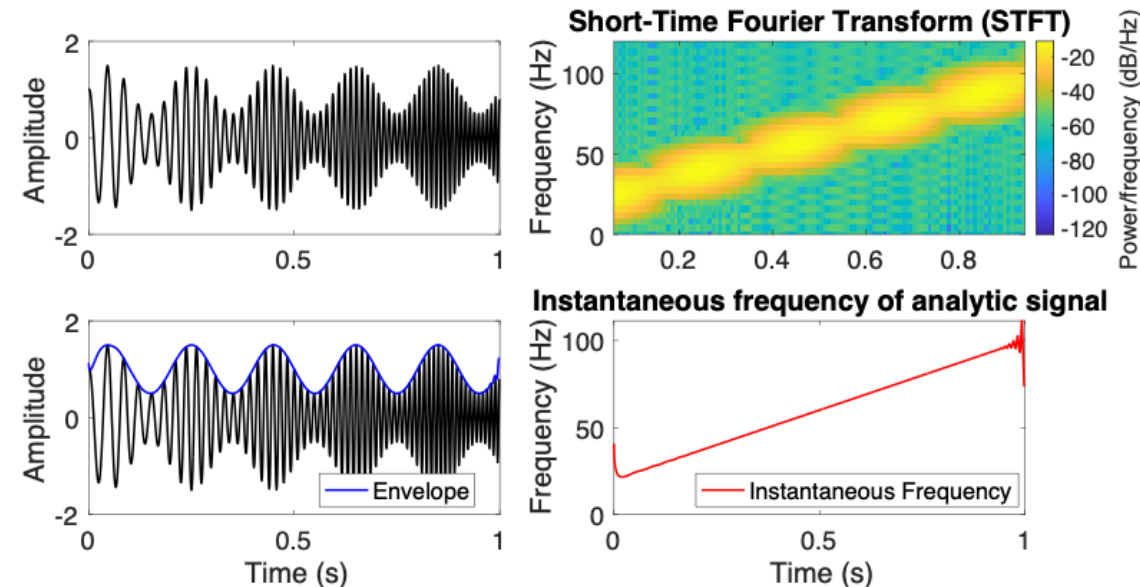
General dynamic signal model:

$$x(t) = \left(1 + \underbrace{g_A(t)}_{\text{Amplitude variation}}\right) \cos\left(2\pi f_0 t + \underbrace{g_\phi(t)}_{\text{Phase variation}}\right)$$

Through the Hilbert transform (\mathcal{H}), we may define the analytic signal:

$$\tilde{x}(t) = x(t) + j \cdot \mathcal{H}[x(t)]$$

$$\tilde{x}(t) = \left(1 + g_A(t)\right) e^{j(2\pi f_0 t + g_\phi(t))}$$



Comparison of STFT and analytic signal for waveform with amplitude modulation (5 Hz, 50%) and a frequency ramp (80 Hz/s)

Functional Basis Analysis (FBA): apply user-engineered dictionaries of common signal dynamics to characterize broadband waveforms

Broadband Signal Analysis

Analytic signal: obs

Bedrosian's theorem:

$$\mathcal{H}[y(t)x(t)] = y(t)\mathcal{H}[x(t)]$$

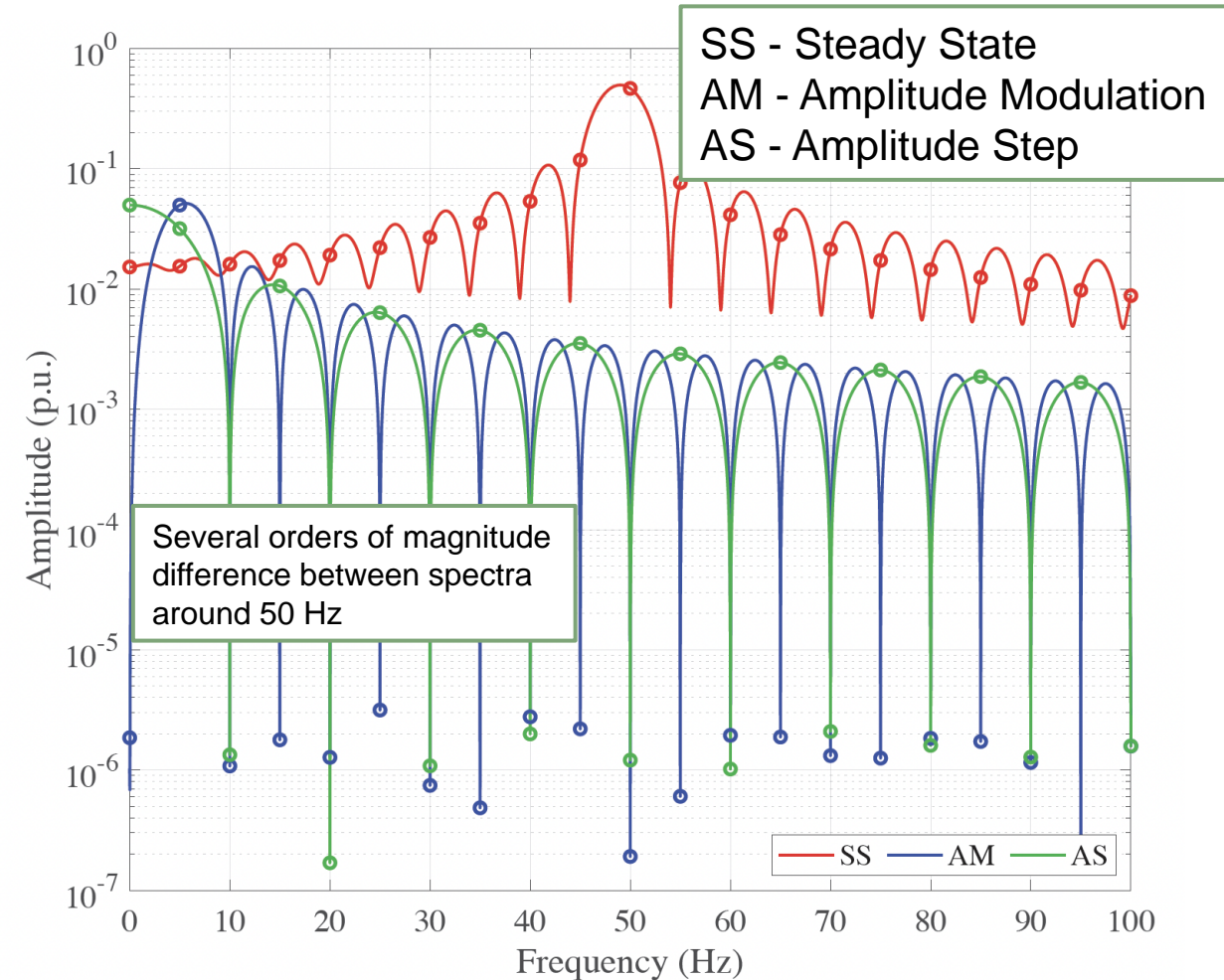
If $x(t)$ and $y(t)$ have non-overlapping spectrum and $y(t)$ is the lower frequency signal.

$$x(t) = (1 + g_A(t))\cos(2\pi f_0 t + g_\phi(t))$$

$$\downarrow$$

$$\tilde{x}(t) = (1 + g_A(t))e^{j(2\pi f_0 t + g_\phi(t))}$$

Holds for slow amplitude and phase modulations but violated for steps.



DFT of SS ($f_0 = 50$ Hz), AM ($f_{AM} = 5$ Hz) and AS ($t_{AS} = \frac{T_W}{2}$, $a_{AS} = 0.1$ p.u.). 200 ms window and $F_s = 10$ kHz

Functional Basis Analysis

Envelope Analysis: $x_A(t) = |\tilde{x}(t)|$

Amplitude Modulation:

$$x_{AM}(t) = A_0(1 + a_{AM} \sin(2\pi f_{AM}t) + \varphi_{AM})$$

\equiv

$$x_{AM}(t) = \gamma_0 + \gamma_1 \sin(2\pi f_{AM}t) + \gamma_2 \cos(2\pi f_{AM}t)$$

Least Squares & Golden Section Search (GSS/LS_{AM})

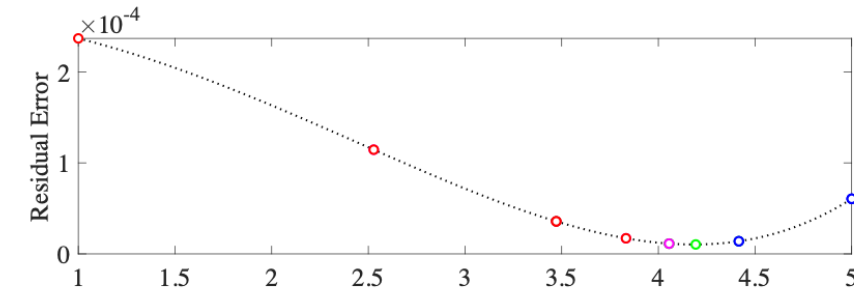
$$\min_{\gamma} \|\mathbf{D}_{AM}(f_{AM})\gamma - \mathbf{x}_A\|_2$$

$$\gamma = [\gamma_0 \quad \gamma_1 \quad \gamma_2]^T, \mathbf{x}_A = [x_A(t_0) \quad \dots \quad x_A(t_{N-1})]^T$$

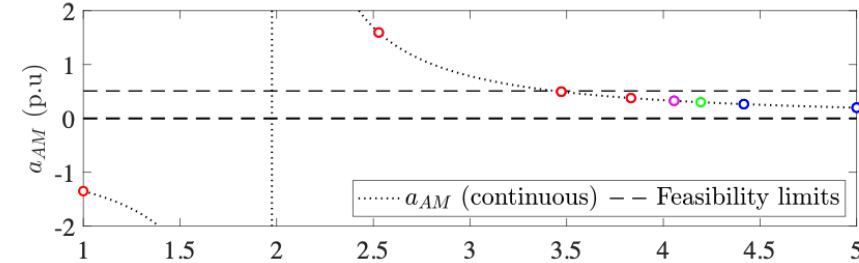
$$\mathbf{D}_{AM}(f_{AM}) = \begin{bmatrix} 1 & \sin(2\pi f_{AM}t_0) & \cos(2\pi f_{AM}t_0) \\ \vdots & \vdots & \vdots \\ 1 & \sin(2\pi f_{AM}t_{L-1}) & \cos(2\pi f_{AM}t_{L-1}) \end{bmatrix}$$

$$\gamma^*(f_{AM}) = (\mathbf{D}_{AM}^T(f_{AM})\mathbf{D}_{AM}(f_{AM}))^{-1} \mathbf{D}_{AM}^T(f_{AM})\mathbf{x}_A$$

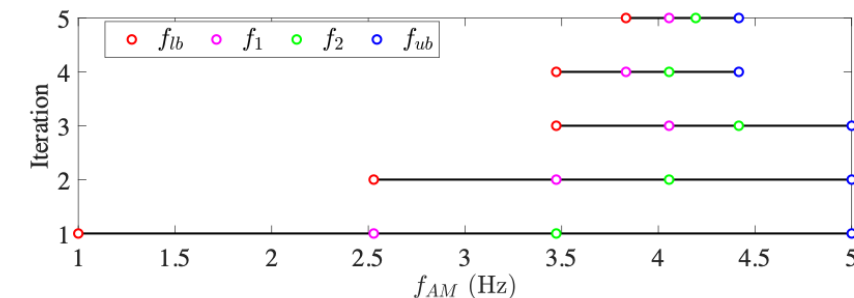
Golden Section Search (GSS) for optimal f_{AM}



Objective Function



Feasibility Check
 $0 < a_{AM} \leq 0.5$,
 $A_0 > 0$



GSS Iterations

Functional Basis Analysis

Argument Analysis: $x_\varphi(t) = \angle \tilde{x}(t)$

Phase Modulation:

$$x_{PM}(t) = \varphi_0 + 2\pi f_0 t + a_{PM} \sin(2\pi f_{AM} t + \varphi_{PM})$$

$$\equiv$$

$$x_{PM}(t) = v_0 + v_1 t + v_2 \sin(2\pi f_{PM} t) + v_3 \cos(2\pi f_{PM} t)$$

Least Squares & Golden Section Search: GSS/LS_{PM}

$$\min_{\boldsymbol{\gamma}} \left\| \mathbf{D}_{PM}(f_{PM}) \boldsymbol{\nu} - \mathbf{x}_\varphi \right\|_2$$

$$\boldsymbol{\gamma} = [v_0 \quad v_1 \quad v_2 \quad v_3]^T, \mathbf{x}_\varphi = [x_\varphi(t_0) \quad \dots \quad x_\varphi(t_{L-1})]^T$$

$$\mathbf{D}_{PM}(f_{PM}) = \begin{bmatrix} 1 & \sin(2\pi f_{PM} t_0) & \cos(2\pi f_{PM} t_0) \\ \vdots & \vdots & \vdots \\ 1 & \sin(2\pi f_{PM} t_{L-1}) & \cos(2\pi f_{PM} t_{L-1}) \end{bmatrix}$$

$$\boldsymbol{\nu}^*(f_{PM}) = \left(\mathbf{D}_{PM}^T(f_{PM}) \mathbf{D}_{PM}(f_{PM}) \right)^{-1} \mathbf{D}_{PM}^T(f_{PM}) \mathbf{x}_\varphi$$

Frequency Ramp:

$$x_{FR}(t) = \varphi_0 + 2\pi f_0 t + R\pi t^2$$

$$\equiv$$

$$x_{FR}(t) = \beta_0 + \beta_1 t + \beta_2 t^2$$

Least Squares: LS_{FR}

$$\min_{\boldsymbol{\gamma}} \left\| \mathbf{D}_{FR} \boldsymbol{\beta} - \mathbf{x}_\varphi \right\|_2$$

$$\boldsymbol{\beta} = [\beta_0 \quad \beta_1 \quad \beta_2]^T$$

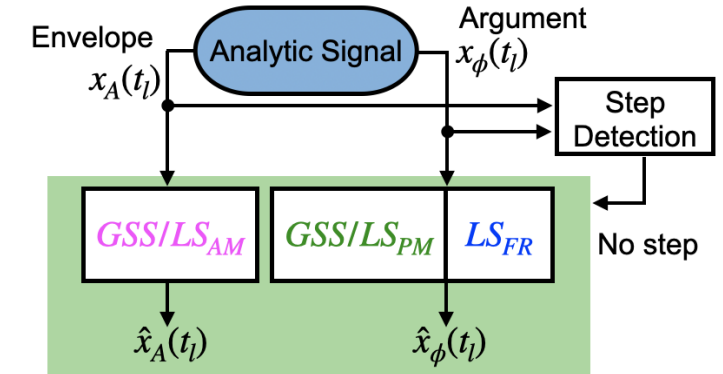
$$\mathbf{D}_{FR} = \begin{bmatrix} 1 & t_0 & t_0^2 \\ \vdots & \vdots & \vdots \\ 1 & t_{L-1} & t_{L-1}^2 \end{bmatrix}$$

$$\boldsymbol{\beta}^* = \left(\mathbf{D}_{FR}^T \mathbf{D}_{FR} \right)^{-1} \mathbf{D}_{FR}^T \mathbf{x}_\varphi$$

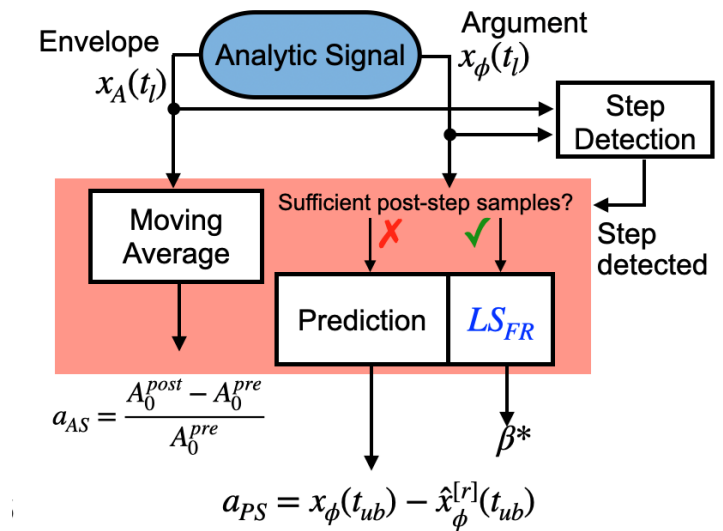
Functional Basis Analysis

Step Detection

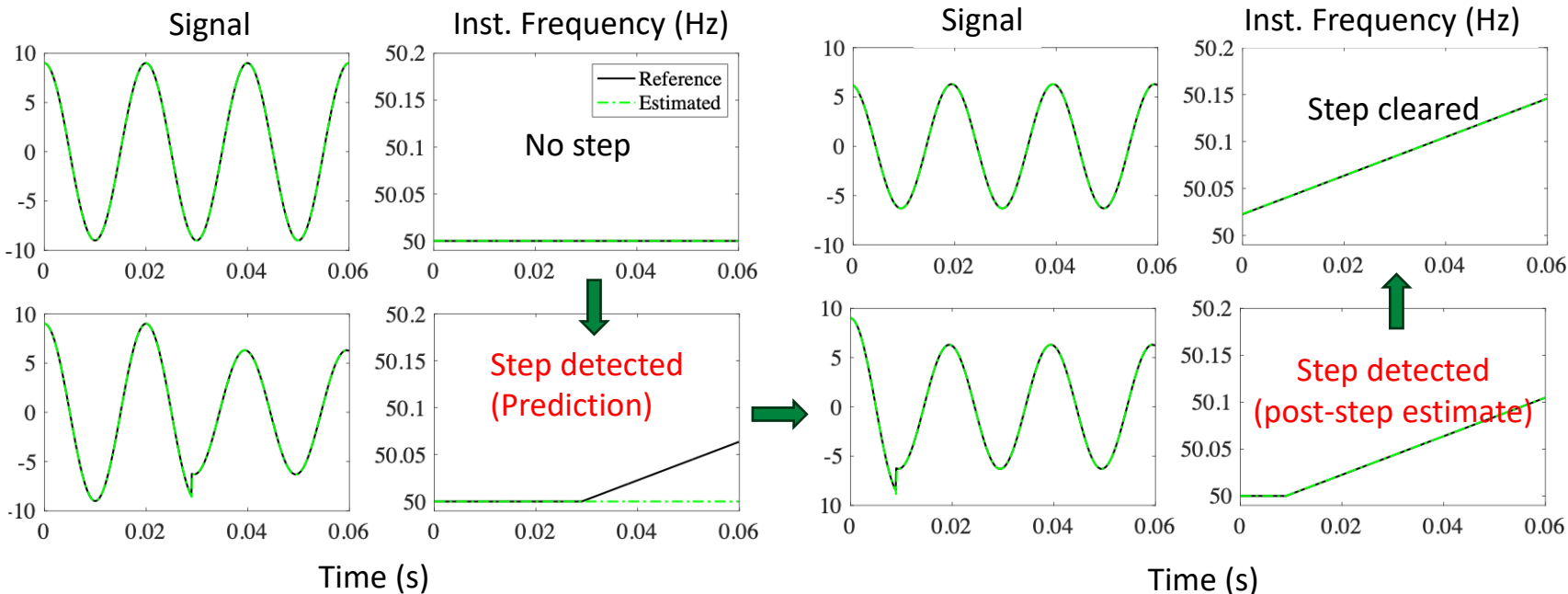
- Thresholds on differential envelope and argument:
 - Amplitude Step: $\Delta x_A(t_l) = x_A(t_l) - x_A(t_{l-1}) \rightarrow |\Delta x_A(t_l) - \Delta \bar{x}_A| > \epsilon_{\Delta A}$
 - Phase Step: $\Delta x_\phi(t_l) = x_\phi(t_l) - x_\phi(t_{l-1}) \rightarrow |\Delta x_\phi(t_l) - \Delta \bar{x}_\phi| > \epsilon_{\Delta \phi}$
- Pre- and post-step states are estimated separately.



FBA when no step is detected



FBA when a step is detected

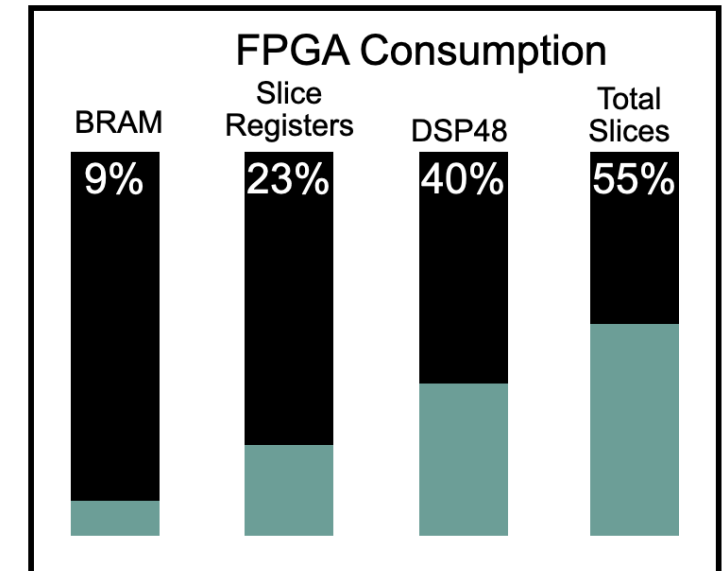


Functional Basis Analysis

Embedded Systems Deployment

- FIR Hilbert filter: low complexity, high performance, high group delay
- Recursive formulation of FR estimation
- Projected to support at least 4 more channels
- Total latency within M Class requirements

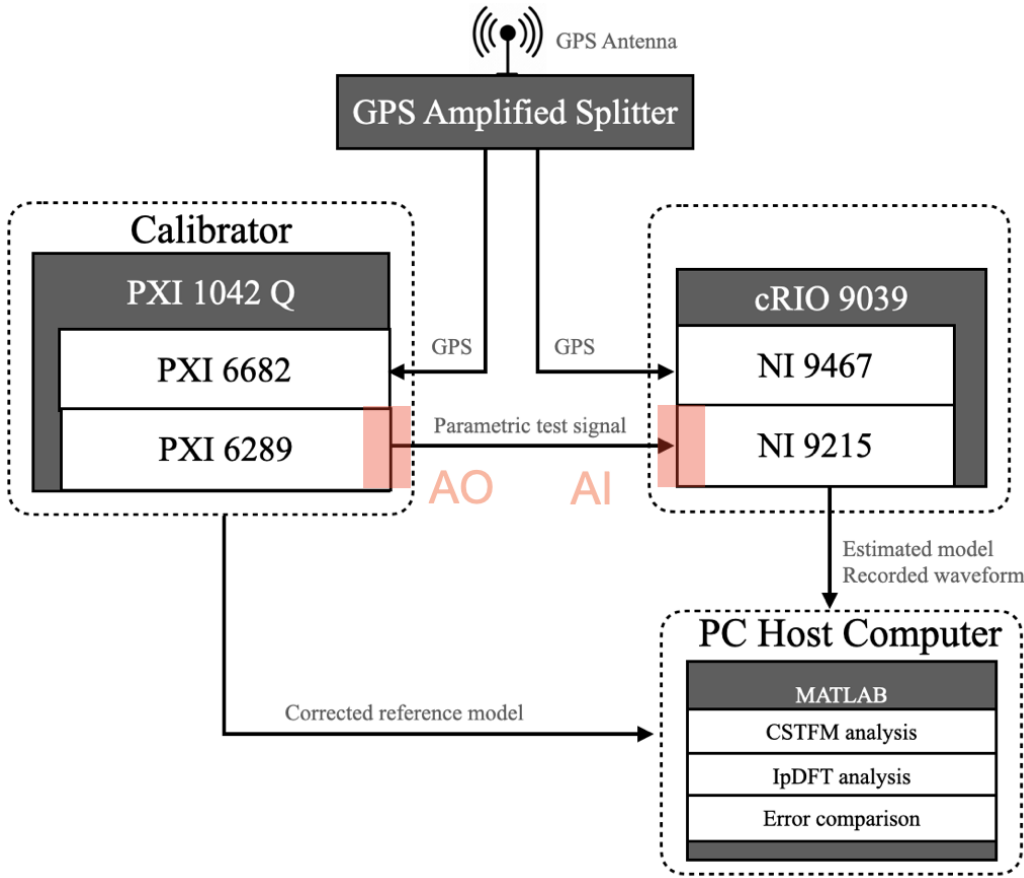
NI cRIO 9039



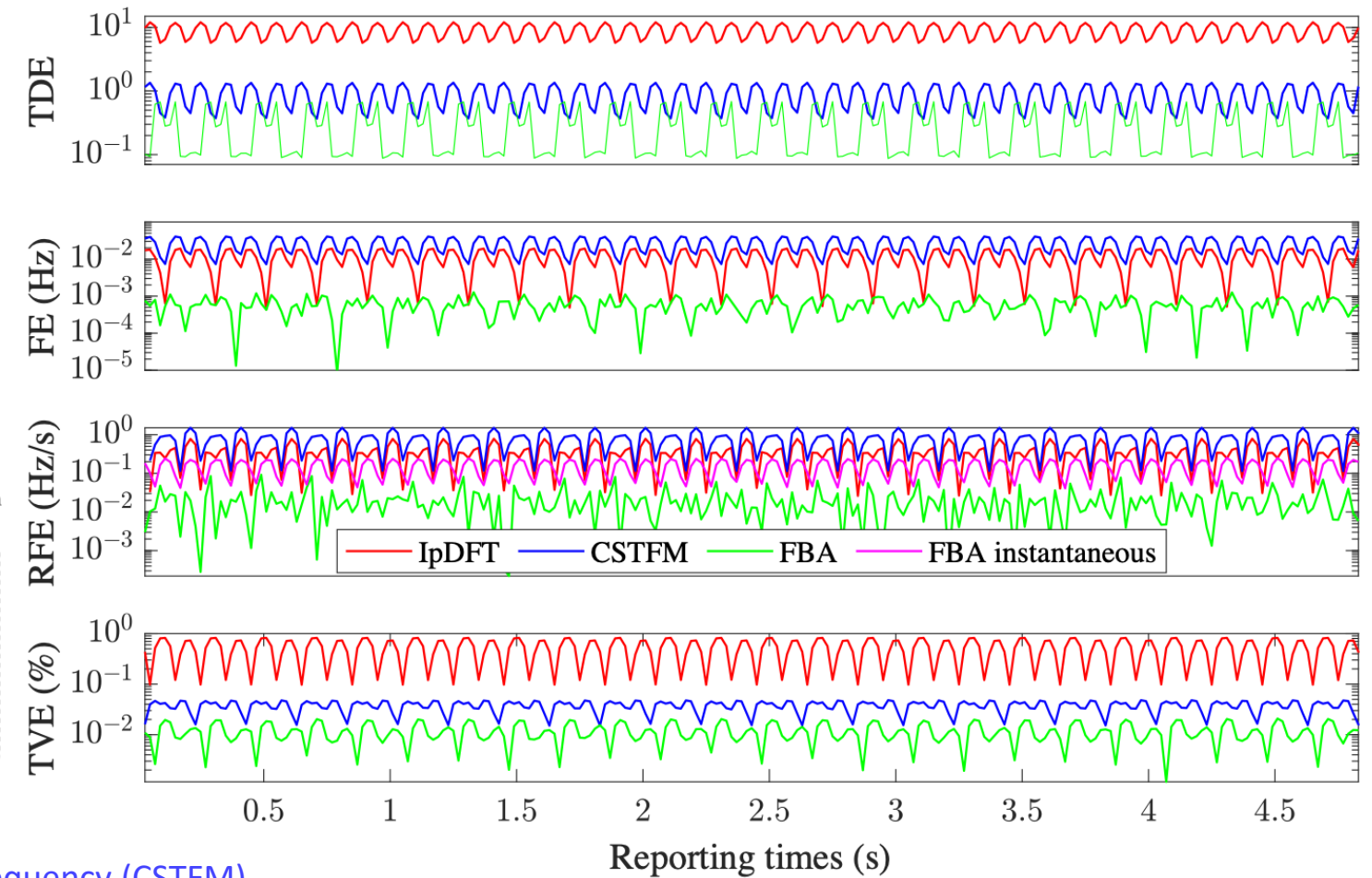
Computation Time	Filter Group Delay	Timestamp delay	Total Latency
0.3 ms	39.9 ms	60 ms/2	70.2 ms

Functional Basis Analysis

Validation



Amplitude and Phase Modulation Test



Static phasor – 3pt, Hann window, iterative IpDFT

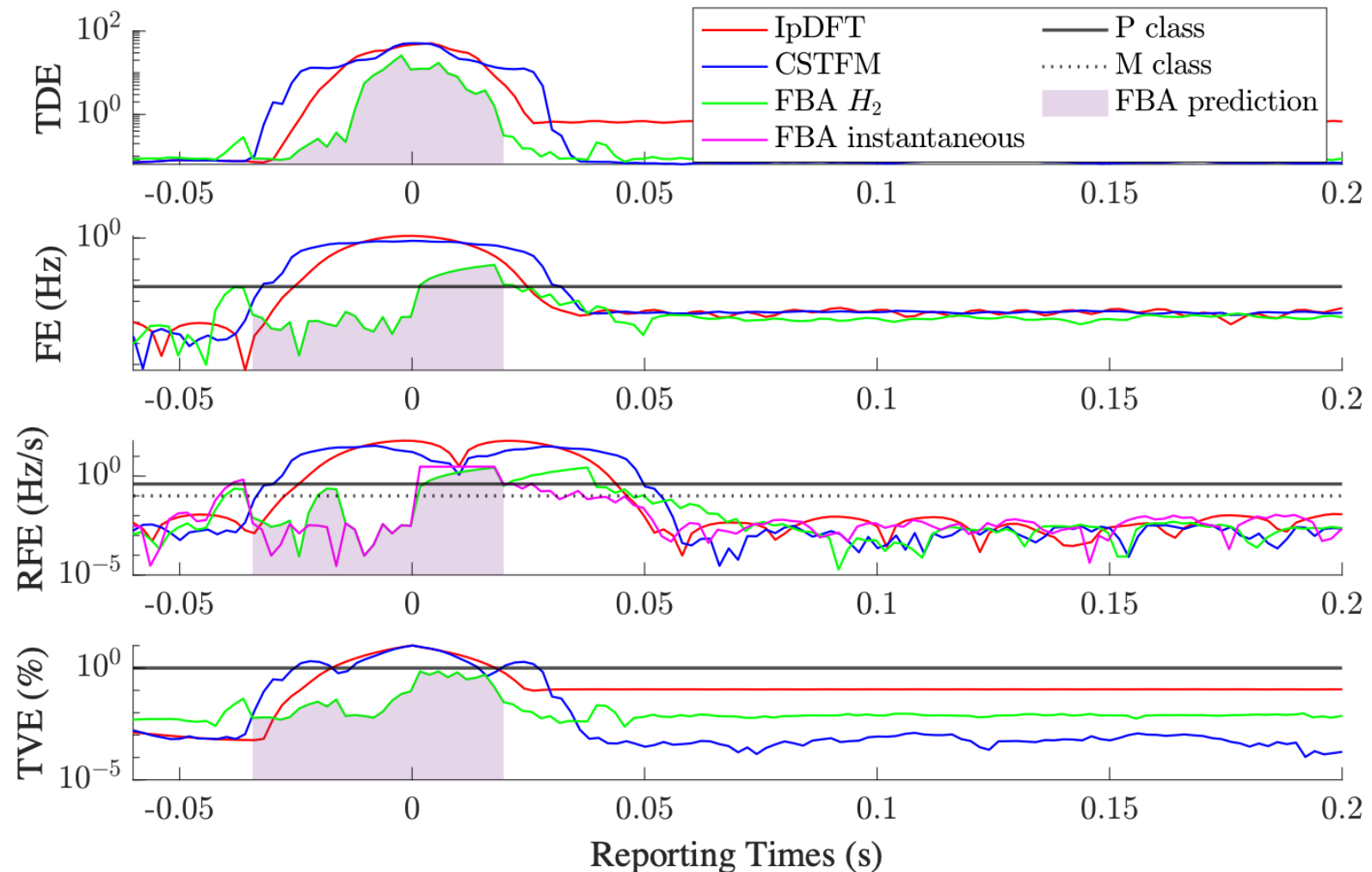
Dynamic phasor – Compressed Sensing Taylor Fourier Multifrequency (CSTFM)

Functional Basis Analysis

Amplitude/Phase Step Followed by a Frequency Ramp (3 Hz/s)

Validation

- The FBA detects step and provides interim prediction (shaded)
- A post-step estimation of the instantaneous frequency variation is provided by FBA before the static/dynamic phasor methods can recover



Conclusions

- Recent events in low-inertia power systems have shown how the **use of phasors may lead to large approximations when modelling signals of electrical quantities in reduced-inertia power grids.**
- The **HT, integrated with the analytical signal representation, may be the appropriate tool for modelling broad-band signals** associated to inertia-less power system dynamics.
- The presentation has shown how the **functional basis analysis (FBA) allows for the extraction of signal parameters and the identification of common dynamics.**
- The **FBA method demonstrated improved performance for common signal dynamics when compared to static and dynamic phasor representations.**
- The FBA can be used for the **compression of time-domain signals and considered as an alternative to point-on-wave data stream.**