Syllabus

Monday, January 23, 2017 8:01 AM

www.ecst.csuchico.edu/~hma - class website

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Office Hours: MTWTh 4:00 - 4:50 pm

Circuits: loop of electrical components

Independent and Dependent Sources

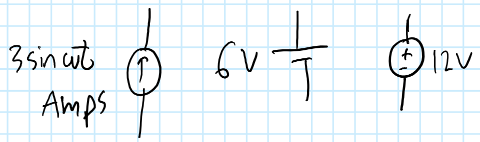
Tuesday, January 24, 2017 8:02 AM

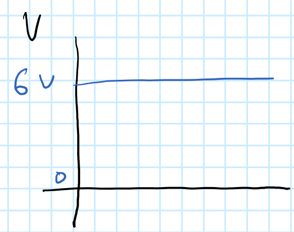
I. Indep Sources

A. Definition and Symbols

Indep Source = source whose value does not depend in any way on the rest of the circuit

Symbols





Ideal voltage source has ∅ resistance

Short Circuit = ∅ resis

Open Circuit = ∞ resis

II. Dependent Sources (controlled sources)

V_s $V_s = \alpha V_x$ → outside

↳ coefficient (unitless)

Voltage Controlled Voltage Source

Dependent Voltage Source

$V_s = \beta I_x$ → outside

↳ coefficient (ohms)

Current Controlled Voltage Source

transistor = transfer resistance

Bit = binary digit

pixel = picture element

$i_s = \Sigma I_x$

↳ sigma

Current Controlled Current Source

$i_s = \gamma V_x$

↳ gamma (mho ℧ → $\frac{1}{ohm}$)

Voltage Controlled Current Source

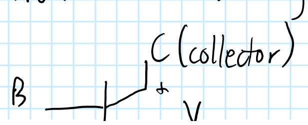
B. Circuit Applications

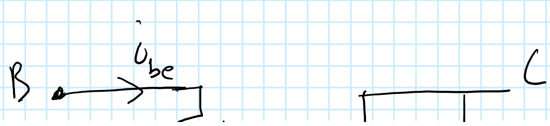
transistors act as dependent sources

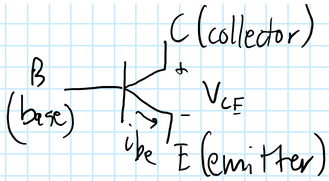
simulate transistors using dependent sources

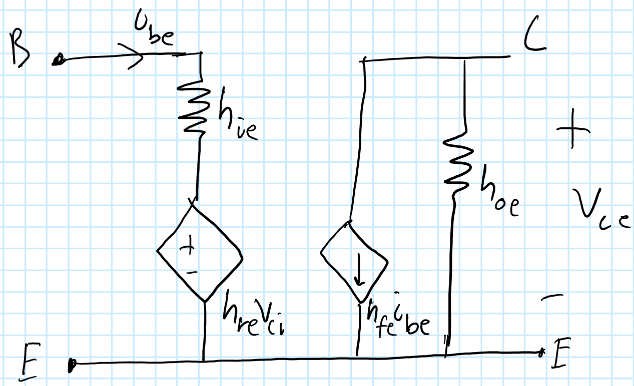
Metal
Oxide
Semiconductor
Field
Effect
Transistor

1. Transistor Small Signal









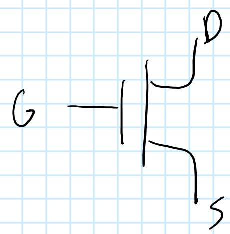
$V_{CE} = h_{fe}\, i_{be}\, h_{oe}$

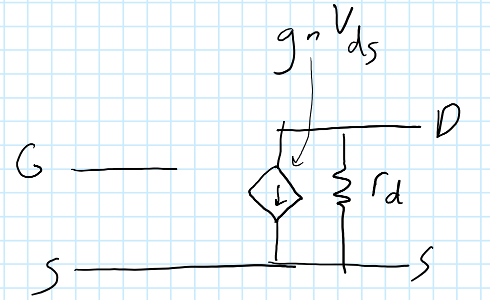
$V_{BE} = i_{be} h_{ie} + h_{re} V_{ci}$

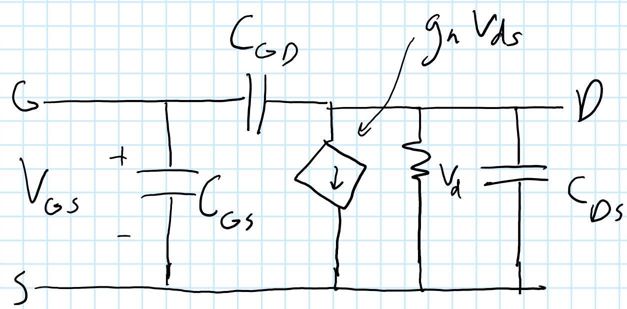
MOSFET

Wednesday, January 25, 2017 8:02 AM

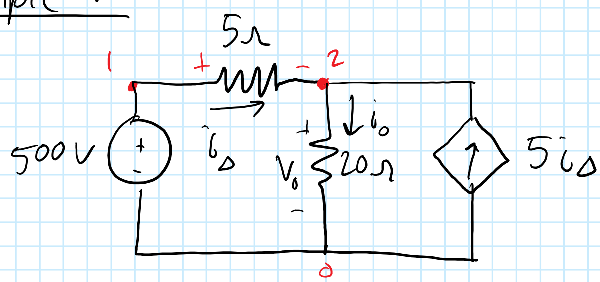
G = Gate
D = Drain
S = Source







Example 1:



$i_\Delta = ?$

$i_o = ?$

$V_o = ?$

Solve: Using KVL

$\sum V = 0$

$\sum V_{rises} = \sum V_{drops}$

$\sum V_{rises} - \sum V_{drops} = 0$

$V_{rises} \rightarrow$ "+" $\quad V_{drops} \rightarrow$ "−"

$500V - 5i_\Delta - V_o = 0 \quad (1)$

$V_o = 20 i_o \quad (2)$

$500 - 5i_\Delta - 20 i_o \quad (3) \quad 2 \rightarrow 1$

Using KCL at node 2: $i_\Delta + 5i_\Delta = i_o \quad (4)$

$6 i_\Delta = i_o \quad (5)$

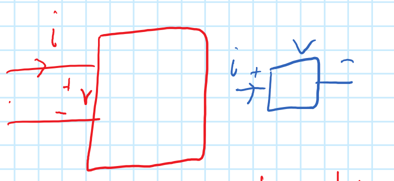
$500 - 5i_\Delta - 20(6i_\Delta) = 0 \quad (6)$

$500 - 125 i_\Delta = 0 \quad (7)$

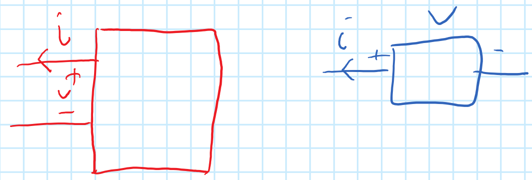
$125 i_\Delta = 500 \quad \boxed{i_\Delta = 4A}$

$i_o = 6 i_\Delta \quad \boxed{i_o = 24A}$

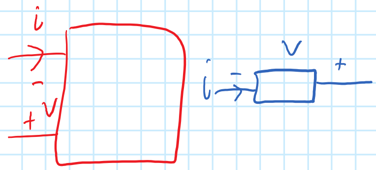
$V_o = 20 i_o \quad \boxed{V_o = 480V}$



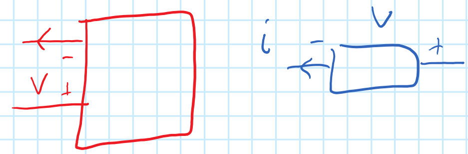
$p = vi$ absorbing



$p = v(-i) = -vi$ delivering



$p = (-v)i = -vi$ delivering



$p = (-v)(-i) = vi$ = absorbing

$P_{500V} = 500V(-i_\Delta) = 500(4) = \boxed{-2000\ W}$ delivering

$P_{5\Omega} = V_{5\Omega}(i_\Delta)$ $\quad V_{5\Omega} = 5\Omega\ i_\Delta$ $\quad P = (5\Omega)(i_\Delta)^2$ $\quad P = 5(4)^2 = \boxed{80\ W}$ absorbing

$P_{20\Omega} = (V_0)(i_0) = 480(24) = \boxed{11{,}520\ W}$ absorbing

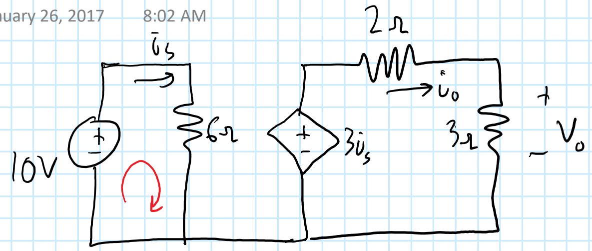
$P_{5i_\Delta} = (V_0)(-5i_\Delta) = 480(-5)(4) = 480(-20) = \boxed{-9600\ W}$ delivery

$P_{500V} + P_{5\Omega} + P_{20\Omega} + P_{5i_\Delta} = -2000 + 80 + 11520 - 9600 = 0$

Dependent Source Analysis

Thursday, January 26, 2017 8:02 AM

Ex:



Solve: KVL at the left loop

Voltage rise = "+"

Voltage drop = "−"

$10V - 6(i_s) = 0 \qquad 6i_s = 10 \qquad i_s = \frac{10}{6}$

$\boxed{i_s = \frac{5}{3} A}$

$P_{10V} = V(-i) = 10(-i_s) = \frac{(10)}{1} \cdot \frac{-5}{3} = \boxed{\frac{-50}{3} W}$

$P_{6\Omega} = Vi = 10(i_s) = \frac{10}{1} \cdot \frac{5}{3} = \boxed{\frac{50}{3} W}$

$P_{3i_s} = V(-i) = \left(\frac{\cancel{3}}{1} \cdot \frac{5}{\cancel{3}}\right) \cdot (-1) = \boxed{-5 W}$

$P_{2\Omega} = i^2 R = (1)^2 \cdot 2 = \boxed{2W}$

$P_{3\Omega} = i^2 R = (1)^2 \cdot 3 = \boxed{3W}$

KVL at the right loop

$3i_s - 2(i_o) - V_o = 0$

$V_o = 3\Omega(i_o)$

$3i_s - 2\Omega(i_o) - 3\Omega(i_o) = 0$

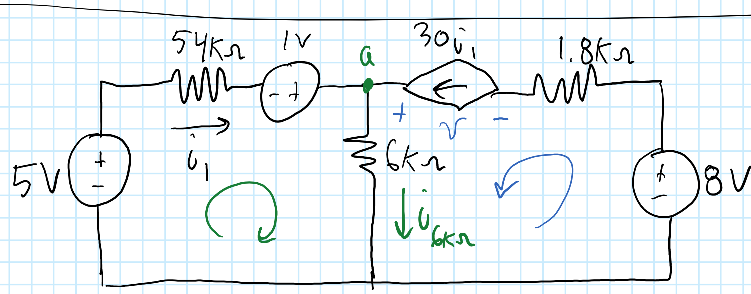
$3i_s - 5i_o = 0 \qquad 5i_o = 3i_s$

$i_o = \frac{3}{5} i_s = \frac{\cancel{3}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{3}}$

$\boxed{i_o = 1 A}$

$V_o = 3(i_o) = 3(1)$

$\boxed{V_o = 3 V}$



Solve: KVL at the left loop

$5V - (54k\Omega)(i_1) + 1V - (6k\Omega)(i_{6k\Omega}) = 0$

$i_{6k\Omega} = i_1 + 30i_1$ (KCL at a)

$5V - (54k\Omega)(i_1) + 1V - (6k\Omega)(31i_1) = 0$

$5 - 54000i_1 + 1 - 186000i_1 = 0$

$6 - 240000i_1 = 0$

$240000i_1 = 6 \qquad i_1 = \frac{6}{240000} = 0.000025A = \boxed{25\mu A = i_1}$

KVL at the right loop:

$8V - (1.8k\Omega)(30i_1) + v - (6k\Omega)(30i_1 + i_1) = 0$

$8V - (1800)(30i_1) + v - 186000i_1 = 0$

$v = -8 + 54000i_1 + 186000i_1$

$= -8 + 54000(25\mu A) + 186000(25\mu A)$

$= -2V$

$P_{5V} = V(-i) = (5V)(-i_1) = (5V)(25\mu A) = -125\mu W$

$P_{54k\Omega} = V(i) = (54k\Omega)(i_1)^2 = (54k\Omega)(25\mu A)^2 = 33.75\mu W$

$P_{1V} = V(-i) = (1V)(-i_1) = (1V)(-25\mu A) = -25\mu W$

$P_{6k\Omega} = V(i) = (6k\Omega)(31i_1)^2 = (6k\Omega)(31 \times 25\mu A)^2 = 3603.75\mu W$

$P_{30i_1} = V(-i) = (-2V)(-30i_1) = 1500\mu W$

$P_{1.8k\Omega} = V(i) = (1.8k\Omega)(30i_1)^2 = 1012.5\mu W$

$P_{8V} = V(-i) = 8V(-30i_1) = -6000\mu W$

Chapter 4: Techniques of Circuit Analysis

Monday, January 30, 2017 8:00 AM

- Read Ch 4.3, 4.4, 4.6, 4.7, 4.8
- Do HW#2, due on Wed. 2/8

Node and Mesh Analysis

I. Node Analysis

A. Define Node - 2 or more branches merged together

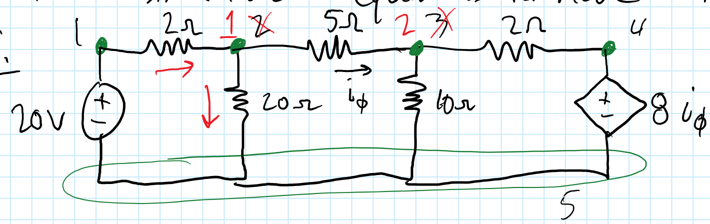
B. Pick Reference Node

C. Define voltages relative to reference

D. Apply KCL to all nodes except reference

E. Solve simultaneous equations for node voltages.

Ex:



5 nodes

$V_1 = 20V$

$V_4 = 8 i_\phi$

$V_5 = 0$ V

Solve: Node 1

Assume current entering "+"
current leaving "-"

$$\frac{20 - V_1}{2\Omega} - \frac{V_1}{20\Omega} - \frac{V_1 - V_2}{5\Omega} = 0$$ ⟸ Use this one (1)

$$\frac{20 - V_1}{2} + \frac{0 - V_1}{20} - \frac{V_1 - V_2}{5} = 0$$

$$\frac{-V_1 - 20}{2} - \frac{V_1}{20} - \frac{V_1 - V_2}{5} = 0$$

$$\frac{-V_1 - 20}{2} + \frac{0 - V_1}{20} - \frac{V_1 - V_2}{5} = 0$$

Node 2:

$$\frac{V_1 - V_2}{5\Omega} + \frac{8i_\phi - V_2}{2\Omega} - \frac{V_2}{10\Omega} = 0 \qquad (2)$$

$$i_\phi = \frac{V_1 - V_2}{5\Omega} \qquad (3)$$

Solve 3 eq. 3 unknowns

cannot do $i_\phi = V_2 - V_1$

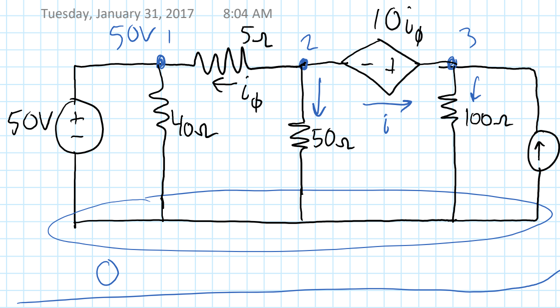
Ask clint to show how to solve eq

cannot do $i_\phi = \frac{V_2 - V_1}{5}$

Ask clint to show how to solve eq on calculator

Supernode

Tuesday, January 31, 2017 8:04 AM



4 nodes

Node #1: $V_1 = 50$ V

Solve: $I_{entering} \to$ "+"

$I_{leaving} \to$ "−"

Node #2: KCL

I. Using assumed i

$$-\frac{V_2-50}{5} - \frac{V_2}{50} - i = 0 \quad (1)$$

Node 3:

$$i + 4A - \frac{V_3}{100\Omega} = 0 \quad (2)$$

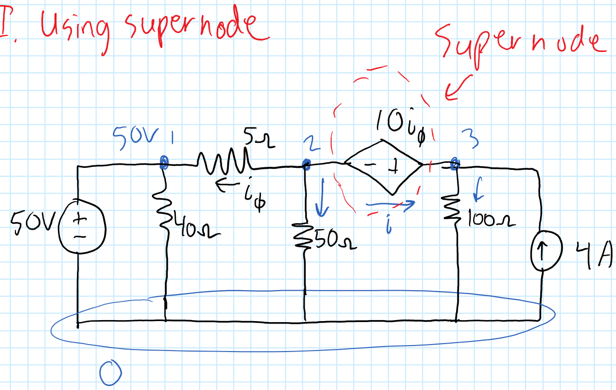
Let (1) + (2) $\Longrightarrow$ $-\frac{V_2-50}{5} - \frac{V_2}{50} \cancel{-i+i} + 4 - \frac{V_3}{100} = 0 \quad (3)$

$$V_3 - V_2 = 10 i_\phi \quad (4)$$

$$\frac{V_2-50}{5\Omega} = i_\phi \quad (5)$$

Use (3), (4), (5) to solve for i_ϕ, V_2, V_3

II. Using supernode



Looking at left side of supernode:

$$-\frac{V_2-50}{5\Omega} - \frac{V_2}{50\Omega} - \frac{V_3}{100} + 4A = 0 \quad \text{Same as (3)}$$

Looking at right side of supernode:

Substitue (5) into (4)

$$V_3 - V_2 = \left(\frac{V_2-50}{5}\right)10 \qquad V_3 - V_2 = 2V_2 - 100$$

$$V_3 = 3V_2 - 100 \quad (6)$$

Substitute (6) into (3): $-\frac{V_2-50}{5} - \frac{V_2}{50} - \frac{3V_2-100}{100} + 4 = 0$

Mult 100 by both sides: $-20V_2 + 1000 - 2V_2 + 400 - 3V_2 + 100 = 0$

$$-25V_2 + 1500 = 0 \qquad \boxed{V_2 = 60 \text{ Volts}}$$

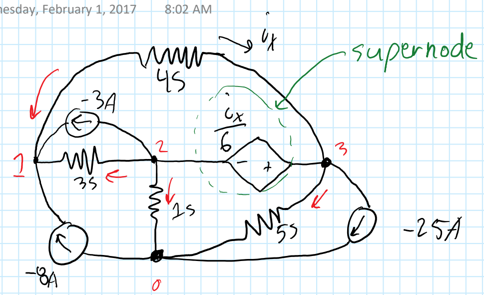
$$V_3 = 3(60) - 100 = 180 - 100$$

$$\boxed{V_3 = 80 \text{ Volts}}$$

$$i_\phi = \frac{V_2 - 50}{5} \qquad i_\phi = \frac{60-50}{5} \qquad i_\phi = \frac{10}{5} \qquad \boxed{i_\phi = 2\,A}$$

More Supernode

Wednesday, February 1, 2017 8:02 AM



Use supernode only for voltage source
-due to not knowing current thru
voltage source

Solution: $I_{entering} \to$ '+' , $I_{leaving} \to$ '−'

At node 1: $(V_2-V_1)3S + (V_3-V_1)4S + (-3A) + (-8A) = 0$

$3(V_2-V_1) + 4(V_3-V_1) - 11 = 0$

$3V_2 - 3V_1 + 4V_3 - 4V_1 - 11 = 0 \Rightarrow -7V_1 + 3V_2 + 4V_3 = 11 \quad 7V_1 - 3V_2 - 4V_3 = -11 \ (1)$

Supernode: $\underbrace{-(-3A) - (V_2-V_1)3S - V_2(1S)}_{\text{node 2}} \ - \underbrace{(V_3-V_1)4S - (V_3)5S - (-25A)}_{3}$

$3 - 3(V_2-V_1) - V_2 - 4(V_3-V_1) - 5V_3 + 25 = 0$

$3 - 3V_2 + 3V_1 - V_2 - 4V_3 + 4V_1 - 5V_3 + 25 = 0$

$7V_1 - 4V_2 - 9V_3 = -28 \quad (2)$

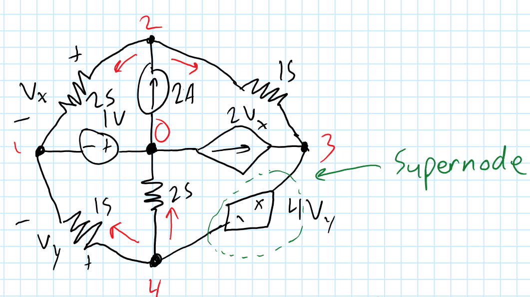
$V_3 - V_2 = \frac{i_x}{8} \Rightarrow -V_2 + V_3 - \frac{i_x}{8} \ (3)$

$(V_1 - V_3)4S = i_x$

$4V_1 - 4V_3 - i_x = 0 \quad (4)$

$V_1 = 1.4314V$
$V_2 = 3.3726V$
$V_3 = 2.7255V$

Ex 2:



Solve: Node 1: $V_1 = -1V \quad (1)$

Node 2: $2A - (V_2-V_1)2S - (V_2-V_3)1S = 0$

$2 - 2V_2 + 2V_1 - V_2 + V_3 = 0$

$2V_1 - 3V_2 + V_3 = 0 \quad (2)$

Supernode: $\underbrace{(V_2-V_3)1S + 2V_x}_{\text{node 3}} \ - \underbrace{V_4(2S) - (V_4-V_1)1S}_{\text{node 4}} = 0$

$V_2 - V_3 + 2V_x - 2V_4 - V_4 + V_1 = 0$

$V_1 + V_2 - V_3 - 3V_4 + 2V_x = 0 \quad (3)$

$V_2 - V_1 = V_x$

$-V_1 + V_2 - V_x = 0 \quad (4)$

$V_3 - V_4 = 4V_y$

$V_3 - V_4 - 4V_y = 0 \quad (5)$

$V_4 - V_1 = V_y \quad (6)$

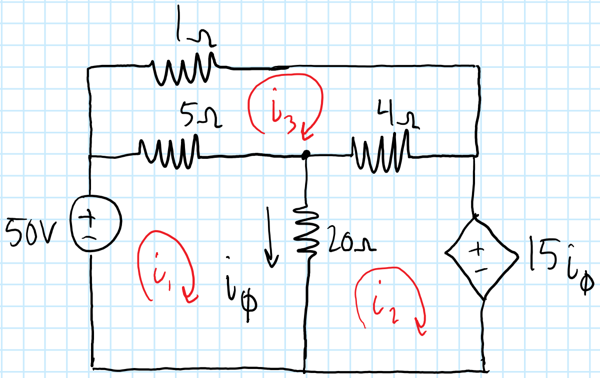
$V_1 = -1V$
$V_2 = \frac{17}{9}V$
$V_3 = \frac{17}{3}V$
$V_4 = -\frac{1}{3}V$

Mesh Analysis

Thursday, February 2, 2017 8:06 AM

Procedure:

a) Define a set of circuit paths
b) Assign a direction to each loop-current
c) Apply KVL to all loops
d) Solve simultaneous equations for loop currents



Solu: $V_{rises} \rightarrow$ '+' ; $V_{drops} \rightarrow$ '−'

Loop 1: $50V - (5\Omega)(i_1 - i_3) - (20\Omega)(i_1 - i_2) = 0$

$50 - 5i_1 + 5i_3 - 20i_1 + 20i_2 = 0$

$\boxed{-25i_1 + 20i_2 + 5i_3 = -50 \quad (1)}$ ✓

Loop 2: $-20\Omega(i_2 - i_1) - 4\Omega(i_2 - i_3) - 15i_\phi = 0$

$-20i_2 + 20i_1 - 4i_2 + 4i_3 - 15i_\phi = 0$

$\boxed{20i_1 - 24i_2 + 4i_3 - 15i_\phi = 0 \quad (2)}$

Loop 3: $-(1\Omega)i_3 - (4\Omega)(i_3 - i_2) - (5\Omega)(i_3 - i_1) = 0$

$-i_3 - 4i_3 + 4i_2 - 5i_3 + 5i_1 = 0$

$\boxed{5i_1 + 4i_2 - 10i_3 = 0 \quad (3)}$ ✓

$i_\phi = i_1 - i_2$

$\boxed{i_1 - i_2 - i_\phi = 0 \quad (4)}$

$20i_1 - 24i_2 + 4i_3 - 15(i_1 - i_2) = 0$

$20i_1 - 24i_2 + 4i_3 - 15i_1 + 15i_2 = 0$

$\boxed{5i_1 - 9i_2 + 4i_3 = 0}$ (2) ✓

$P_{50V} = (50V)(-i_1)$

$P_{5\Omega} = i^2R = (5\Omega)(i_1 - i_3)^2$

$P_{20\Omega} = (20\Omega)(i_1 - i_2)^2$

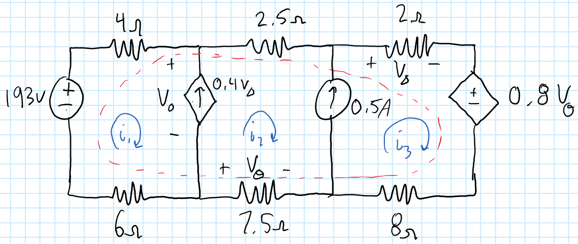
$P_{4\Omega} = (4\Omega)(i_3 - i_2)^2$ ← order does not matter, due to the square

$P_{1\Omega} = (1\Omega)(i_3)^2$

$P_{15 i_\phi} = (15 i_\phi)(i_2)$

Supermesh

Monday, February 6, 2017 8:02 AM



Solution: $V_{rise} \Rightarrow$ '+' ; $V_{drop} \Rightarrow$ '−'

Supermesh: $193V - (4\Omega)i_1 - (2.5\Omega)i_2 - (2\Omega)i_3 - 0.8V_\theta - (8\Omega)i_3 - \underbrace{(7.5\Omega)i_2}_{} - (6\Omega)i_1 = 0$

or V_θ, but $V_\theta = (7.5)(-i_2)$

$193 - 4i_1 - 2.5i_2 - 2i_3 - 0.8V_\theta - 8i_3 - 7.5i_2 - 6i_1 = 0$

$193 - 10i_1 - 10i_2 - 10i_3 - 0.8V_\theta = 0$

$10i_1 + 10i_2 + 10i_3 + 0.8V_\theta = 193$ (1)

$V_\theta = -7.5i_2$ (2)

$i_3 - i_2 = 0.5A$ (3)

$-i_1 + i_2 = 0.4V_\Delta$ (4)

$V_\Delta = 2i_3$ (5)

i_2

if i_2 is assumed to go the opposite direction

Supermesh:

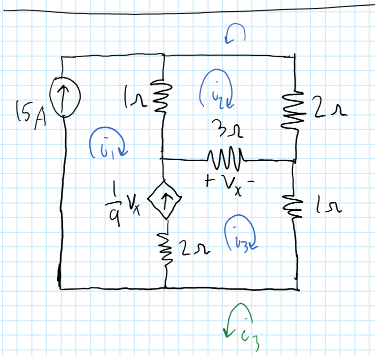
$193 - (4\Omega)i_1 + (2.5\Omega)i_2 - (2\Omega)i_3 - 0.8V_\theta - (8\Omega)i_3 + (7.5\Omega)i_2 - (6\Omega)i_1 = 0$ (1)

$V_\theta = (7.5\Omega)\, i_2$ (2)

$i_2 + i_3 = 0.5A$ (3)

$-i_1 - i_2 = 0.4V_\Delta$ (4)

$V_\Delta = 2i_3$ (5)



$i_1 = 15A$

Solu:

Mesh 2: $(-1\Omega)(i_2 - 15A) - 2\Omega(i_2) - 3\Omega(i_2 - i_3) = 0$ (1)

$i_3 - i_1 = \frac{1}{9}V_x$ (2)

$V_x = (3\Omega)(i_3 - i_2)$ (3)

$\boxed{\begin{aligned} i_1 &= 15A \\ i_2 &= 11A \\ i_3 &= 17A \end{aligned}}$

Mesh 3: $i_3 + i_1 = -\frac{1}{9}V_x$ (1)

$V_x = (-3\Omega)(i_2 + i_3)$ (3)

Mesh 2: $(-1\Omega)(i_2 - 15A) - 2\Omega(i_2) - 3\Omega(i_2 + i_3) = 0$

$-i_2 + 15 - 2i_2 - 3i_2 - 3i_3 = 0 \Longrightarrow -6i_2 - 3i_3 = -15$ (2)

$i_3 + i_1 = \frac{-1}{9}(-3i_2 - 3i_3)$ $\quad i_3 + i_1 = \frac{1}{3}i_2 + \frac{1}{3}i_3$ $\quad \frac{2}{3}i_3 + i_1 - \frac{1}{3}i_2 = 0$

$15 - \frac{1}{3}i_2 + \frac{2}{3}i_3 = 0$ (3)

$i_1 = 15A$

$i_2 = 11A$

$i_3 = -17A$

Mesh 2: $(-2\Omega)i_2 - (1\Omega)(i_2 + 15A) - (3\Omega)(i_2 + i_3) = 0$

$-2i_2 - i_2 - 15 - 3i_2 - 3i_3 = 0 \Longrightarrow -6i_2 - 3i_3 = 15$ (1)

<u>Mesh 3:</u> $i_3 - i_1 = \frac{1}{9} V_x$ (2)

$i_1 = 15A$

$i_2 = -11A$

$i_3 = 17A$

$V_x = 3\Omega(i_2 + i_3)$ (3)

$i_3 - i_1 = \frac{1}{9}(3i_2 + 3i_3)$

$i_3 - i_1 = \frac{1}{3} i_2 + \frac{1}{3} i_3$

$\frac{2}{3} i_3 - \frac{1}{3} i_2 - 15 = 0$ (4)

Thevenin's and Norton's Theorem

Tuesday, February 7, 2017 8:09 AM

1) Read Section 4.11-4.13
2) Do Hw#3, which will be due on $\boxed{\text{Monday 2/20}}$

   Hw#3: Ch 4 #79, 80, 81, 87, 88, 89, 97, 98, 99
3) Quiz #1 Monday 2/13
4) Test #1 Tuesday 2/28

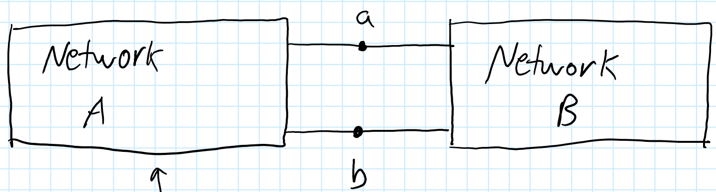
Thevenin's Theorem

Given any linear circuit, rearrange it in the form of two networks A and B connected by two resistanceless conductors. If either network contains a dependent source, its control variable must be in the same network. Define a voltage V_{oc} as the open-circuit voltage which would appear across the terminals of A if B were disconnected so that no current is drawn from A. Then, all the currents and voltages in B will remain unchanged if all independent voltage sources in A are short circuited and all independent current sources in A are open circuits, and an independent voltage source V_{oc} is connected with proper polarity in series with the inactive A network.

Pictorial Description of Thevenin and Norton Theorem

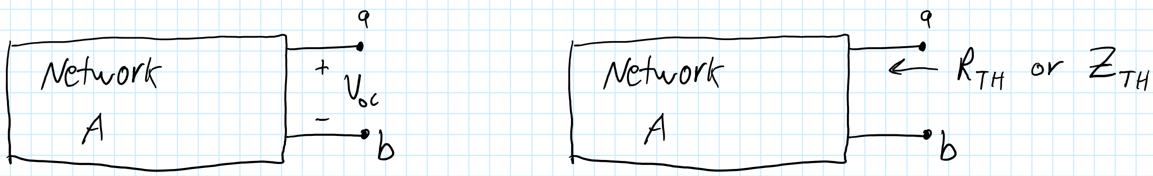
Wednesday, February 8, 2017 8:07 AM

1) Divide network into smaller networks A and B



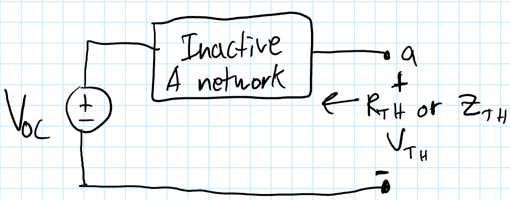
If dependent source here, its control variable must be here also

2) Remove network A and tie V_{oc} and inactive A network (resistor for dc, impedance for ac)



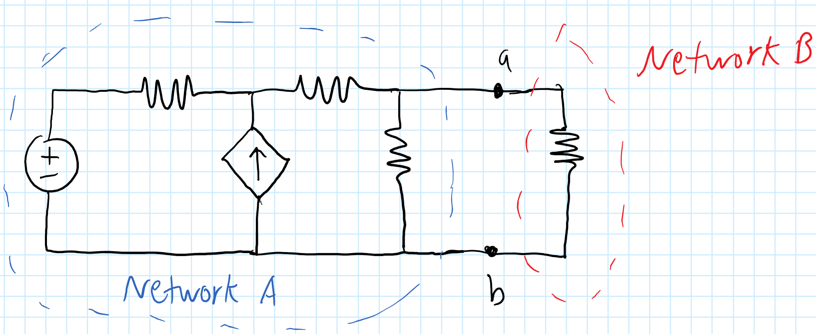
short circuits all independent voltage sources and open circuits all indep. current sources

3) Connect V_{oc} and Inactive network A in series.



$V_{oc} = V_{TH}$ since no current thru inactive A network (i.e. no voltage drop)

Ex:

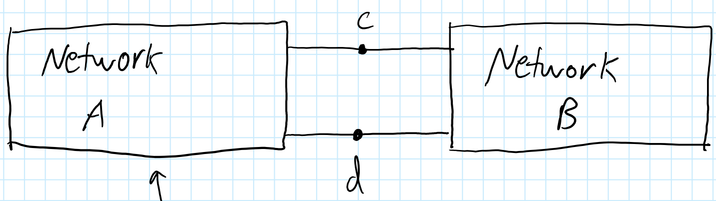


Norton's Theorem

Given any linear circuit, rearrange it in the form of two networks A and B that are connected together by two resistanceless conductors. If either network contains a dependent source, its control variable must be in that same network. Define a current i_{sc} as the short-circuit current which would appear at the terminals of A if B were short-circuited so that no voltage is provided by A. Then all the voltages and currents in B will remain unchanged if all independent voltage sources in A are short-circuited and all independent current sources in A are open-circuited, and an independent current source i_{sc} is connected with proper polarity in parallel with the inactive A network.

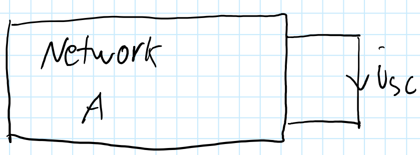
Pictorial Description of Norton Theorem

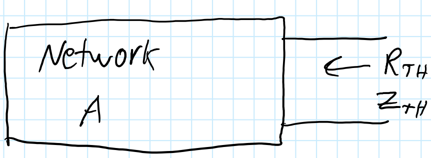
1) Divide network into smaller networks A and B



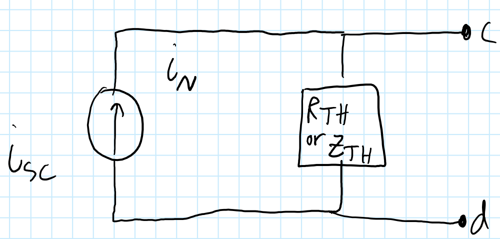
If dependent source here, its control variable must be here also

2) Remove Network A and find i_{sc} and inactive network A (resistor for dc, impedance for ac).





3) Connect i_{sc} and inactive A network in parallel.

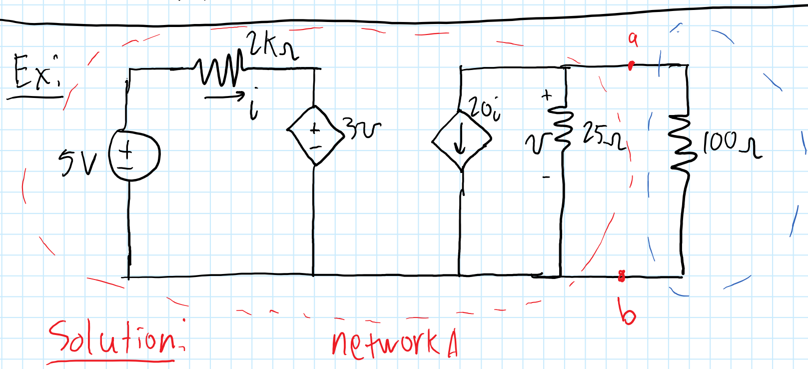


Relationship btwn Thevenin and Norton theorems are:

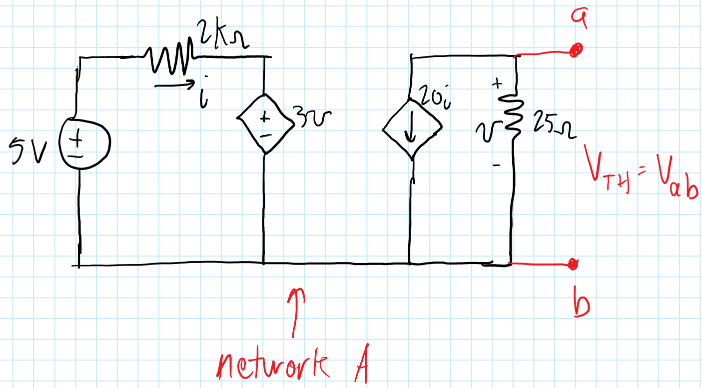
$V_{TH} = i_N * R_N$

$$i_N = \frac{V_{TH}}{R_{TH}}$$

Ex: Find its Thevenin equivalent circuit



Solution:



$$V_{TH} = V_{ab} = (-20i)(25\Omega)$$

$$i = \frac{5V - 3v}{2k\Omega} = \frac{5V - 3V_{TH}}{2k\Omega}$$

$$i = \frac{5 - 3V_{TH}}{2000}$$

$$V_{TH} = -20\left(\frac{5 - 3V_{TH}}{2000}\right)(25)$$

$$V_{TH} = \left(\frac{-5}{100} + \frac{3V_{TH}}{100}\right)25$$

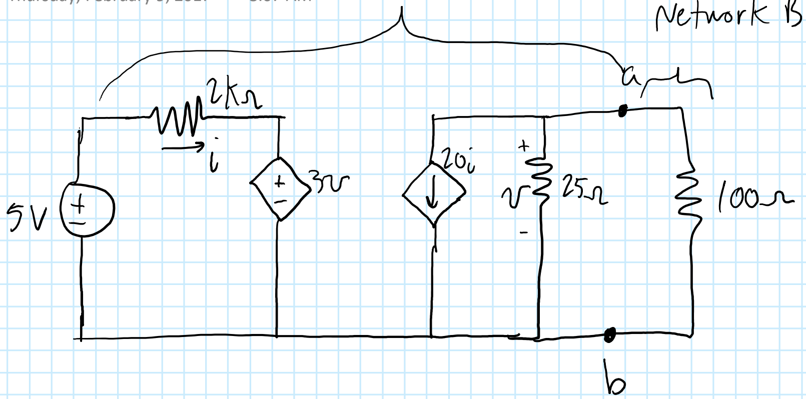
$$V_{TH} = \frac{-5}{4} + \frac{3V_{TH}}{4}$$

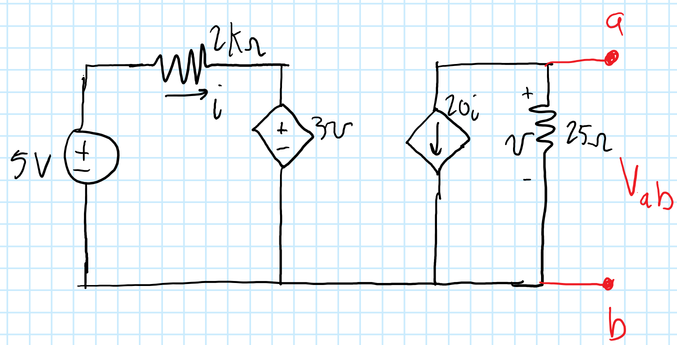
$$4V_{TH} = -5 + 3V_{TH}$$

$$\boxed{V_{TH} = -5\ V}$$

Thevenin Example

Thursday, February 9, 2017 8:07 AM





$V_{ab} = V_{oc} = V_{TH}$

$V_{TH} = V_{ab} = (-20i)(25\Omega)$ (1)

$i = \frac{5V - 3v}{2000\Omega} = \frac{5 - 3V_{TH}}{2000}$ (2)

Substitute (2) $\rightarrow$ (1)

$V_{TH} = \left(-20 \cdot \frac{5 - 3V_{TH}}{2000}\right)(25)$

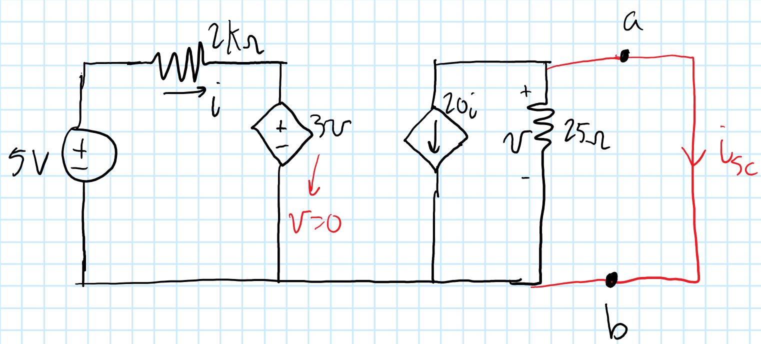
$V_{TH} = -\left(\frac{5 - 3V_{TH}}{4}\right)$

$4V_{TH} = -5 + 3V_{TH}$

$V_{TH} = -5V$

R_{TH}

(1) $R_{TH} = \frac{V_{TH}}{i_{sc}}$



$v = \emptyset$ (shorted)

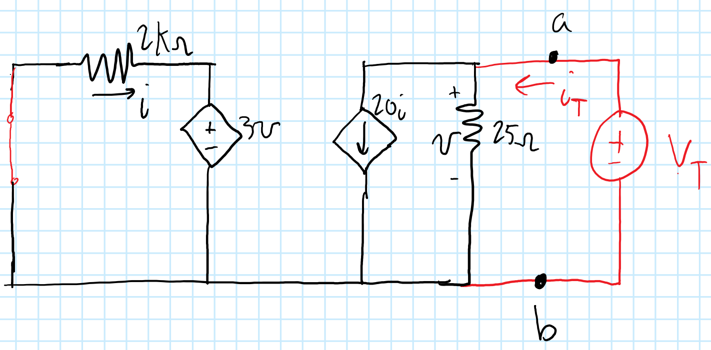
$i = \frac{5V}{2000\Omega} = 2.5mA$

$i_{sc} = -20i = -20(2.5mA) = -50mA$

$R_{TH} = \frac{-5V}{-50mA} = 100\Omega$

(2) $R_{TH} = V_T$ (using test voltage, V_T)

② $R_{TH} = \frac{V_T}{i_T}$ (using test voltage V_T)



$3v = 3V_T$

$i_{enter} = '+'$; $i_{leave} = '-'$

KCL at a:

$$i_T - \frac{V_T}{25\Omega} - 20i = 0 \quad (1)$$

$$i = \frac{-3V_T}{2000\Omega} \quad (2)$$

Substitute (2) into (1)

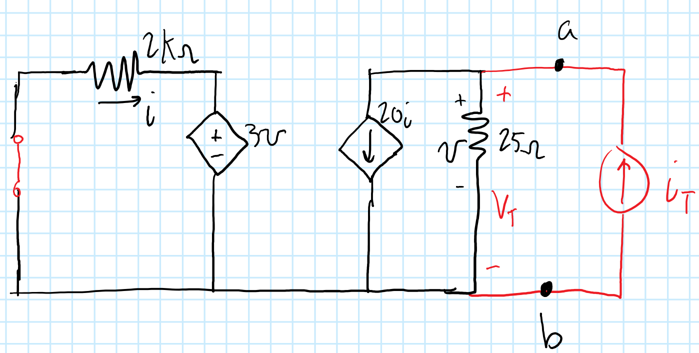
$$i_T = \frac{V_T}{25} + 20\left(\frac{-3V_T}{2000}\right)$$

$$i_T = \frac{V_T}{25} - \frac{3V_T}{100}$$

$$i_T = \frac{4V_T - 3V_T}{100}$$

$$i_T = \frac{V_T}{100} \qquad \frac{V_T}{i_T} = 100\Omega = R_{TH}$$

③ $R_{TH} = \frac{V_T}{i_T}$ (using test current i_T)



$$V_T = 25\Omega(i_T - 20i)$$

$$V_T = (25)(i_T - 20i) \quad (1)$$

$$i = \frac{-3V_T}{2k\Omega} \quad (2)$$

$$V_T = 25\left(i_T - 20\left[\frac{-3V_T}{2000}\right]\right)$$

$$V_T = 25 i_T + 25(3V_T)$$

$$V_T = 25 i_T + 25\left(\frac{3V_T}{100}\right)$$

$$V_T = 25 i_T + \frac{3V_T}{4}$$

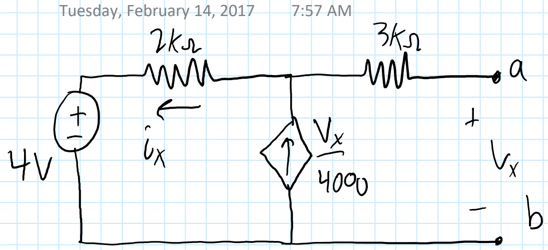
$$V_T - \frac{3V_T}{4} = 25 i_T$$

$$\frac{1}{4} V_T = 25 i_T$$

$$\frac{V_T}{i_T} = 100\ \Omega$$

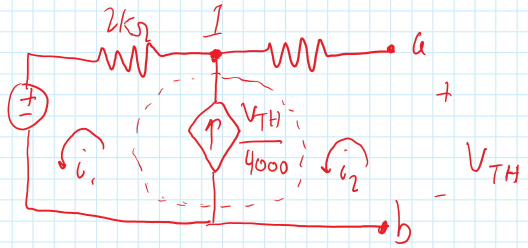
Another Thevenin Example

Tuesday, February 14, 2017 7:57 AM



Find Thevenin equivalent for the circuit

Solu:



Using Supermesh:

V_{rise} = '+' ; V_{drop} = '−'

I_{enter} = '+' ; I_{leave} = '−'

$$-4V - (2k\Omega)i_1 - (3k\Omega)i_2 + V_{TH} = 0$$

$$-4 - 2000 i_1 - 3000 i_2 + V_{TH} = 0 \quad (1)$$

$$i_2 = 0 \quad (2)$$

At node 1:

$$i_x - \frac{V_{TH}}{4000} - i_2 = 0 \quad (3)$$

$$i_1 = i_x \quad (4)$$

Substitute (4) into (3) $\quad i_1 - \frac{V_{TH}}{4000} - i_2 = 0 \quad (5)$

substitute (2) into (5)

$$i_1 - \frac{V_{TH}}{4000} = 0 \quad (6)$$

Substitute (2) into (1)

$$-4 - 2000 i_1 + V_{TH} = 0 \quad (7)$$

$$i_1 = \frac{V_{TH}}{4000} \quad (8)$$

Substitute (8) into (7)

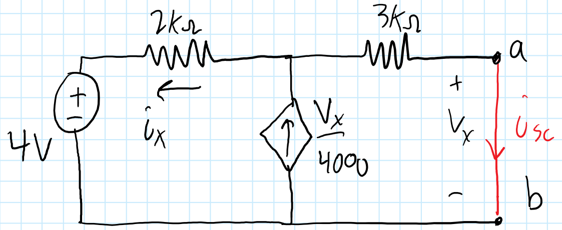
$$-4 - 2000\left(\frac{V_{TH}}{4000}\right) + V_{TH} = 0$$

$$-4 - \frac{V_{TH}}{2} + V_{TH} = 0$$

$$\frac{V_{TH}}{2} = 4 \qquad \boxed{V_{TH} = 8V}$$

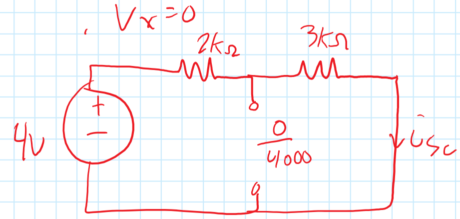
R_{TH}:

1) $R_{TH} = \frac{V_{TH}}{i_{SC}} = \frac{8V}{0.8mA} = \boxed{10K\Omega}$

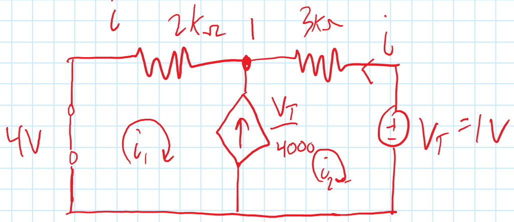


$i_{sc} = \frac{4}{2k+3k} = 0.8mA$

$\therefore V_x = 0$



2) $R_{TH} = \frac{V_T}{i}$



$V_{drop} =$ '+' ; $V_{rise} =$ '−' ; $I_{enter} =$ '+' ; $I_{leave} =$ '−'

Supermesh: $2k\Omega(i_1) + 3k\Omega(i_2) + 1 = 0$

$2000 i_1 + 3000 i_2 = -1$ (1)

at node 1: $i_1 + \frac{1}{4000} - i_2 = 0$

$i_1 = i_2 - \frac{1}{4000}$ (2)

Substitute (2) into (1)

$2000\left(i_2 - \frac{1}{4000}\right) + 3000 i_2 = -1$

$2000 i_2 - \frac{1}{2} + 3000 i_2 = -1$

$5000 i_2 = -\frac{1}{2}$

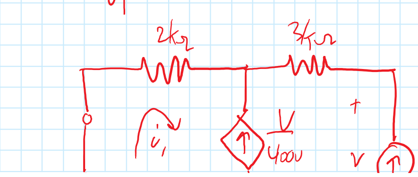
$i_2 = \frac{-1}{10{,}000}$ A

$i = -i_2 = \frac{1}{10{,}000}$

$R_{TH} = \frac{V_T}{i} = \frac{1V}{\frac{1}{10{,}000}A} = \boxed{10k\Omega}$

3) $R_{TH} = \frac{V}{i_T}$

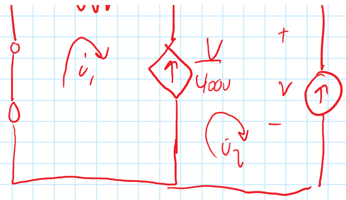
$i_T = 1A$



Supermesh:

$2k\Omega(i_1) + (3k\Omega) i_2 + V = 0$

$2000 i_1 + 3000 i_2 + V = 0$ (1)



$2k\Omega(i_1) + (3k\Omega)i_2 + V = 0$

$2000 i_1 + 3000 i_2 + V = 0 \qquad (1)$

Node i_1

$$i_1 + \frac{V}{4000} - i_2 = 0$$

$$i_1 = i_2 - \frac{V}{4000} \qquad (2)$$

sub (2) into (1)

$$2000\left(i_2 - \frac{V}{4000}\right) + 3000 i_2 + V = 0$$

$$2000 i_2 - \frac{V}{2} + 3000 i_2 + V = 0$$

$$5000 i_2 + \frac{V}{2} = 0$$

$$\frac{V}{2} = -5000 i_2$$

$$i_2 = -i_T = -1$$

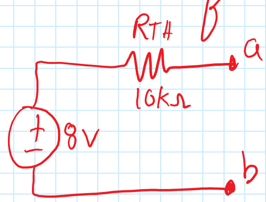
$$\frac{V}{2} = 5000$$

$V = 10{,}000\ V$

$$R_{TH} = \frac{V}{i_T}$$

$$= \frac{10000}{1} = \boxed{10k\Omega}$$

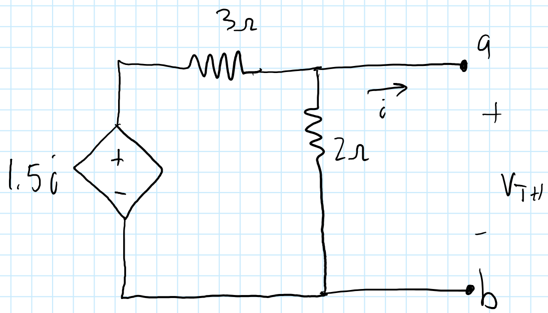
Thevenin Eqivalent



Superposition

Wednesday, February 15, 2017 8:02 AM

Ex: Find the Thevenin Equivalent of the circuit shown below.



Solu: $V_{TH} = V_{ab} = 0$

$\therefore i = 0$

or

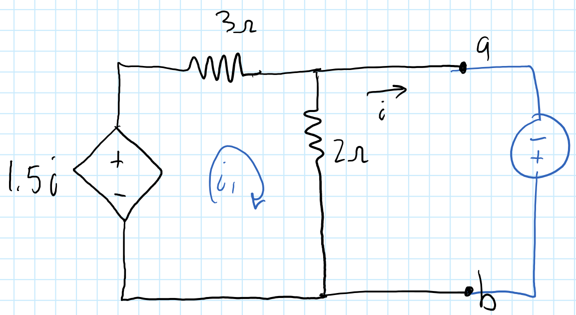
$V_{TH} = (2\Omega)(i_1 - i) = 0$

$i = 0$

$i_1 = 0$

$R_{TH} = ?$

$R_{TH} = \frac{V_T}{i}$



$V_T = 1V$

$V_{rises} = '-'$; $V_{drops} = '+'$

Left Mesh: $-1.5i + 3\Omega(i_1) + 2\Omega(i_1 - i) = 0$

$-1.5i + 3i_1 + 2i_1 - 2i = 0$

$5i_1 - 3.5i = 0$ (1)

Right Mesh: $2\Omega(i - i_1) - 1V = 0$

$2i - 2i_1 - 1 = 0$

$2i - 2i_1 = 1$ (2)

$i_1 = \frac{2i - 1}{2}$ (3)

Substitute (3) into (1)

$5\left(\frac{2i-1}{2}\right) - 3.5i = 0$

$\frac{10i - 5}{2} - 3.5i = 0$

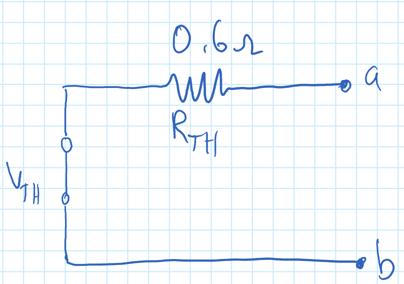
$10i - 5 - 7i = 0$

$3i = 5$ $\quad i = 5/3\,A$

$R_{TH} = \frac{V_T}{i}$

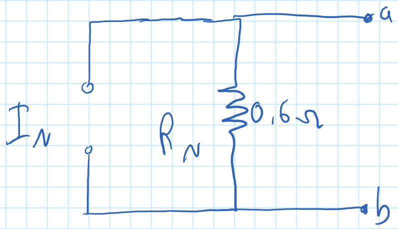
$R_{TH} = \frac{1V}{5/3 A}$

$R_{TH} = 0.6\Omega$



$\Rightarrow$ Thevenin Equivalent

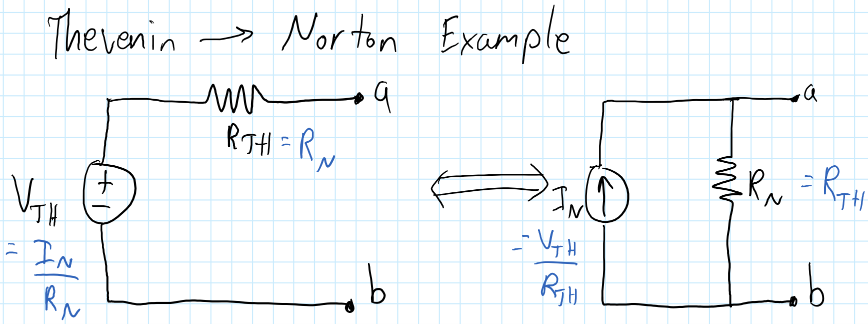
$V_{TH} = I_N R_N = (0)(0.6\Omega) = 0V$

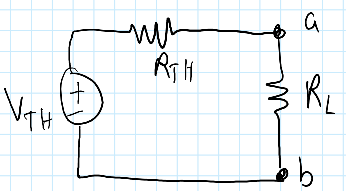


⇒ Norton Equivalent

$$I_n = \frac{V_{TH}}{R_{TH}} = \frac{0\,V}{0.6\,\Omega} = 0\,A$$

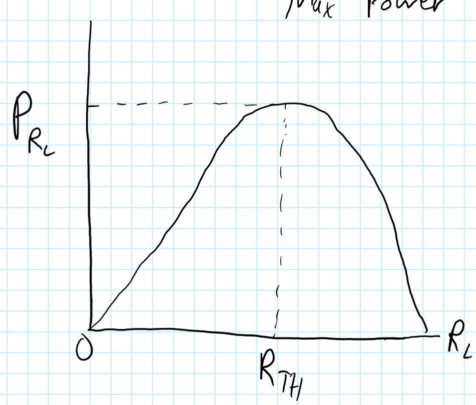
Thevenin → Norton Example





$$P_{R_L} = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

Max Power transfer when $R_L = R_{TH}$



Superposition Theorem

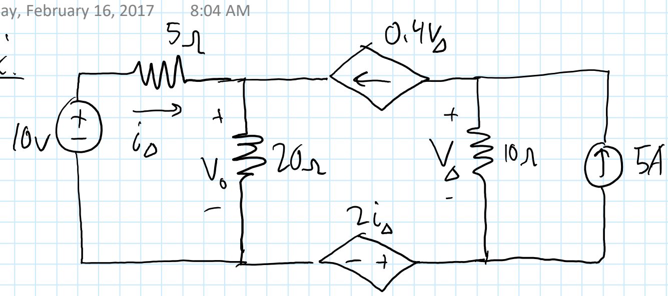
In any linear network containing several sources, the voltage across or the current through any passive element or source may be calculated by adding algebraically all the individual voltages or currents caused by each independent source acting alone, with all independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits. Dependent sources must be left in the network.

* Cannot short dependent sources because we do not know the inner resistance

Superposition Examples

Thursday, February 16, 2017 8:04 AM

Ex:



Use the principle of superposition to find V_o.

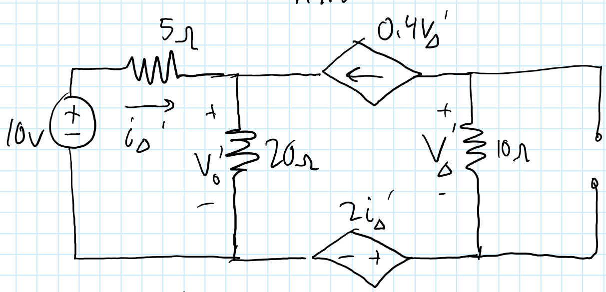
$V_\Delta' = -0.4V_\Delta'(10)$

$V_\Delta' = -4V_\Delta'$

$V_\Delta(5) = 0$

Solution:

Consider 10 V



at V_0' (using node)

$$\frac{10\,V - V_o'}{5\Omega} - \frac{V_o'}{20\Omega} + 0.4V_\Delta' = 0 \quad (1)$$

$$V_\Delta' = -0.4V_\Delta'(10\Omega)$$

$$V_\Delta' = 0 \quad (2)$$

Substitute (2) into (1)

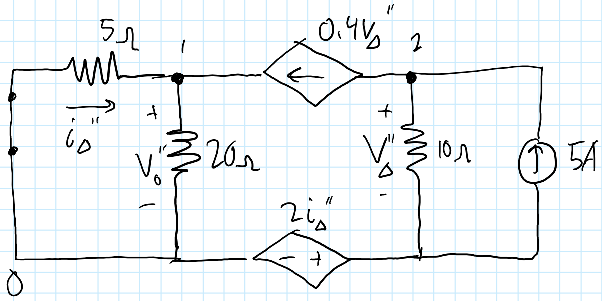
$$\frac{10 - V_o'}{5} - \frac{V_o'}{20} + 0.4(0) = 0$$

$$4(10 - V_o') - V_o' = 0$$

$$40 - 4V_0' - V_0' = 0$$

$$5V_o' = 40 \qquad \boxed{V_o' = 8V}$$

Consider 5A only:



$$i_\Delta'' = \frac{-V_o''}{5\Omega} \quad (3)$$

Node 1: I_{enter} = '+' ; I_{leave} = '−'

$$i_\Delta'' - \frac{V_o''}{20\Omega} + 0.4V_\Delta'' = 0 \quad (1)$$

Node 2:

$$V_\Delta'' = V_2 - 2i_\Delta''$$

$$-0.4V_\Delta'' - \frac{V_\Delta''}{10\Omega} + 5A = 0$$

$$-4V_\Delta'' - V_\Delta'' + 50 = 0$$

$$V_\Delta'' = 10 \qquad (2)$$

Substitute (2) and (3) into (1)

$$\frac{-V_0''}{5\Omega} - \frac{V_0''}{20\Omega} + 0.4(10) = 0$$

$$-4V_0'' - V_0'' + 80 = 0$$

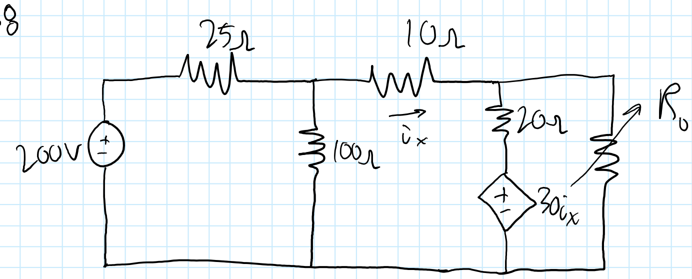
$$5V_0'' = 80 \qquad \boxed{V_0'' = 16V}$$

$$V_0 = V_0' + V_0''$$

$$V_0 = 8V + 16V$$

$$\boxed{V_0 = 24V}$$

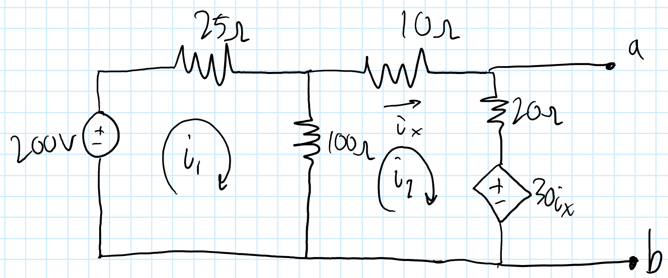
4.88



The R_0 is adjusted until the $P_{R0} = 250\,W$

Find $R_0 = ?$

Solution:



$V_{th} = ?$

$R_{th} = ?$

$$P_{R0} = \left(\frac{V_{th}}{R_{th} + R_0}\right)^2 R_0$$

$$250 = \_\_\_\_$$

Mesh

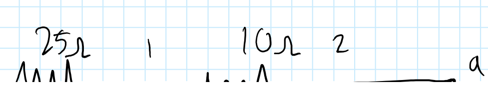
Loop 1: $200V - 25\Omega(i_1) - 100\Omega(i_1 - i_2) = 0$

Loop 2: $-100\Omega(i_2 - i_1) - 10\Omega(i_2) - 20\Omega(i_2) - 30i_x = 0$

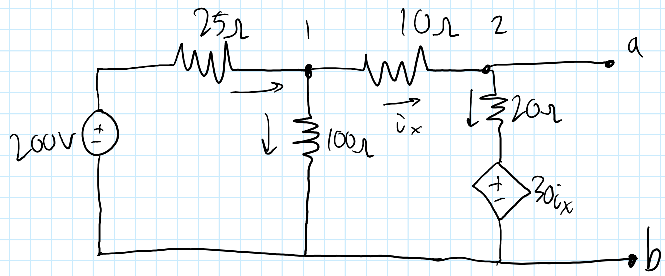
$i_x = i_2$

$V_{th} = (20\Omega)(i_2) + 30\underset{i_2}{i_x} = C$

Node



Node



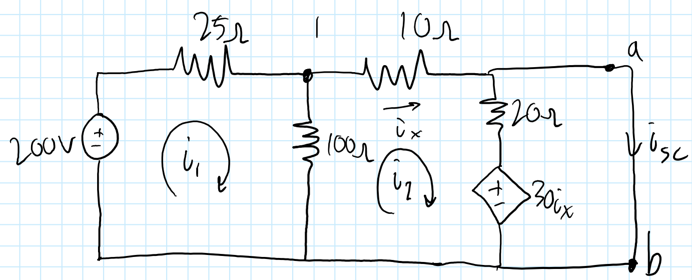
Node 1:
$$\frac{200 - V_1}{25\Omega} - \frac{V_1}{100\Omega} - \frac{V_1 - V_2}{10\Omega} = 0 \quad (1)$$

Node 2:
$$\frac{V_1 - V_2}{10\Omega} - \frac{V_2 - 30i_x}{20\Omega} = 0 \quad (2)$$

$$i_x = \frac{V_1 - V_2}{10\Omega} \quad (3)$$

$$V_2 = V_{th} = C$$

R_{Th}:



C, d, e are just values when you solve the numbers

Node:
$$\frac{200 - V_1}{25\Omega} - \frac{V_1}{100\Omega} - \frac{V_1}{10\Omega} = 0$$

$$V_1 = d$$

$$i_x = \frac{V_1}{10} = i_{sc} = e$$

$$R_{Th} = \frac{V_{th}}{i_{sc}} = \frac{C}{e}$$

Mesh: Loop 1:
$$200 - 25i_1 - 100(i_1 - i_2) = 0$$

Loop 2:
$$-100(i_2 - i_1) - 10(i_2) = 0$$

$$i_{sc} = i_2$$

Intro to RL and RC Circuits

Monday, February 20, 2017 8:05 AM

No class this wednesday

1.) Read Ch 7.

2.) Do HW #4 – Chapter 7: 1, 2, 3, 7, 8, 14, 15, 21, 23, 25, 28, 31, 35, 39, 44, 45, 60, 61, 85

3) HW #4 due on Thursday, 3/09

4) Test 1: HW #1-3

Ch. 7: RL + RC Circuits

I. Basic Definition + concepts

A. Equivalent terminology

Natural response = transient response

= source frequency = complementary function

B. Linear constant coefficient Differential Equations

1. General Form

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_0 y = m(t) \qquad (1)$$

Where $a_0, a_1, \ldots, a_{n-1}$ are all constants

If $m(t) = 0$, equation (1) is called homogeneous, if not, its called nonhomogeneous

Example of RL Circuit

Tuesday, February 21, 2017 8:01 AM

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \dots + a_n y = m(t)$$

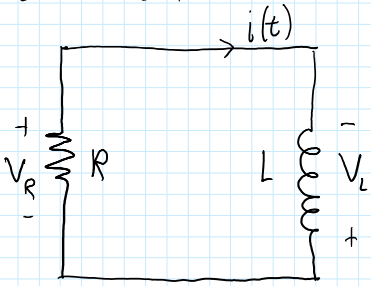
2. Solution of equation (1)

   ① Expression for $y(t)$ that satisfies for all $t > 0$.

   ② $y(t)$ satisfies the given initial conditions at $t=0$.

II. Source-Free RL circuit

A. Circuit Model



Let $i(0) = I_0$ constant

$$V_L = L\frac{di}{dt}$$

KVL:

$$V_R + V_L = 0 = iR + \underbrace{L\frac{di}{dt}}_{\text{Lenz' Law}} = 0$$

$$L\frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0 \Rightarrow \text{1st order homogenous d.e.} \qquad (1)$$

B. Solution for $i(t)$

1. Method 1 - Substitute into general solution.

   $$i(t) = I_0 e^{-(R/L)t} \quad \text{for } t \geq 0$$

   comes from $y(t) = Ae^{-st}$

2. Method 2 - Assume a solution with unknown coefficients

   Let $i(t) = Ae^{s_1 t}$ where A, s_1 are unknown

   Substitute $i(t)$ into (1)

   $$\frac{dAe^{s_1 t}}{dt} + \frac{R}{L}Ae^{s_1 t} = 0$$

   $$s_1 Ae^{s_1 t} + \frac{R}{L}Ae^{s_1 t} = 0$$

$$dt \qquad L$$

$$s_1 A e^{s_1 t} + \frac{R}{L} A e^{s_1 t} = 0$$

$$\left(s_1 + \frac{R}{L}\right) A e^{s_1 t} = 0$$

$$s_1 + \frac{R}{L} = 0$$

$$s_1 = -\frac{R}{L}$$

$$\therefore\ i(t) = A e^{(-R/L)t} \quad \text{for } t \geqslant 0$$

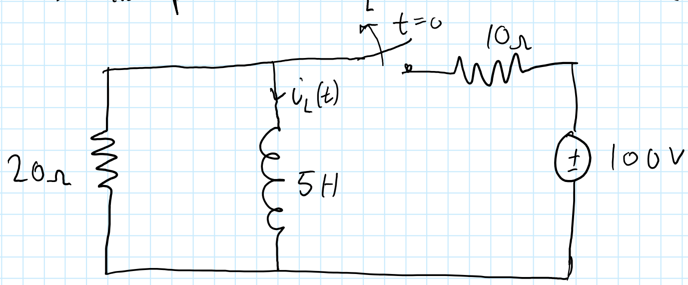
at $t=0$, $i(0) = A e^{(-R/L)0} = A = I_0$

$$\therefore\ i(t) = I_0\, e^{(-R/L)t},\ t \geqslant 0$$

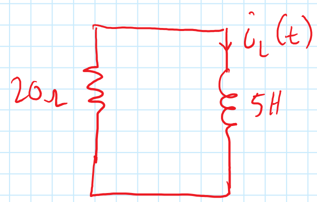
C. RL Circuit Sample Problems

1. Case 1: Problem reduces to one resistor and one inductor.

a) Example 1: Find $i_L(t)$ for $t \geqslant 0$ in the circuit below

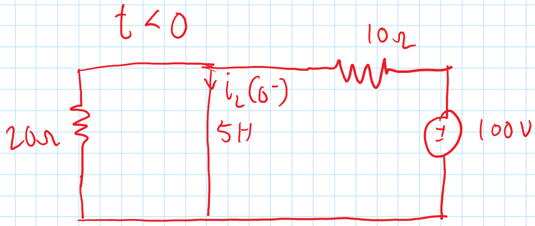


Solution: $t \geqslant 0$



$$i_L(t) = I_0 e^{\left(-\frac{R}{L}\right)t} = I_0\, e^{\frac{-20}{5}t} = I_0\, e^{-4t},\ t \geqslant 0$$

$t < 0$



inductor is short circuit since $\frac{di}{dt}$ is $\emptyset$, meaning V_{drop} across L is $\emptyset$, meaning it is a short.

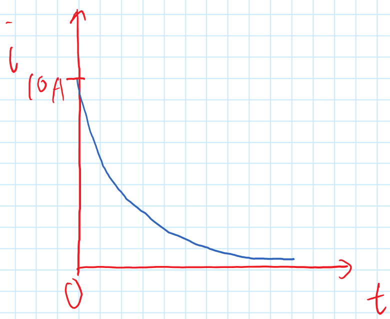
$$i_L(0^-) = \frac{100V}{10\Omega} = 10A = I_0$$

Current cannot be changed instantanously on an inductor, but voltage can

$$\boxed{i_L(t) = 10e^{-4t},\ t \geqslant 0}$$

$$\frac{R}{} = \tau$$

$$\boxed{i_L(t) = 10e^{-4t}, \quad t \geq 0}$$



$$\frac{R}{L} = \tau$$

finished after 5 τ, since e^{-5} is less than 1%

$$\underbrace{i_L(t) = 10\left(\frac{1}{e^{4t}}\right)}$$

never goes to $\emptyset$

(1) $t = 0.25$

$e^{-1} = 0.67$

(5) $t = 1.25$

$e^{-5} = 0.0067$

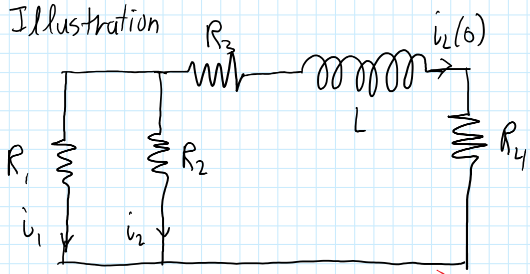
More RL Examples

Thursday, February 23, 2017 8:05 AM

2.) Case 2: One inductor and several resistors

a) Approach - find equivalent resistance and time constant

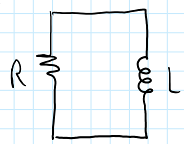
b) Illustration



Let $t=0$ and $i_L(0) = I_0$

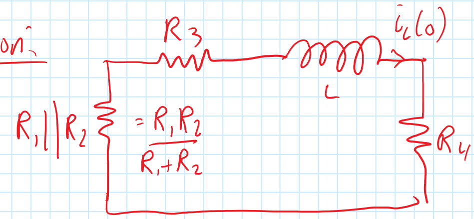
Find $i_1(t)$ and $i_2(t)$ for $t \geq 0$

Remember:

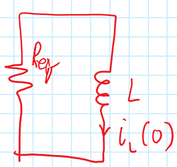


$$\frac{di}{dt} + \frac{R}{L} i = 0$$

Solution:



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_4$$



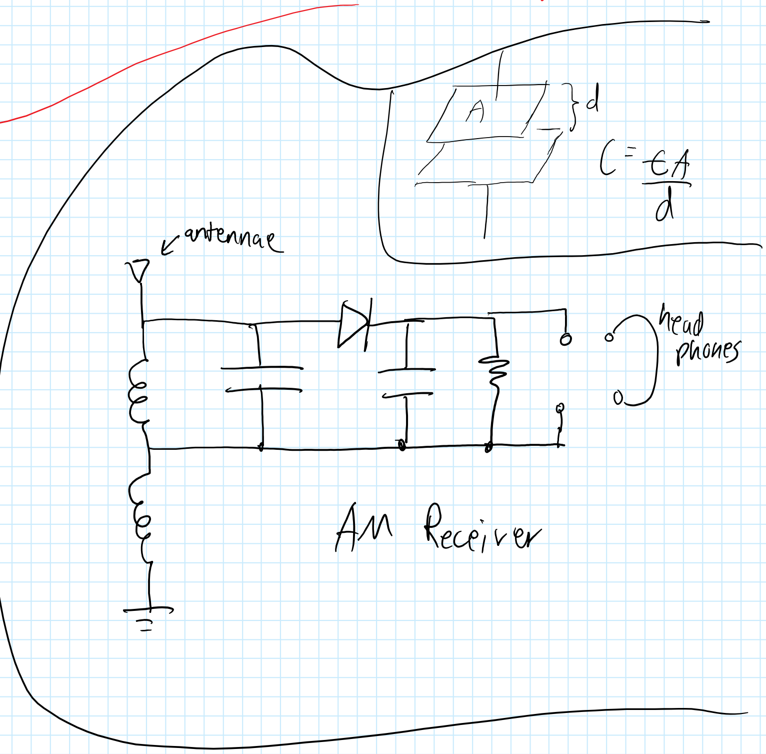
$$i_L(t) = i_L(0) e^{-t/\tau} \qquad \tau = \frac{L}{R_{eq}}$$

$$i_L(t) = i_L(0) e^{\left(-\frac{R_{eq}}{L}\right) t}, \quad t \geq 0$$

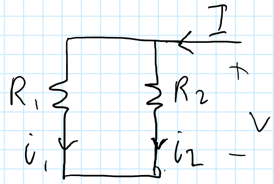
$$i_1(t) = \frac{R_2}{R_1 + R_2} i_L(t)$$

$$= \boxed{\frac{R_2}{R_1 + R_2}\left[i_L(0) e^{-t/\tau}\right] A, \quad t \geq 0}$$

$$i_2(t) = \boxed{\frac{R_1}{R_1 + R_2}\left[i_L(0) e^{-t/\tau}\right] A, \quad t \geq 0}$$



Current Divider



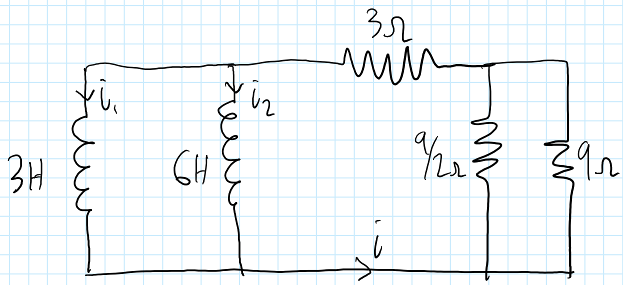
$$I = i_1 + i_2$$

$$i_1 = \frac{V}{R_1} = \frac{(R_1 || R_2) I}{R_1} = \frac{\frac{R_1 R_2}{R_1 + R_2} I}{R_1} = \frac{R_2}{R_1 + R_2}(I)$$

$$i_2 = \frac{V}{R_2} = \frac{(R_1 || R_2) I}{R_2} = \frac{R_1 R_2}{R_1 + R_2}(I) \quad R$$

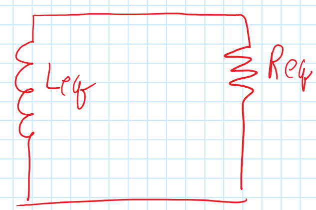
$$i_2 = \frac{V}{R_2} = \frac{(R_1 || R_2) I}{R_2} = \frac{\frac{R_1 R_2}{R_1 + R_2}(I)}{R_2} = \frac{R_1}{R_1 + R_2}(I)$$

3) <u>Case 3:</u> Serial Inductors and Resistors reduceable to one inductor and resistor



Find $i(t) = ?$ for $t \geq 0$
when $i_1(0) = 1A$, $i_2(0) = 2A$

<u>Solution:</u>



$$L_{eq} = 3H || 6H = \frac{3 \times 6}{3+6} = 2H$$

$$R_{eq} = \left(\tfrac{9}{2}\Omega \,||\, 9\Omega\right) + 3\Omega = 3 + \left(\frac{\frac{9}{2} \cdot 9}{\frac{9}{2} + 9}\right)$$

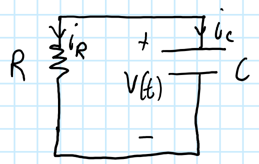
$$= 3 + \left(\frac{\frac{81}{2}}{\frac{27}{2}}\right) = 3 + 3 = 6\Omega$$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{2H}{6\Omega} = \frac{1}{3}$$

$$i(t) = i(0)\, e^{-t/\tau}$$
$$= i(0)\, e^{-t/\tau}$$
$$= (i_1(0) + i_2(0))\, e^{-3t}$$
$$= (1+2)\, e^{-3t}$$
$$= \boxed{3e^{-3t}\ A, \quad t \geq 0}$$

III. Source-Free RC Circuits

A. Circuit Model



$Q = CV$

$I = \frac{Q}{t} = C\frac{V}{t}$

$\frac{di}{dt} = C\frac{dV}{dt}$

$i_R + i_c = 0$ (KCL)

$\frac{V}{R} + C\frac{dV}{dt} = 0$

$\frac{dV}{dt} + \frac{1}{RC}(V) = 0$

where $V(0) = V_0$

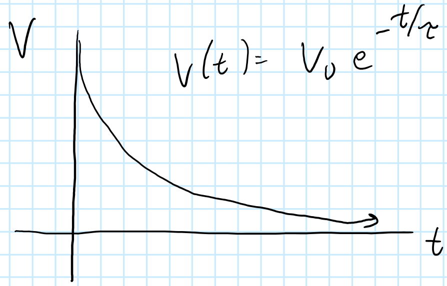
B. Solution for V(t)

$V(t) = V_0 e^{-t/\tau} = V_0 e^{-t/RC}, \; t \geq 0$

$\tau = RC \rightarrow$ time constant, how long it takes to charge/discharge [s]

[s] [Ω][F]

$= \frac{V}{I} \cdot \frac{Q}{V} = \frac{Q}{I} = t$

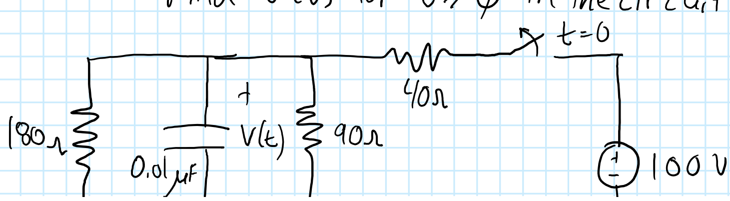


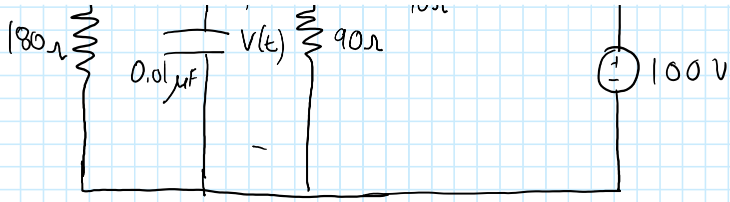
C. RC Circuit Sample Problem

1. Case 1: One Capacitor and one or more resistors

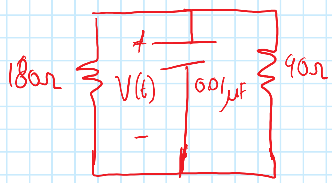
a) Example 1:

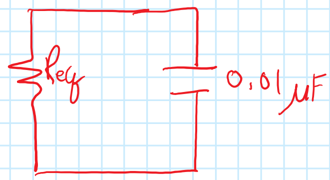
Find $V(t)$ for $t \geq 0$ in the circuit below





Solu: $t \geq 0$



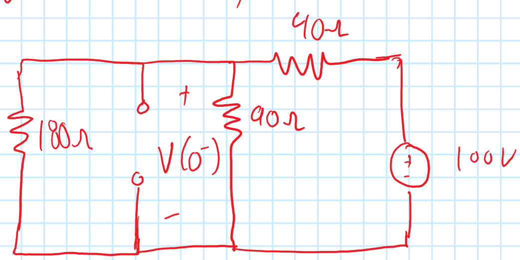


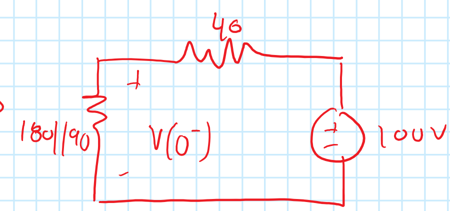
$$R_{eq} = 180\Omega \| 90\Omega = \frac{180 \cdot 90}{180+90} = 60\Omega$$

$$\tau = R_{eq}(C) = (60\Omega)(0.01\times10^{-6}\ F) = 0.6\,\mu s$$

$$v(t) = V_0 e^{-t/0.6\times10^{-6} s}, \quad t \geq 0$$

$t < 0$





$$V(0^-) = 100V\left(\frac{180\|90}{(180\|90)+40}\right)$$

$$= 100\left(\frac{60}{60+40}\right) = 60V = V(0)$$

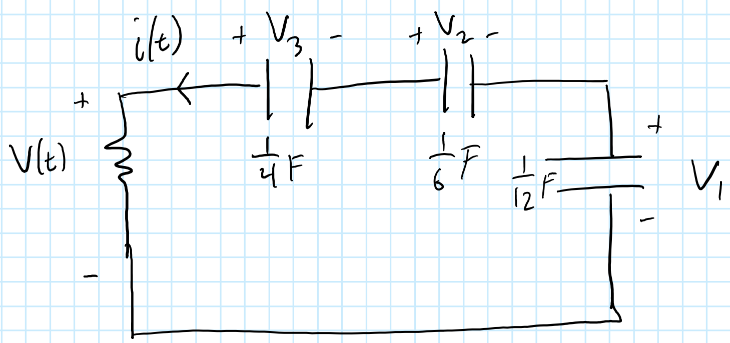
$$v(t) = 60\, e^{-t/0.6\times10^{-6}}, \quad t \geq 0$$

2) Case 2: Several Capacitors and Resistors reducible to one capacitor and resistor

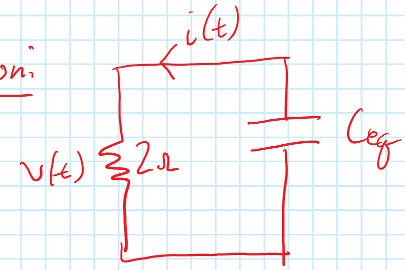
a) Example

Let $V_1(0) = 6V$, $V_2(0) = 4V$, $V_3(0) = 1V$

Find $V_1(t)$, $V_2(t)$, $V_3(t)$, $i(t)$ for $t \geq 0$



Solution:



$$\frac{1}{C_{eq}} = \frac{1}{1/4\,F} + \frac{1}{1/6\,F} + \frac{1}{1/12\,F}$$

$$= 4 + 6 + 12 = 22\,F$$

$$C_{eq} = \frac{1}{22}\,F$$

$$\tau = RC_{eq} = (2\Omega)\left(\frac{1}{22}F\right) = \frac{1}{11}\,s$$

$$V(t) = V(0)e^{-t/\tau} = V(0)e^{-t/1/11}$$

$$= V(0)e^{-11t}$$

$$= [V_1(0) + V_2(0) + V_3(0)]\,e^{-11t}$$

$$= [6+4+1]\,e^{-11t}\ V,\ t \geq 0$$

$$i(t) = \frac{V(t)}{R} = \frac{11e^{-11t}}{2} = \boxed{\frac{11}{2}e^{-11t}\ A,\ t \geq 0}$$

$Q = CV$

$$i(t) = C_1\frac{dV_1}{dt}$$

$$i(t) = -\frac{1}{12}\frac{dV_1}{dt} \Rightarrow 12i(t) = \frac{-dV_1}{dt} \qquad 12i(t)dt = -dV_1$$

$V_1(t) = \int^t$

$$12 \qquad dt \qquad\qquad \overline{dt}$$

$$V_1(t) = -\int_0^t 12 i(t) dt + V_1(0)$$

$$= -\int_0^t 12 \cdot \frac{11}{2} e^{-11t} dt + 6$$

$$= \frac{(12)\, 11 e^{-11t}}{-(2)(11)} \Big|_0^t + 6$$

$$= 6\left(e^{-11t} - 1\right) + 6$$

$$= 6e^{-11t} \quad V, \; t \geqslant 0$$

$$V_2(t) = -\int_0^t 6 i(t) dt + V_2 0$$

$$= -\int_0^t 6 \cdot \frac{11}{2} e^{-11t} + 4$$

$$= 3e^{-11t} + 1 \quad V, \; t \geqslant 0$$

$$V_3(t) = -\int_0^t 4 i(t) dt + V_3(0)$$

$$= -\int_0^t 4 \cdot \frac{11}{2} e^{-11t} dt + 1$$

$$= 2e^{11t} - 1 \quad V, \; t \geqslant 0$$

Complete Response for RL Circuits

Thursday, March 2, 2017 8:03 AM

I. General Form of Complete Response

A. Definition

Complete Response = natural response + forced response

For example: $i = i_n + i_f$ or $V = V_n + V_f$

where i_n = natural response
i_f = forced response
i = complete response

B. Relation to Differential Equation

$$\frac{d^k i}{dt^k} + a_{k-1}\frac{d^{k-1} i}{dt^{k-1}} + \cdots + a_0 i = f(t)$$

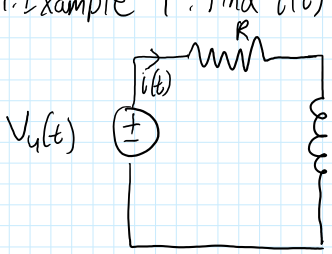
Solution consists of homogenous (transient) part $= i_n$ plus particular integral (forced response) part $= i_f$.

C. Analysis Procedure

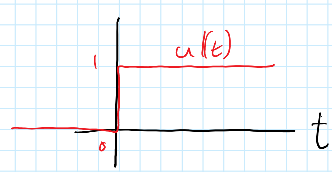
1. Find general form of natural response (i_n) without solving all unknown coefficients (set all independent sources to $\emptyset$).
2. Find forced or steady-state response (i_f)
3. Add the two expressions; $i = i_n + i_f$
4. Adjust unknown coefficients to satisfy initial conditions

D. RL Example

1. Example 1: Find $i(t)$ for $t \geq 0$ in the circuit below

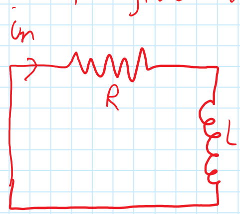


$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$



Solution:

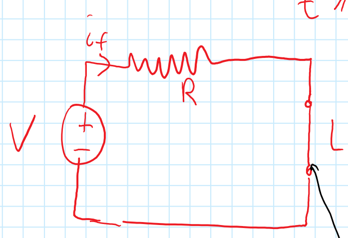
step 1: Set sources = 0
This gives the network below



$i_n(t) = Ae^{-\frac{R}{L}t}, t \geq 0$ $\quad \tau = \frac{L}{R}$

step 2: Find forced response

$t \geq 0$



$$i_f = \frac{V}{R}$$

in steady state $V_L = L\frac{di}{dt}$

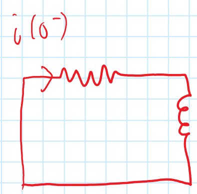
$$= L\frac{di_f}{dt}$$

$$= L \times 0 = 0$$

step 3: $i = i_n + i_f$

$$= \left(Ae^{[-R/L]t}\right) + \left(\frac{V}{R}\right) \qquad t \geq 0$$

step 4: $t < 0$



$$i(0^-) = 0 = i(0)$$

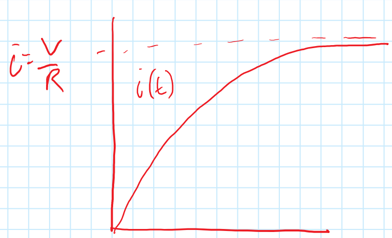
$$i(0) = Ae^{-R/L(0)} + \frac{V}{R} = 0$$

$$A + \frac{V}{R} = 0$$

$$A = -\frac{V}{R}$$

$$i(t) = -\frac{V}{R}e^{-R/L(t)} + \frac{V}{R}$$

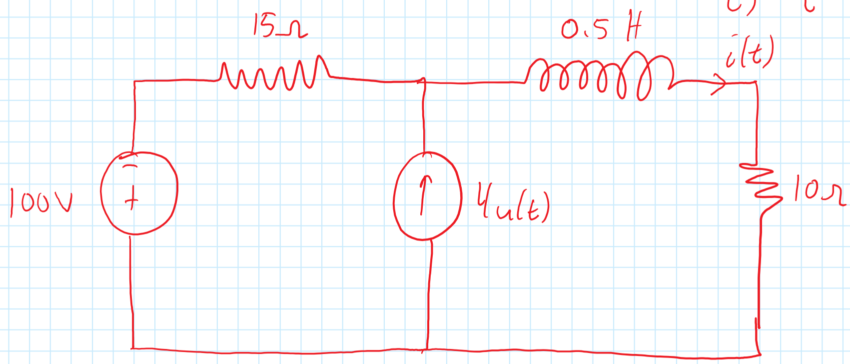
$$= \frac{V}{R}\left(1 - e^{-R/L(t)}\right) A, \quad t \geq 0$$



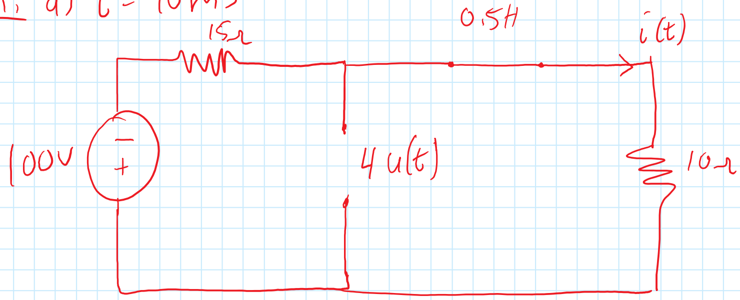
Example 2: Find energy stored in the inductor at a) $t = -10\,ms$

b) $t = 20\,ms$

c) $t = 5\,s$



Solution: a) $t = -10\,ms$



$$i = \frac{-100V}{15\Omega + 10\Omega}$$

$$i = -4A$$

$$\therefore W_L(-10\,ms) = \frac{1}{2} L i^2$$

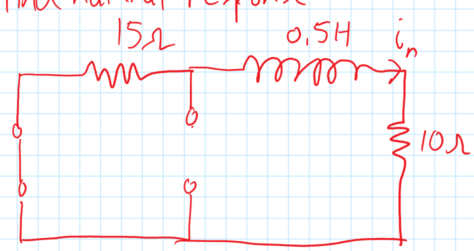
$$= \frac{1}{2}(0.5H)(-4A)^2$$

$$= 4\,J \text{ stored}$$

(b) and (c) must find complete response

$(\because t \geq 0)$

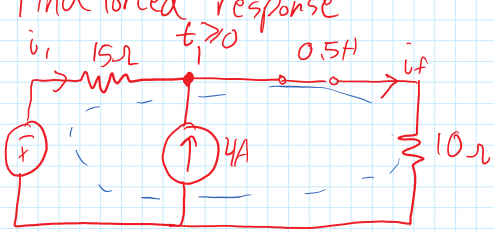
Step 1: find natural response



$$\tau = \frac{L}{R_{eq}} = \frac{0.5H}{15\Omega + 10\Omega} = 0.02\,s$$

$$i_n = Ae^{-t/\tau} = Ae^{-50t}$$

Step 2: Find forced response



Supermesh

$$100V + 15i_1 + 10i_f = 0$$

KCL at 1

$$i_1 + 4A = i_f$$

$$i_f = -1.6A$$

Step 3: $i = i_n + i_f = Ae^{-50t} - 1.6 \quad t \geq 0$

Step 4: Solve for A from initial conditions

$$i(0^-) = -4A = i(0) \quad \text{from step 1}$$

$$i(0) = -4 = A - 1.6$$

$$A = -2.4$$

$$i(t) = -2.4e^{-50t} - 1.6\ A, \quad t \geq 0$$

at $t = 20ms$

$$i(20ms) = -2.4e^{-50(0.02)} - 1.6$$

$$= -2.483\ A$$

$$W_L(20\ ms) = \frac{1}{2} L i^2 = \frac{1}{2}(0.5H)(-2.483A)^2$$

$$= 1.541\ J$$

at $t = 5s$

$$i(5sec) = -2.4e^{-50(5)} - 1.6$$

$$= -1.6A$$

$$W_L(5s) = \frac{1}{2} L i^2 = \frac{1}{2}(0.5H)(-1.6A)^2$$

$$= 0.64\ J$$

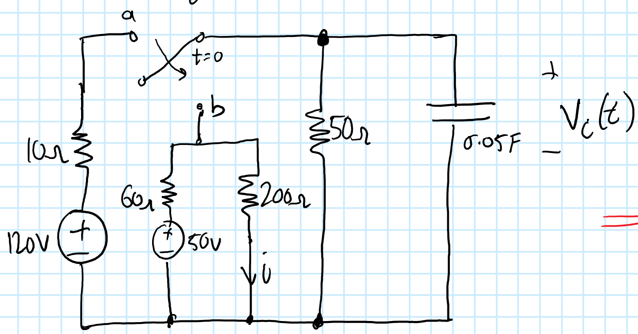
RC Examples

Monday, March 6, 2017 8:08 AM

E: RC Examples

1. Example 1:

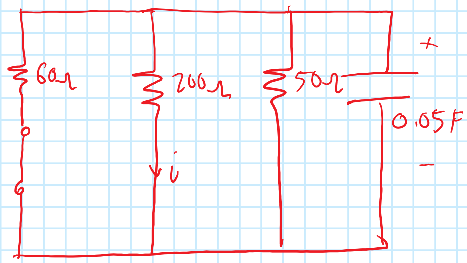
Find $V_c(t)$ for $t \geq 0$ in the circuit below



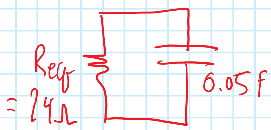
Solution:

Source free

Step 1: Find natural response with switch in position b



$$\left(\frac{1}{60}+\frac{1}{200}+\frac{1}{50}\right)^{-1}=R_{eq}$$

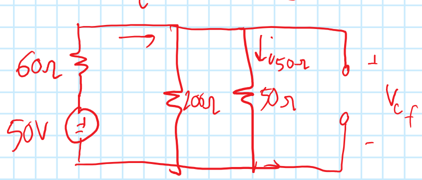


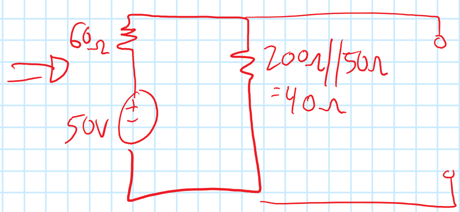
$$\tau = R_{eq}C = (24\Omega)(0.05F) = 1.2\,sec$$

$$V_{c_n}(t) = Ae^{-t/\tau},\ t \geq 0$$

Step 2: Find forced or steady-state response

$t > 0$





$$i = \frac{50V}{60\Omega + (200\Omega + 50\Omega)} = 0.5A$$

$$V_{cf} = 50V\left(\frac{40\Omega}{40\Omega+60\Omega}\right) = 20V$$

$$i_{50\Omega} = i \cdot \left(\frac{200\Omega}{200\Omega+50\Omega}\right) = 0.5(4/5) = 0.4\ A$$

$$V_{cf} = (50\Omega)(i_{50\Omega}) = 50(0.4) = 20V$$

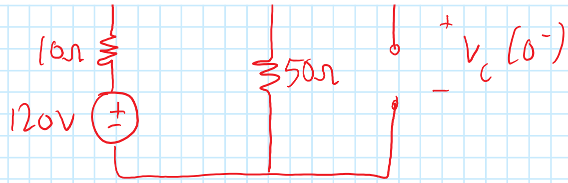
Step 3: Add the two expressions

$$V_c = V_{cn} + V_{cf} = Ae^{-t/\tau} + 20$$

Step 4: Determine A from initial conditions

At $t = 0^-$

$$V_c(0^-) = 120V\left(\frac{50\Omega}{10\Omega+50\Omega}\right) = 100V = V_c(0^+) = V_c(0)$$



$$v_c(0^-) = 120\,V\left(\frac{50\Omega}{10\Omega+50\Omega}\right) = 100\,V = v_c(0^+) = v_c(0)$$

$$V_c(0) = 100 = Ae^{-0} + 20$$

$$= A + 20$$

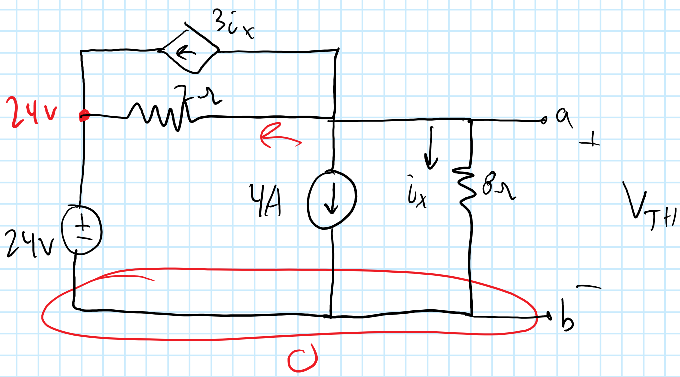
$$A = 80\ V$$

$$\boxed{V_c(t) = 80e^{-t/1.2} + 20 \text{ Volts},\quad t \geq 0}$$

Test 1 Review

Tuesday, March 7, 2017 8:03 AM

Hw#4 due 3/23



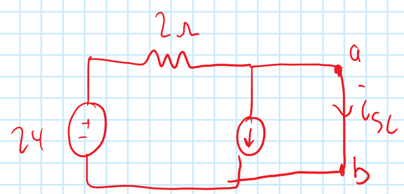
Solu: at $V_{TH} = V_{ab}$

$$\frac{V_{TH}}{8\Omega} + 4A + 3i_x + \frac{V_{TH} - 24}{2\Omega} = 0 \quad (1)$$

$$i_x = \frac{V_{TH}}{8\Omega} \quad (2)$$

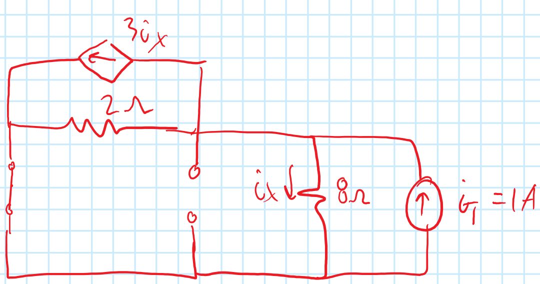
$$V_{TH} = 8V$$

$$R_{TH} = \frac{V_{TH}}{i_{sc}}$$



$$4A + i_{sc} - \frac{24}{2\Omega} = 0 \qquad i_{sc} = 8A$$

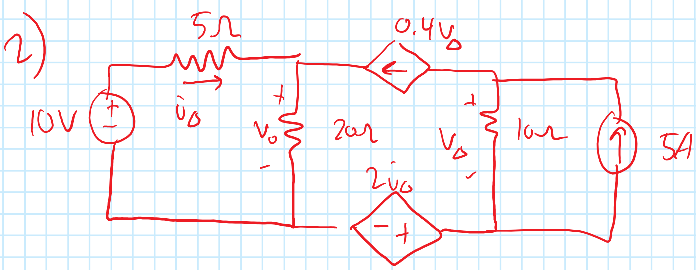
$$R_{TH} = \frac{8V}{8A} = \boxed{1\Omega}$$



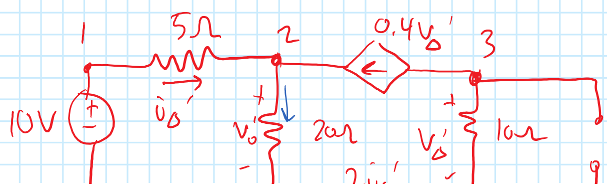
$$\frac{V_T}{2\Omega} + 3i_x + \frac{V_T}{8\Omega} - 1A = 0$$

$$i_x = \frac{V_T}{8\Omega} \qquad V_T = 1V \qquad R_{TH} = \frac{V_T}{i_T} = \boxed{1\Omega}$$

2)

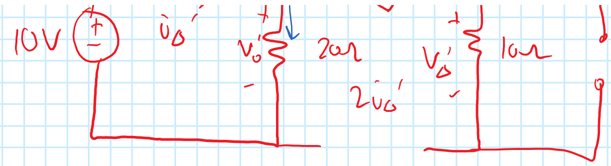


Consider 10V only



Node 1 = 10V

Node 2: $\frac{V_1 - V_2}{} - \frac{V_2}{} + 0.4V_x' = 0 \quad (1)$



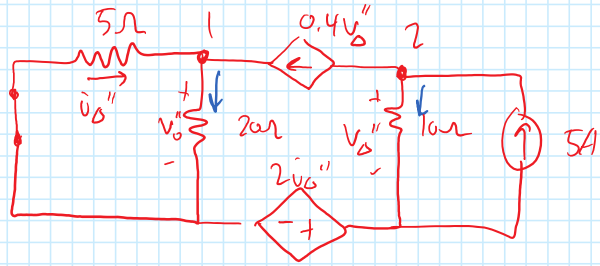
Node 2: $\dfrac{V_1 - V_2}{5\Omega} - \dfrac{V_2}{20\Omega} + 0.4V_\delta' = 0 \quad (1)$

$V_\delta' = -0.4V_\delta'(10\Omega) \quad (2)$

$V_\delta' = 0$

$V_2 = 8V = V_o'$

Consider 5A only:



Node 1: $\dfrac{V_1}{20\Omega} - \dfrac{0 - V_1}{5\Omega} - 0.4V_\delta'' = 0$

Node 2: $0.4V_\delta'' + \dfrac{V_2 - 2i_\delta''}{10\Omega} - 5 = 0$

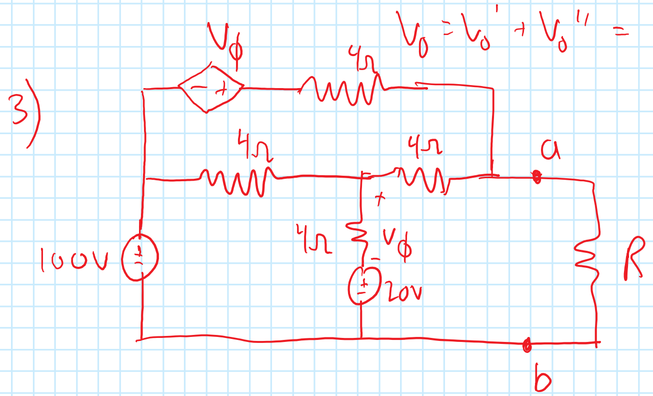
$V_\delta'' = V_2 - 2i_\delta''$

$i_\delta'' = \dfrac{-V_1}{5\Omega}$

$V_1 = 16V = V_o''$

$V_o = V_o' + V_o'' = 24V$

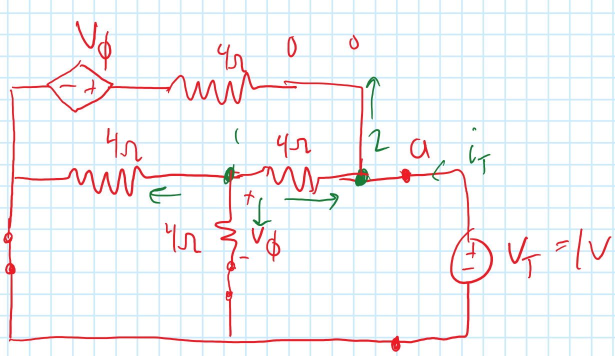
3)



R = ? if max power delivered

$R = R_{TH}$

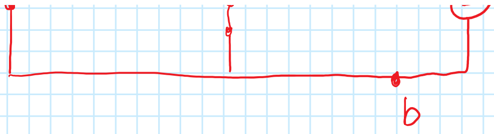
Solution:



$V_\phi = 1V$, since $v_\phi \parallel V_T$

Node 1: $\dfrac{V_1}{4\Omega} + \dfrac{V_1}{4\Omega} + \dfrac{V_1 - V_T}{4\Omega} = 0 \quad (1)$

Node 2: $-\left(\dfrac{V_1 - V_T}{4}\right) - i_T + \dfrac{V_T - V_\phi}{} = 0 \quad (1)$



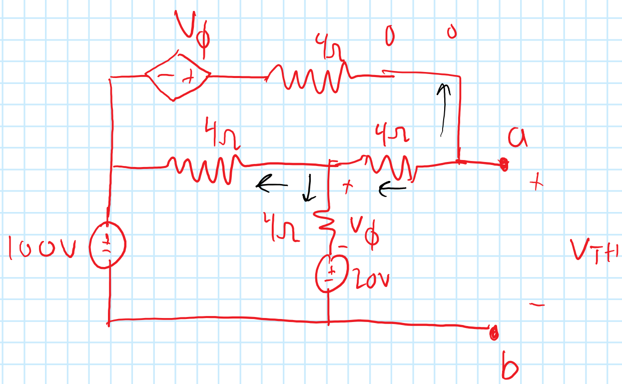
Node 2: $-\left(\frac{V_1 - V_T}{4\Omega}\right) - i_T + \frac{V_T - V_\phi}{4\Omega} = 0$ (1)

$V_1 = 0.333V$

$i_T = 0.333A$

$R_{TH} = \frac{V_T}{i_T} = \frac{1V}{0.33A} = 3\Omega$

$R_{TH} = \frac{V_{TH}}{i_{sc}}$

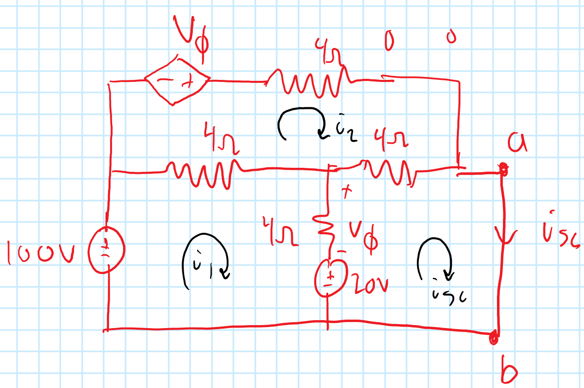


At V_{TH}: $\frac{V_{TH} - (100V + V_\phi)}{4\Omega} + \frac{V_{TH} - V_1}{4\Omega} = 0$ (1)

at V_1: $\frac{V_1 - 100}{4\Omega} + \frac{V_1 - 20}{4\Omega} + \frac{V_1 - V_{TH}}{4\Omega} = 0$ (2)

$V_\phi = V_1 - 20V$ (3)

$V_{TH} = 120V$



Mesh 1: $-100 + 4(i_1 - i_2) + V_\phi + 20 = 0$

Mesh 2: $-V_\phi + 4i_2 + 4(i_2 - i_{sc}) + 4(i_2 - i_1) = 0$

Mesh i_{sc}: $-20 - V_\phi + 4(i_{sc} - i_2) = 0$

$V_\phi = 4(i_1 - i_{sc})$

$i_{sc} = -40A$

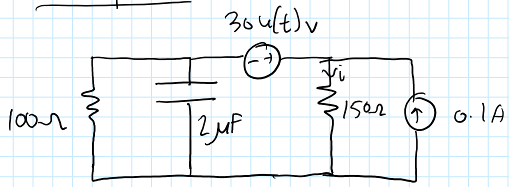
$i_1 = 45A$

$i_2 = 30A$

Complete Response for RC Circuits

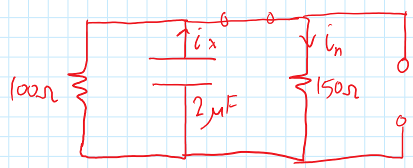
Wednesday, March 8, 2017 8:12 AM

Example 2:

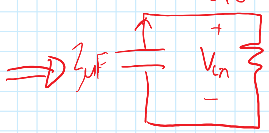


Find $i(t)$ for $t \geq 0$

Solu: Step 1: Natural response



$\Rightarrow$



$$i_x = -C\frac{dV_{cn}}{dt} = (-2\times10^{-6})\frac{d}{dt}Ae^{-t/\tau} = (2\times10^{6}) = \frac{A}{\tau}e^{-t/\tau}$$

$$R_{eq} = 100\Omega \,\|\, 150\Omega = 60\Omega$$

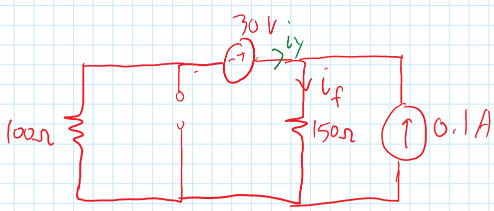
$$\tau = R_{eq}C = (2\mu F)(60\Omega) = 1.2\times10^{-4}\text{ sec}$$

$$V_{cn} = Ae^{-t/\tau} = Ae^{-t/1.2\times10^{-4}\text{ sec}}$$

$$i_n = i_x \cdot \frac{100\Omega}{100\Omega+150\Omega} = \frac{4\times10^{-2}A}{6}e^{-t/\tau}$$

$$i_n = 0.8\times10^{-6}\frac{A}{\tau}e^{\frac{-t}{\tau}}$$

Step 2: Forced Response

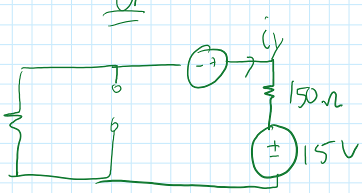


Mesh: $100i_y - 30V + 150i_f = 0$

KCL: $i_y + 0.1 = i_f$

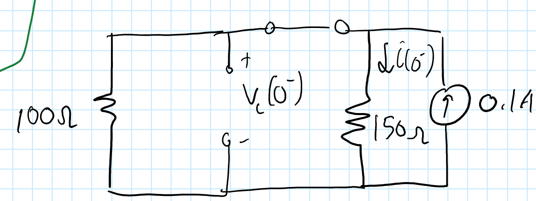
$i_f = 0.16A$

or (Source Trans)



Step 3: $i = i_n + i_f = 0.8\times10^{-6}\frac{A}{\tau}e^{-t/\tau} + 0.16 \quad t \geq 0$

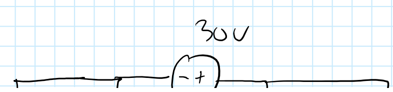
Step 4: Solve for A from initial conditions $(t = 0^-)$



$$i(0^-) = \frac{V_c(0^-)}{150\Omega} = 0.04A$$

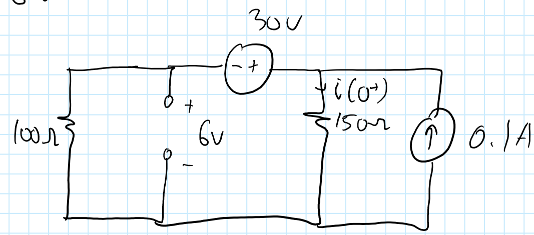
$$V_c(0^-) = (100\Omega \,\|\, 150\Omega)(0.1A) = 6V$$

$i(0^+)$



$= 6V$

$t = (0^+)$



$$6V + 30V - (150)\,i(0^+) = 0$$

$$i(0^+) = \frac{36V}{150\Omega} = 0.24A = i(0)$$

Pick up $i(0^+)$ since looking for $t \geq 0$

greater than or equal to

$$i(0) = 0.24 = 0.8 \times 10^{-6} \frac{A}{\tau} + 0.16$$

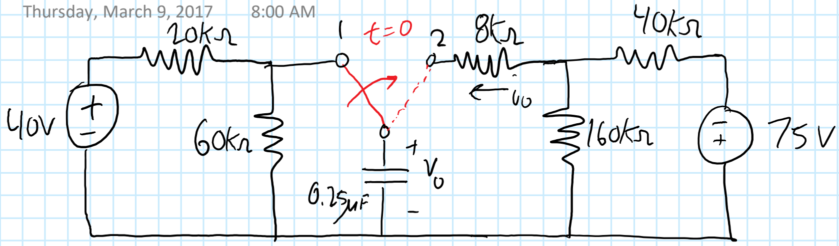
$$\frac{A}{\tau} = 10^5$$

$$i(t) = 0.8 \times 10^{-6} (10^5) e^{-t/1.2 \times 10^{-4}} + 0.16$$

$$\boxed{i(t) = 0.08 e^{-t/1.2 \times 10^{-4}} + 0.16 \text{ A} \quad , t \geq 0}$$

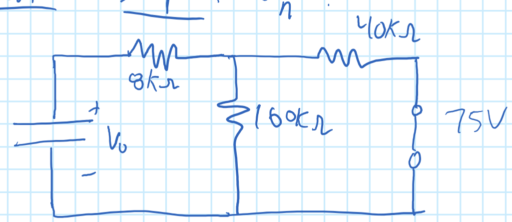
More RC Examples

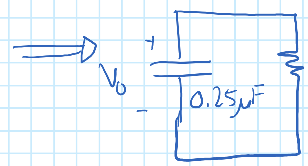
Thursday, March 9, 2017 8:00 AM



Find $V_o(t)$ and $i_o(t)$ for $t \geq 0$.

Solution — Step 1: $V_n = ?$ for $t \geq 0$





$R_{eq} = 8\,k\Omega + (160\,k\Omega \,//\, 40\,k\Omega)$

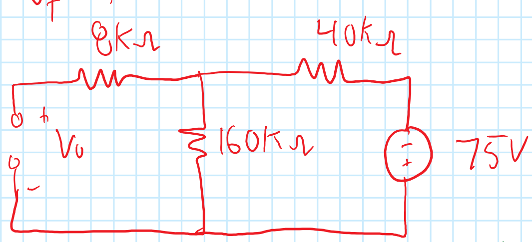
$= 40\,k\Omega$

$\tau = RC = 40\,k\Omega\,(0.25\,\mu F)$

$= 10 \times 10^{-3} = 0.01$

$V_n = Ae^{-t/0.01} = Ae^{-100t}$

Step 2: $V_f = ?$



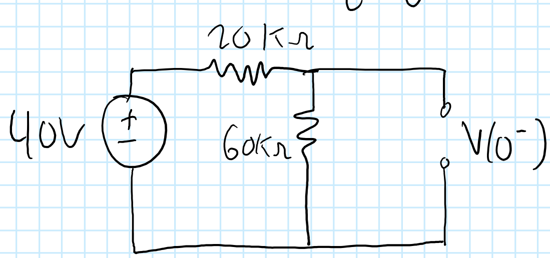
$$V_f = -75\left(\frac{160\,k\Omega}{160\,k\Omega + 40\,k\Omega}\right)$$

$= -60\,V$

Step 3: $V_o = V_n + V_f = Ae^{-100t} - 60, \; t \geq 0$

Step 4: Find A from initial conditions

$t = 0^-$



$$V(0^-) = 40\left(\frac{60\,k\Omega}{20\,k\Omega + 60\,k\Omega}\right)$$

$= 30\,V = V(0)$

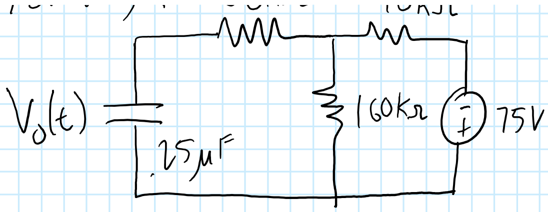
$V(0) = 30 = Ae^{0} - 60$

$30 = A - 60 \quad A = 90$

$$\boxed{V(0) = 90e^{-100t} - 60 \text{ Volts}, \; t \geq 0}$$

For i(t):

$i_o(t) = C\,\dfrac{dv_o(t)}{dt}$



$$i_0(t) = C\frac{dv_0(t)}{dt}$$

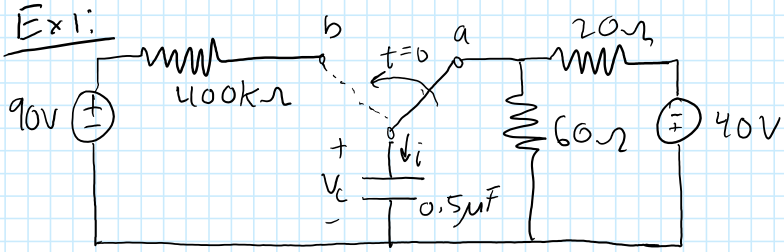
$$= (0.25\mu F)\frac{d}{dt}\left(90e^{-100t} - 60\right)$$

$$\left(-9000e^{-100t}\right)\left(0.25\times10^{-6}\right)$$

RC With Dependent Source

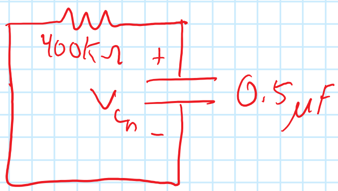
Monday, March 20, 2017 8:10 AM

Ex1:



$v_c(t)$ and $i(t)$ = ? $t \geq 0$

Solution:

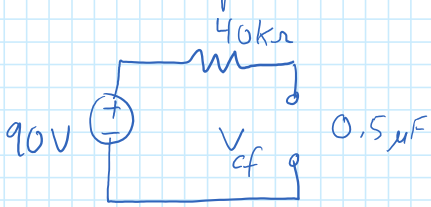


Step 1: Natural Response

$\tau = 400k\Omega(0.5\mu F) = 200\ ms$

$V_c(t) = Ae^{-t/\tau} = Ae^{-5t}$ Volts

Step 2: Forced Response

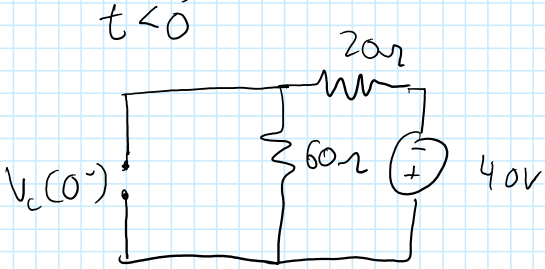


$V_f = 90\ V$

Step 3: $V_c(t) = V_{cn} + V_{cf} = Ae^{-5t} + 90V, \quad t \geq 0$

Step 4: Using initial conditions to adjust unknown parameter

$t < 0$



$V_c(0^-) = (-40V)\left(\frac{60\Omega}{20\Omega + 60\Omega}\right) = -30V$

$-30V = V_c(0)$

$V_c(0) = Ae^0 + 90 = -30$

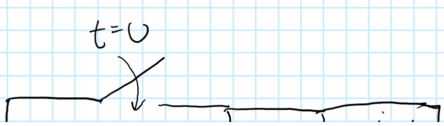
$A = -120$

$$\boxed{V_c(t) = 90 - 120e^{-5t} \text{ Volts}, \quad t \geq 0}$$

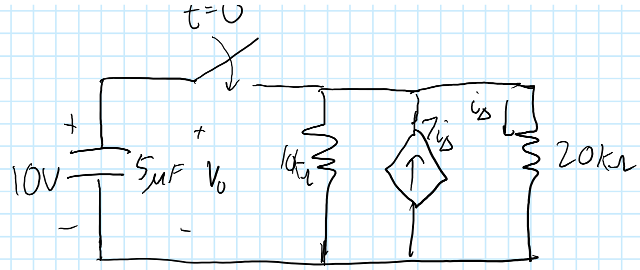
$i(t) = C\frac{dV_c(t)}{dt} = (0.5\mu F)\frac{d}{dt}\left(90 - 120e^{-5t}\right) = (0.5\mu F)(600e^{-5t})$

$i(t) = (300 \times 10^{-6})e^{-5t}$ Amps, $t \geq 0$

Ex 7.13



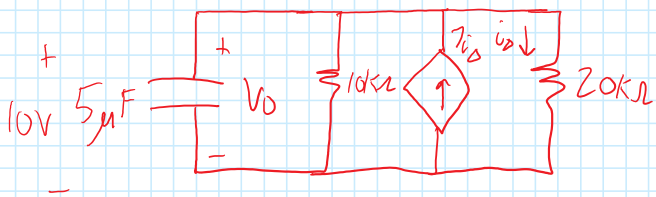
$V(t) = ?\ \ t \geq 0$

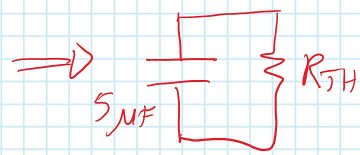


$v_0(t) = ?\ \ t \geq 0$

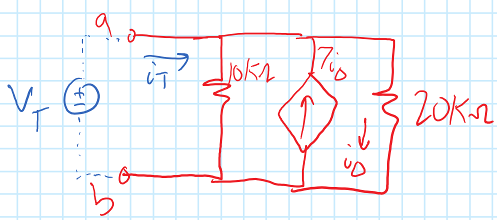
Solu:

Step 1: Natural Response V_{on}





$\tau = (5\mu F)(R_{TH})$



Node A:

$$i_T - \frac{V_T}{10k} + 7i_\Delta - i_\Delta = 0$$

$$i_\Delta = \frac{V_T}{20k}$$

$$i_T - \frac{V_T}{10k} + \frac{6V_T}{20k} = 0$$

$$i_T + \frac{3V_T}{10k\Omega} - \frac{V_T}{10k} = 0$$

$$10k\, i_T + 2V_T = 0$$

$$2V_T = -10k\, i_T \qquad \frac{V_T}{i_T} = \frac{-10k}{2} = -5k$$

$$R_{TH} = \frac{V_T}{i_T} = -5000\Omega$$

$$\tau = (5\mu F)(-5000) = -0.025$$

$$V_{on} = Ae^{-t/\tau} = Ae^{40t} \text{ Volts},\ t \geq 0$$

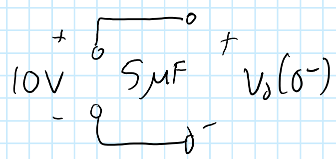
Step 2: Forced Response V_{of}

$V_{of} = 0\ V \rightarrow$ no indep source

Step 3: $V_{on} + V_{of} = Ae^{40t}$ Volts, $t \geq 0$

Step 4: Use initial conditions to adjust A

$t < 0$



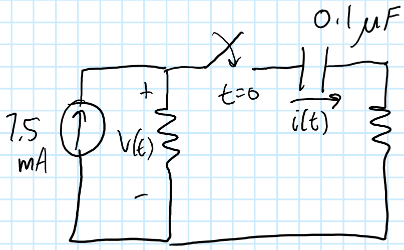
$V_o(0^-) = V_c(0^-) = 10V = V_o(0)$

$V_o(0) = 10V = Ae^0 \quad A = 10$

$$\boxed{V_o(t) = 10e^{40t}\ \text{Volts},\ t \geq 0}$$

Find V(t) Across Resistor

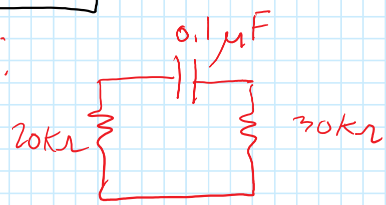
Tuesday, March 21, 2017 8:29 AM



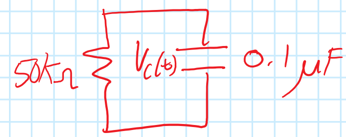
$i(t) = ?$ for $t \geq 0$

$V(t) - ?$ for $t \geq 0$

Solu: step 1:



$\Rightarrow$

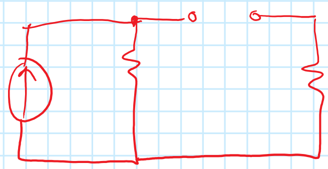


$\tau = R_{eq}(C) = 50k\Omega(0.1\mu F)$

$\tau = 5ms$

$V_c n = Ae^{-200t}$

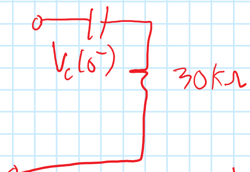
Step 2:



$V_c f = 7.5mA(20k\Omega) = 150V$

Step 3: $V_c(t) = V_c n + V_c f = Ae^{-200t} + 150$ Volts, $t \geq 0$

Step 4: $t < 0$

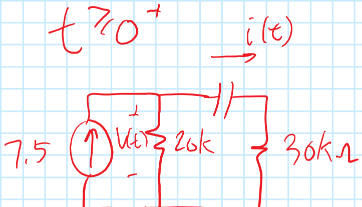


$V_c(0^-) = 0 = V_c(0)$

$0 = A + 150 \quad A = -150$

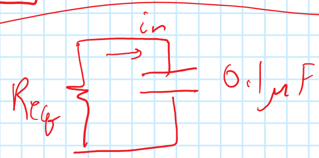
$V_c(t) = 150 - 150e^{-200t}$ Volts, $t \geq 0$

$t \geq 0^+$



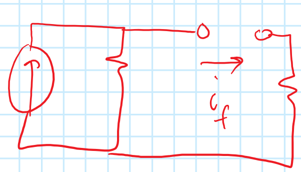
$V(t) = V_c(t) + i(t)\,30k\Omega$

Step 1) $i_n = ?$



$i_n = Be^{-200t}$

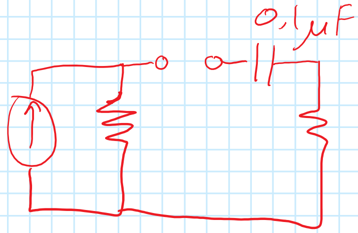
Step 2) $i_f = ?$



$i_f = 0$ Amps

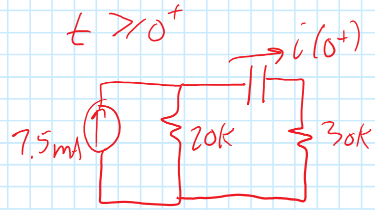
Step 3) $i(t) = i_n + i_f = Be^{-200t}$

Step 4)



$i_0^- = 0$ Amps $\neq i_0^+$

$t \geq 0^+$



$i(0^+) = 7.5mA\left(\frac{20k\Omega}{50k\Omega}\right) = 3mA$

$i(0^+) = 3mA = Be^{0^+}$

$B = 3mA$

$i(t) = 3e^{-200t}$ mA, $t \geq 0^+$

$V(t) = 150 - 150e^{-200t} + (0.003e^{-200t})(30,000)$

$V(t) = 150 - 150e^{-200t} + 90e^{-200t} = \boxed{150 - 60e^{-200t} \text{ Volts}, t \geq 0^+}$

Intro to RLC Circuits

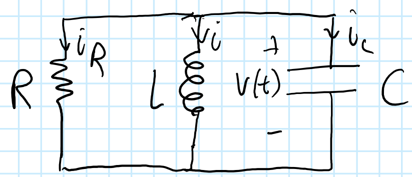
Wednesday, March 22, 2017 8:04 AM

1) HW#5 due on Tuesday, 4/11 — Ch 8: 6, 8, 10, 14, 19, 27, 30, 31, 35, 37, 42, 45, 47, 49, 52, 53
2) Read Ch. 8
3) Test #2 on Tuesday, 4/18

(16 problems, do 4 a day)

Ch 8: Transient RLC Networks

I. Source free Parallel Circuits



$v = L\frac{di}{dt}$

$Vdt = L\,di$

$di = \frac{1}{L} Vdt \Rightarrow i = \frac{1}{L}\int_0^t vdt + i(0)$

A. Differential Equations

KCL: $i_R + i_L + i_C = 0$

$$\underbrace{\frac{V(t)}{R}}_{i_R} + \underbrace{C\frac{dV(t)}{dt}}_{i_C} + \underbrace{\frac{1}{L}\int_0^t Vdt + i(0)}_{i} = 0$$

$$C\frac{d^2V(t)}{dt^2} + \frac{1}{R}\frac{dV(t)}{dt} + \frac{1}{L}V(t) = 0$$

$$\boxed{\frac{d^2V(t)}{dt^2} + \frac{1}{RC}\frac{dV(t)}{dt} + \frac{1}{LC}V(t) = 0}$$

3 Types of Oscillations

Thursday, March 23, 2017 8:00 AM

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0 \qquad (1)$$

B. Solution of Eq. (1)

1. Characteristic Roots, (eigenvalue)

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad (2)$$

$$ax^2 + bx + c$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_{1,2} = \frac{\frac{-1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2(1)} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{4}\left(\frac{1}{RC}\right)^2 - \frac{4}{4}\left(\frac{1}{LC}\right)} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad (3)$$

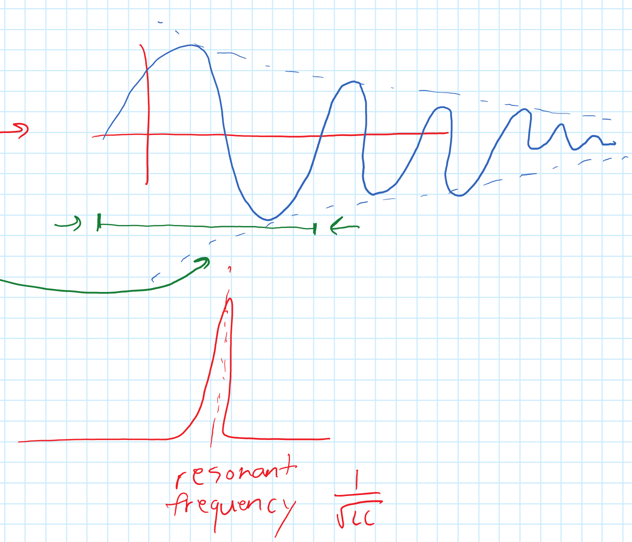
$$s_2 = \frac{-1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad (4)$$

2. Three Possible Solution Forms

a) Case 1: Overdamped $(\alpha > \omega_0)$

$\alpha = \frac{1}{2RC} \Rightarrow$ "exponential damping factor"

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow$ "resonant frequency"



Eq (3) and (4) have distinct real roots

$$\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC} > 0 \qquad \alpha^2 > \omega_0^2$$

$$\boxed{V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \quad t \geq 0}$$

where $s_1 < 0$ and $s_2 < 0$ (both real)

A_1 and A_2 determined by initial conditions

b) Case 2: Critically Damped

Eq (3) and (4) have same value

$(s_1 = s_2$ or $\alpha = \omega_0)$

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$$

Solutions to (1) for this case is

$$\boxed{v(t) = (A_1 t + A_2)\, e^{-\alpha t}}$$

where A_1 and A_2 are determined by initial conditions

c) Case 3: Underdamped $(\alpha < \omega_0)$

Eqs (3) and (4) are complex numbers

$$\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC} \quad \text{or} \quad \alpha < \omega_0$$

$s_1 = s_2^*$ (complex conjugates)

$j = \sqrt{-1}$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$\boxed{v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)}$$

Overdamped RLC Parallel

Monday, March 27, 2017 8:00 AM

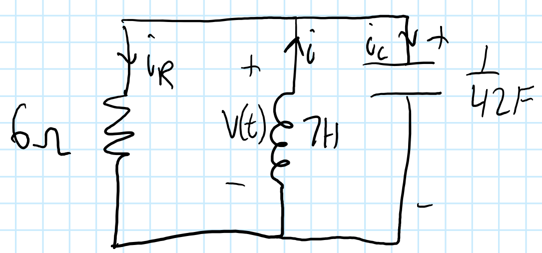
II. Circuit Example for Source-Free RLC Networks

A. Overdamped Case

1. Example 1

Let $v(0) = 15V$, $i(0) = -6A$

Find A_1, A_2, $v(t)$ and V_{min} for the following circuit



Solu:

$$\frac{dv^2}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$\frac{dv^2}{dt^2} + \frac{1}{6(\frac{1}{42})} \cdot \frac{dv}{dt} + \frac{1}{7(1/42)}v = 0$$

$$\frac{dv^2}{dt^2} + 7\frac{dv}{dt} + 6v = 0$$

$$s^2 + 7s + 6$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)} = \frac{-7 \pm \sqrt{25}}{2} = \frac{-7 \pm 5}{2}$$

$$s_1 = \frac{-7+5}{2} = \frac{-2}{2} = -1$$

$$s_2 = \frac{-7-5}{2} = \frac{-12}{2} = -6$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(6)(1/42)} = \frac{7}{2} = 3.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7(1/42)}} = \frac{1}{\sqrt{1/6}} \approx 2.4$$

$\alpha > \omega_0$ Overdamped

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= A_1 e^{-t} + A_2 e^{-6t}$$

$$v(0) = 15 = A_1 + A_2$$
$$A_1 + A_2 = 15$$

$$i_c = C\frac{dv}{dt} = C\frac{d}{dt}\left(A_1 e^{-t} + A_2 e^{-6t}\right)$$
$$= C\left[-A_1 e^{-t} - 6A_2 e^{-6t}\right]$$

$$i = i_R + i_c$$
$$i_c = i - i_R$$
$$i_c(0) = i(0) - i_R(0)$$

Shortcut: $i_c = C\frac{dv}{dt}$

$$\frac{dv}{dt} = \frac{i_c}{C} = \frac{i - i_R}{C}$$

$$\boxed{\frac{dv}{dt} = \frac{i - i_R}{C}}$$ at $t=0$

$$\frac{1}{42}\left[-A_1 - 6A_2\right] = -6 - \left(\frac{15\,V}{6\,\Omega}\right)$$
$$-A_1 - 6A_2 = -6(42) - \left[\frac{15}{6}\cdot 42\right]$$
$$-A_1 - 6A_2 = -252 - 105$$
$$-A_1 - 6A_2 = -357$$

$$A_1 + A_2 = 15 \qquad (1)$$
$$-A_1 - 6A_2 = -357 \qquad (2)$$
$$A_1 = \frac{-267}{5} = -53.4$$
$$A_2 = \frac{342}{5} = 68.4$$

$$\boxed{v(t) = -53.4e^{-t} + 68.4e^{-6t} \text{ Volts}, \ t \geq 0}$$

natural response of circuit

V_{min} : $V'(t) = 0$

$$\frac{dV}{dt} = 53.4e^{-t} - 6(68.4)e^{-6t}$$

$$53.4e^{-t} - 410.4e^{-6t} = 0$$

$$53.4e^{-t} = 410.4e^{-6t}$$

$$53.4 = 410.4e^{-5t}$$

$$e^{-5t} = \frac{53.4}{410.4}$$

$$-5t = \ln\left[\frac{53.4}{410.4}\right]$$

$$-5t = -2.0393$$

$t = 0.40786$ sec (time at V_{min})

$$V(0.40786) = -53.4e^{-0.40786} + 68.4e^{-6(0.40786)}$$

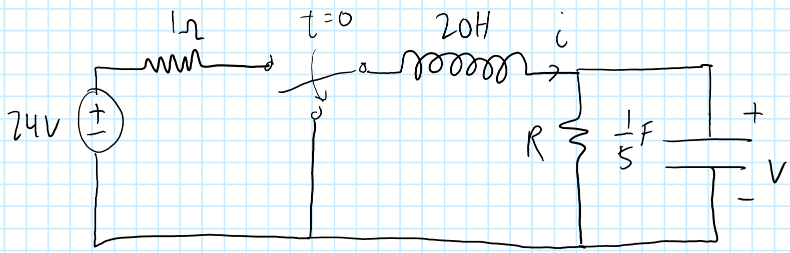
$$\boxed{V_{min} = -29.6 \text{ Volts}}$$

Critically Damped RLC Parallel

Tuesday, March 28, 2017 8:06 AM

B. Critically Damped

1. Example 1



Given that the circuit is critically damped,
Find R, $i(0^+)$, $V(0^+)$, $v(2)$

Solu: $\alpha = \omega_0$

$$\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{(2RC)^2} = \frac{1}{LC}$$

$$LC = 4R^2C^2$$

$$R^2 = \frac{L\cancel{C}}{4C^{\cancel{2}}} = \frac{L}{4C}$$

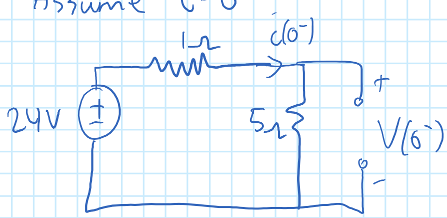
$$R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{20}{1/5}} = 5\Omega$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)(1/5)} = 0.5$$

$$V(t) = (A_1 t + A_2)e^{-\alpha t}$$

$$V(t) = (A_1 t + A_2)e^{-0.5t}$$

Assume $t = 0^-$

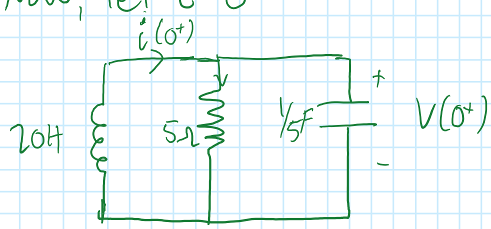


$$i = \frac{24V}{6\Omega} \qquad i(0^-) = 4A$$

$$V = 24V\left(\frac{5\Omega}{5\Omega + 1\Omega}\right) = 24\left(\frac{5}{6}\right) = 20V$$

$$V(0^-) = 20V$$

Now, let $t = 0^+$



$$\boxed{i(0^+) = i(0^-) = 4A = i(0)}$$

$$\boxed{V(0^+) = V(0^-) = 20V = V(0)}$$

$$V(0) = 20 = (A_1 \cdot 0 + A_2)e^{-\alpha \cdot 0}$$

$$A_2 = 20$$

$$V(t) = (A_1 t + 20)e^{-0.5t}$$

$$i_C = C\frac{dV}{dt} \Rightarrow \left.\frac{dV}{dt}\right|_{t=0} = \frac{i_C(0)}{C} = \frac{i(0) - i_R(0)}{C} = \frac{4A - \frac{20V}{5\Omega}}{1/5F} = 0$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(A_1 t + 20\right)e^{-0.5t} = A_1\left[t(-0.5e^{-0.5t}) + e^{-0.5t}\right] + 20(-0.5)e^{-0.5t}\Big|_{t=0}$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(A_1 t + 20\right)e^{-0.5t} = A_1\left[t\left(-0.5e^{-0.5t}\right) + e^{-0.5t}\right] + 20(-0.5)e^{-0.5t}\Big|_{t=0}$$

$$= A_1 - 10 = 0$$

$$A_1 = 10$$

$$V(t) = \left(10t + 20\right)e^{-0.5t} \text{ Volts, } t \geq 0$$

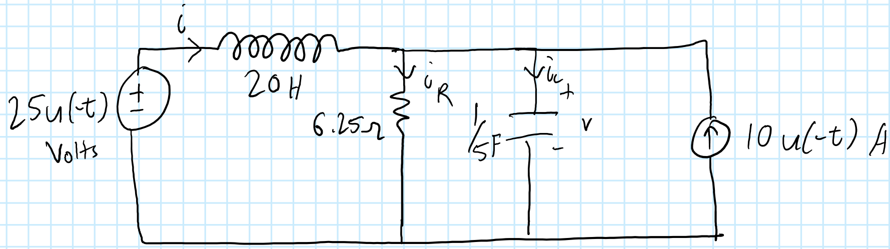
$$\boxed{V(2) = (10 \cdot 2 + 20)e^{-0.5(2)} = 14.72\,V}$$

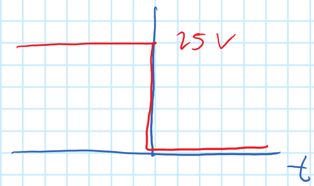
Underdamped RLC Parallel

Wednesday, March 29, 2017 8:03 AM

C. Underdamped

1. Example 1





Solution: Let t > 0, then u(-t) = 0

$\alpha = \frac{1}{2RC} = \frac{1}{2(6.25\Omega)(1/5F)} = 0.4$

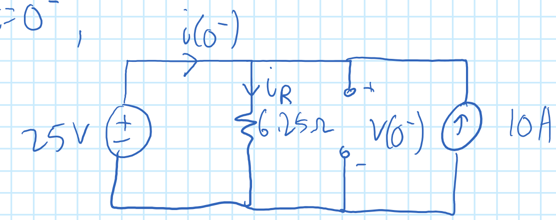
$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20H(1/5F)}} = 0.5$

Since $\alpha < \omega_0$, circuit is underdamped

$V(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3$

At $t = 0^-$,



$V(0^-) = 25V$

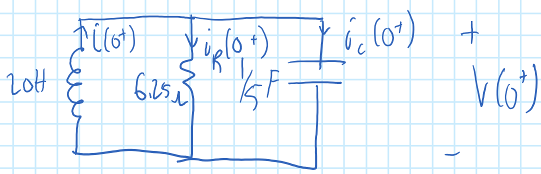
$i_R(0^-) = \frac{25V}{6.25\Omega} = 4A$

$i(0^-) + 10A - i_R(0^-) = 0$

$i(0^-) = i_R(0^-) - 10 = 4 - 10$

$i(0^-) = -6A$

Let $t = 0^+$



$\boxed{i(0^-) = i(0^+) = i(0) = -6A}$

$\boxed{V(0^+) = V(0^-) = V(0) = 25V}$

$v(0) = e^0(B_1 \cos 0 + B_2 \sin 0) = 25$

$B_1 = 25$

$v(t) = e^{-0.4}(25 \cos 0.3t + B_2 \sin 0.3t)$

$i_c = C\,dv$

$\overline{dt}$

$$\left.\frac{dv}{dt}\right|_{t=0} = \frac{i_c(0)}{C} = \frac{i(0) - i_R(0)}{C} = \frac{-6 - \frac{25V}{6.25\Omega}}{1/5 F} = -50$$

$$\frac{dv}{dt} = \frac{d}{dt}\left[e^{-0.4t}(25\cos 0.3t + B_2 \sin 0.3t)\right]$$

$$= e^{-0.4t}(-7.5\sin 0.3t + 0.3B_2\cos 0.3t) - 0.4e^{-0.4t}(25\cos 0.3t + B_2\sin 0.3t)$$

at $t=0$, $\frac{dv}{dt} = 0.3B_2 - 0.4(25)$

$$0.3B_2 - 0.4(25) = -50$$

$$B_2 = -133.3$$

$v(t) = e^{-0.4t}(25\cos(0.3t) - 133.3\sin(0.3t))$ Volts, $t \geq 0$

$$v(2) = e^{-0.4(2)}(25\cos(0.3\cdot 2) - 133.3\sin(0.3\cdot 2))$$

$\boxed{v(2) = -24.5 \text{ Volts}}$

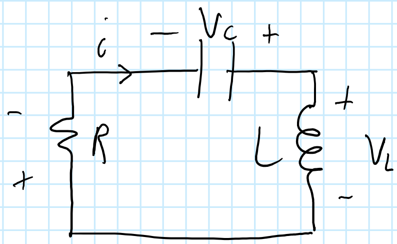
Overdamped RLC Series

Thursday, March 30, 2017 8:14 AM

III. Source-Free Series Circuit

A. Differential equation

Let $V_c(t_0) = V_0$, $i(t_0) = I$



$$\underbrace{L\frac{di}{dt}}_{V_L} + \underbrace{Ri}_{V_R} + \underbrace{\frac{1}{C}\int_0^t i\,dt + V_c(t_0)}_{V_C} = 0$$

KVL: $CV = Q \qquad V = \frac{1}{C}Q$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \qquad (1)$$

B. Solution of Equ (1)

1. Characteristic Roots

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Roots are

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

or $$s_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (2)$$

$$s_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (3)$$

$\alpha = \frac{R}{2L}$ exponential damping factor

$\omega_0 = \frac{1}{\sqrt{LC}}$ resonant frequency

2. Three Possible Solutions

a) Case 1: Overdamped $(\alpha > \omega_0)$

(2) and (3) have distinct real roots

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

b) Case 2: Critically Damped $(\alpha = \omega_0)$

$i(t) = (A_1 t + A_2)\, e^{-\alpha t}$

c) Case 3: Underdamped $(\alpha < \omega_0)$

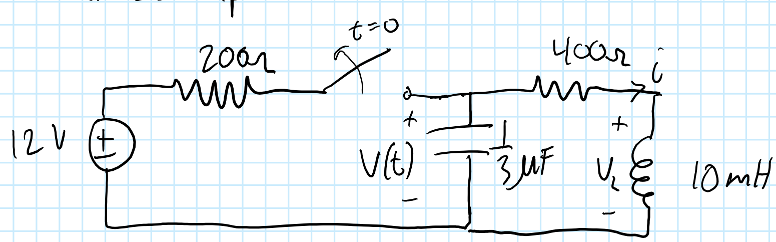
$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

IV. Circuit Example for Source-Free series RLC network

A. Overdamped Case

1. Example 1



Find $V(0^+)$, $i(0^+)$, $V(0.1\,ms)$

Solu: By (1) when $t > 0$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

check with case:

$$\alpha = \frac{R}{2L} = \frac{400\Omega}{2(10 \times 10^{-3} H)} = 20{,}000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10\times10^{-3} H)(1/3 \times 10^{-6} F)}} = 17{,}320.5$$

$\therefore \alpha > \omega_0 \longrightarrow$ overdamped

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

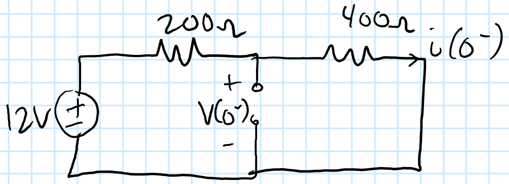
where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2\times10^4 + \sqrt{(2\times10^4)^2 - (1.73205\times10^4)^2} = -1\times10^4$

where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2\times10^4 + \sqrt{(2\times10^4)^2 - (1.73205\times10^4)^2} = -1\times10^4$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -2\times10^4 + \sqrt{(2\times10^4)^2 - (1.73205\times10^4)^2} = -3\times10^4$

Determine initial conditions

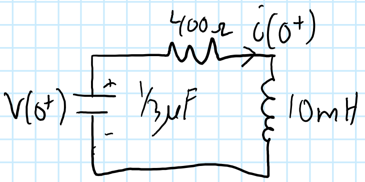
$t = 0^-$



$$V(0^-) = 12V\left(\frac{400\Omega}{400\Omega + 200\Omega}\right) = 8V$$

$$i(0^-) = \frac{12V}{200\Omega + 400\Omega} = 0.02A$$

$t = 0^+$



$$\boxed{\begin{aligned} V(0^+) &= V(0^-) = 8V = V(0) \\ i(0^+) &= i(0^-) = 0.02A = i(0) \end{aligned}}$$

$i(0) = 0.02 = A_1 e^0 + A_2 e^0$

$A_1 + A_2 = 0.02$

$A_2 = 0.02 - A_1$

$i(t) = A_1 e^{s_1 t} + (0.02 - A_1) e^{s_2 t}$

$$V_L = L\frac{di}{dt} \qquad \left.\frac{di}{dt}\right|_{t=0} = \left.\frac{1}{L}V_L\right|_{t=0} = \left.\frac{V_L}{L}\right|_{t=0} = \frac{V(0) - 400\Omega\left[i(0^+)\right]}{10\times10^{-3}H} = \frac{8 - 400(0.02)}{10mH} = 0$$

Critically Damped RLC Series

Monday, April 3, 2017 8:03 AM

$$i(t) = A_1 e^{s_1 t} + \underbrace{(0.02 - A_1)}_{A_2} e^{s_2 t}$$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_L(0)}{L} = \frac{V(0) - 400\Omega\,(i_L(0))}{0.01\ H} = \frac{8 - (400)(0.02)}{0.01} = 0$$

also:
$$\left.\frac{di}{dt}\right|_{t=0} = \left.\frac{d}{dt}\left(A_1 e^{s_1 t} + (0.02 - A_1)e^{s_2 t}\right)\right|_{t=0}$$

$$= \left. s_1 A_1 e^{s_1 t} + (0.02 - A_1) s_2 e^{s_2 t}\right|_{t=0}$$

$$= s_1 A_1 + (0.02 - A_1) s_2$$

$$0 = -10^4 A_1 + (0.02 - A_1)(-3 \times 10^4)$$

$$A_1 + (0.02 - A_1)3 = 0$$

$$A_1 = 0.03$$

$$\boxed{i(t) = 0.03 e^{-10^4 t} - 0.01 e^{-3\times10^4 t} \quad \text{Amps}, \ t \geq 0}$$

$$V_L(t) = L\frac{di}{dt} = (0.01 H)\frac{d}{dt}\left(0.03 e^{-10^4 t} - 0.01 e^{(-3\times10^4)t}\right)$$

$$= (0.01)\left[-3\times10^2 e^{-10^4 t} + 3\times10^2 e^{(-3\times10^4)t}\right]$$

$$V_L(t) = 3\left(-e^{-10^4 t} + e^{(-3\times10^4)t}\right)$$

$$V(t) = (400\Omega)\, i(t) + V_L(t)$$

$$= 400\left[0.03e^{-10^4 t} - 0.01 e^{(-3\times10^4)t}\right] + 3\left(-e^{-10^4 t} + e^{(-3\times10^4)t}\right)$$

$$V(t) = 9e^{-10^4 t} - e^{(-3\times10^4)t} \quad \text{Volts}, \ t \geq 0$$

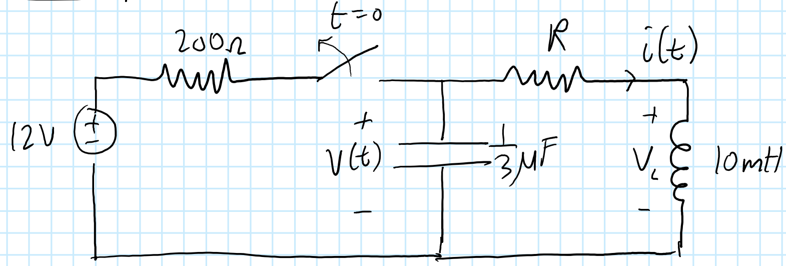
$$V(0.1ms) = 9e^{-10^4(0.1\times10^{-3})} - e^{(-3\times10^4)(0.1\times10^{-3})}$$

$$= 3.31083 \text{ Volts} - 0.049787 \text{ Volts}$$

$$\boxed{V(0.1ms) = 3.261\ V}$$

B. Critically Damped Case

1. Example 1:



Find $R = ?$

$i(t) = ?$ $t \geq 0$

Solu: $\alpha = \omega_0$

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

$$R = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{10\times10^{-3}H}{1/3\times10^{-6}F}}$$

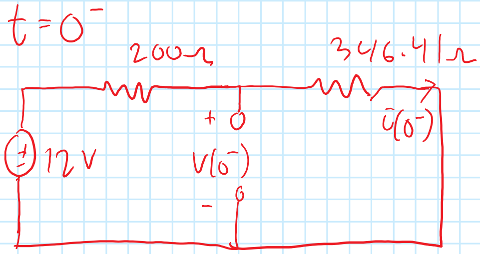
$$\boxed{R = 346.41\Omega}$$

Since critically damped:

$$i(t) = (A_1 t + A_2)e^{-\alpha t}, \quad t \geq 0$$

where $\alpha = \frac{R}{2L} = \frac{346.41\Omega}{2(10\times10^{-3}H)} = 17{,}320.5$

$t = 0^-$



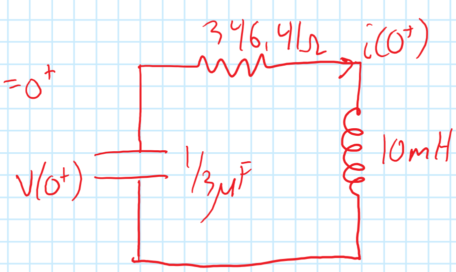
$$V(0^-) = 12V\left(\frac{346.41\Omega}{200\Omega + 346.41\Omega}\right)$$

$$V(0^-) = 7.6077\ V$$

$$i(0^-) = \frac{12V}{200\Omega + 346.41\Omega}$$

$$i(0^-) = 0.022A$$

$t = 0^+$



$$V(0^+) = V(0^-) = 7.6077V = V(0)$$

$$i(0^+) = i(0^-) = 0.022A = i(0)$$

$$i(0) = 0.022A = (A_1(0) + A_2)e^{-0}$$

$A_2 = 0.022$

$$\therefore\ i(t) = (A_1 t + 0.022)e^{-17{,}320.5t}$$

$$\left.\frac{di}{dt}\right|_{t=0} = \left.\frac{V_L}{L}\right|_{t=0} = \frac{V(0) - (346.41\Omega)\,i(0)}{L} = \frac{7.6077 - 346.41(0.022)}{0.01} = -1.332$$

$$\frac{di}{dt} = \frac{d}{dt}\left[(A_1 t + 0.022)e^{-17{,}320.5t}\right]$$

$$= (A_1 t + 0.022)(-17{,}320.5)e^{-17{,}320.5t} + A_1 e^{-17{,}320.5t}\Big|_{t=0}$$

$$= -381.05 + A_1$$

$$-381.05 + A_1 = -1.332$$

$$A_1 = 379.7$$

$$\boxed{i(t) = (379.7t + 0.022)e^{-17{,}320.5t} \quad \text{Amps}, \ t \geq 0}$$

IV. Complete Response of RLC Circuits

A. Analysis Procedure

1. Find general form of natural response (v_n or i_n) without solving for all unknown coefficients (set all independent sources = ∅)
2. Find forced or steady-state response (v_f or i_f)
3. Add the two expressions

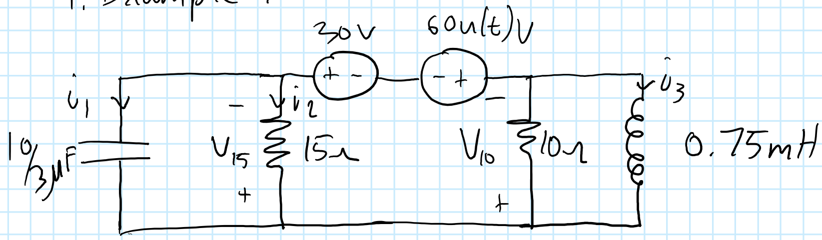
$$v = v_n + v_f$$

or, $$i = i_n + i_f$$

4. Adjust unknown coefficients to satisfy initial condition.

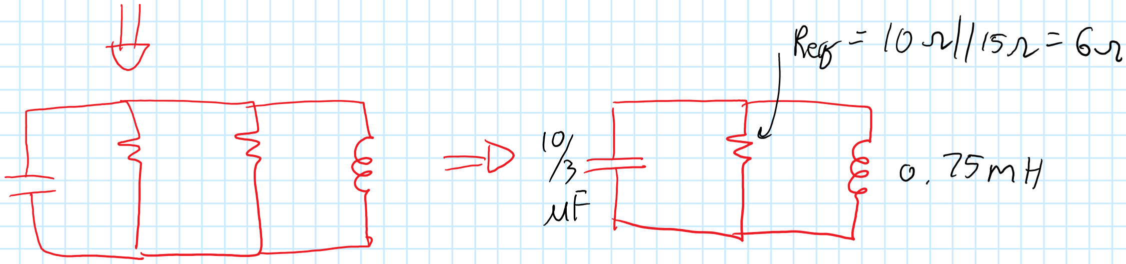
B. Sample Problems

1. Example 1



Solve for i_1, i_2, i_3, $t \geq 0$

Solu: Step 1: Natural Response for $t > 0$ parallel RLC form



Parallel RLC, so $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(6\Omega)(10/3\,\mu F)} = 2.5\times10^4$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.75mH)(10/3\,\mu F)}} = 2\times10^4$$

Since $\alpha > \omega_0$, response is overdamped

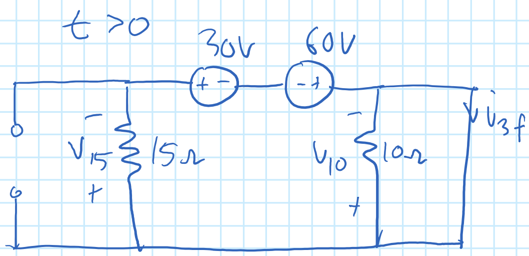
$$v_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -(2.5\times10^4) + \sqrt{(2.5\times10^4)^2 - (2\times10^4)^2} = -10^4$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -(2.5\times10^4) - \sqrt{(2.5\times10^4)^2 - (2\times10^4)^2} = -4\times10^4$$

$$V_n(t) = A_1 e^{(-10^2)t} + A_2 e^{(-4\times10^4)t}$$

Step 2: Find forced response



$V_{10_f} = 0\ V$, due to short

$V_{15f} = 30\ V$

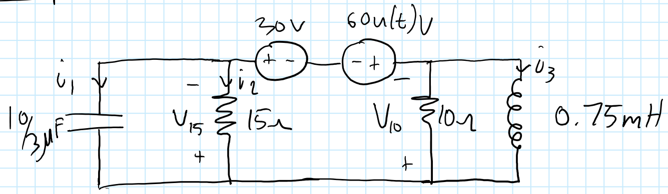
$$i_{3f} = \frac{(-30V)-(-60V)}{15\Omega} = \frac{30}{15} = 2A$$

$i_{2f} = -i_{3f} = -2A$

$i_{1f} = 0$

Wednesday, April 5, 2017 8:04 AM

Example 1:



Solve for i_1, i_2, i_3, $t \geq 0$

Solution:

Step 1: Natural response

$$\alpha = \frac{1}{2RC} = 2.5 \times 10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2 \times 10^4$$

$\alpha > \omega_0$ overdamped

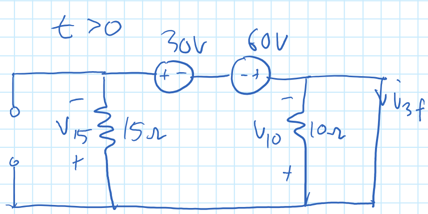
$$s_1 = \alpha + \sqrt{\alpha^2 - \omega_0^2} = -10^4$$

$$s_2 = \alpha - \sqrt{\alpha^2 - \omega_0^2} = -4 \times 10^4$$

$$V_n(t) = A_1 e^{-10^4 t} + A_2 e^{(-4 \times 10^4)t}$$

Step 2: Forced Response

$t > 0$



$$V_{10f} = 0V$$

$$V_{15f} = 30V$$

$$i_{3f} = 2A$$

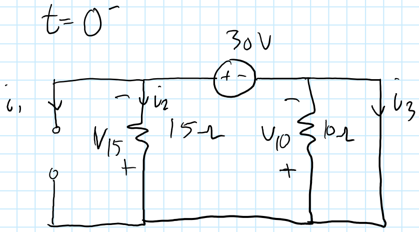
Step 3: Add responses

$$V_{10}(t) = V_{10f} + V_n = A_1 e^{-10^4 t} + A_2 e^{(-4 \times 10^4)t}$$

$$V_{15}(t) = V_{15f} + V_n = A_1' e^{-10^4 t} + A_2' e^{(-4 \times 10^4)t} + 30$$

Step 4: Adjust A_1, A_2, A_1', A_2' to satisfy initial conditions

$t = 0^-$

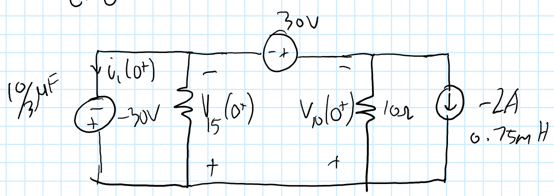


$$i_3(0^-) = \frac{-30V}{15\Omega} = -2A$$

$$V_{15}(0^-) = -30V$$

$$V_{10}(0^-) = 0V$$

$t = 0^+$



$i_3(0^+) = -2A$

$V_{15}(0^+) = -30V$

$-V_{15}(0^+) + 30V + V_{10}(0^+) = 0 \quad (KVL)$

$-(-30V) + 30V + V_{10}(0^+) = 0$

$V_{10}(0^+) = -60V$

$$V_{10}(0^+) = A_1 e^{0^+} + A_2 e^{0^+} = -60$$

$$A_1 + A_2 = -60$$

$$A_2 = -A_1 - 60$$

$$V_{10}(t) = A_1 e^{-10^4 t} - (A_1 + 60) e^{(-4\times10^4)t}$$

$$V_{15}(0^+) = 30 + A_1' e^{0^+} + A_2' e^{0^+} = -30$$

$$30 + A_1' + A_2' = -30$$

$$A_1' + A_2' = -60$$

$$A_2' = -A_1' - 60$$

$$V_{15}(t) = 30 + A_1' e^{-10^4 t} - (A_1' + 60) e^{(-4\times10^4)t}$$

$$i_1 = -C \frac{dV_{15}}{dt} \Rightarrow \left.\frac{dV_{15}}{dt}\right|_{t=0^+} = \frac{-i_1(0^+)}{C} = \frac{-\left[\frac{V_{15}(0^+)}{15\Omega} + \frac{V_{10}(0^+)}{10\Omega} + 2\right]}{10/3 \times 10^{-6}} = \frac{-\left[\frac{-30V}{15\Omega} + \frac{-60V}{10\Omega} + 2\right]}{10/3 \times 10^{-6}}$$

$$= 1.8\times10^6$$

$$\frac{dV_{15}}{dt} = \frac{d\left(30 + A_1' e^{-10^4 t} + A_2 e^{(-4\times10^4)t}\right)}{dt} = \left.(-10^4)(A_1')e^{-10^4 t} + A_2(-4\times10^4)e^{(-4\times10^4 t)}\right|_{t=0}$$

$$= 3\times10^4 A_1' + 2.4\times10^6$$

$$3\times10^4 A_1' + 2.4\times10^6 = 1.8\times10^6 \Rightarrow A_1' = -20$$

Finished Complete RLC Response

Thursday, April 6, 2017 8:02 AM

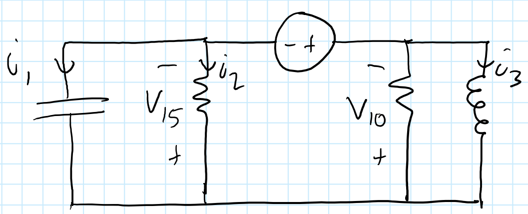
$$\boxed{V_{15}(t) = 30 - 20e^{-10^4 t} - 40e^{(-4\times10^4)t} \text{ Volts, } t \geq 0}$$

$$i_2(t) = \frac{-V_{15}(t)}{15\Omega} = \frac{30 - 20e^{-10^4 t} - 40e^{(-4\times10^4)t}}{15}$$

$$\boxed{i_2(t) = -2 + \frac{4}{3}e^{-10^4 t} - \frac{8}{3}e^{(-4\times10^4)t} \text{ Amps, } t \geq 0}$$

$$i_1(t) = C\frac{dV_{15}}{dt} = \left(\frac{-10}{3}\times10^6\right)\frac{d}{dt}\left(30 - 20e^{-10^4 t} - 40e^{(-4\times10^4)t}\right)$$

$$\boxed{i_1(t) = \frac{-2}{3}e^{-10^4 t} - \frac{16}{3}e^{(-4\times10^4)t} \text{ Amps, } t \geq 0}$$



Since $V_{10}(t) = -L\frac{di_3}{dt}$ negative due to polarity of 10Ω resistor

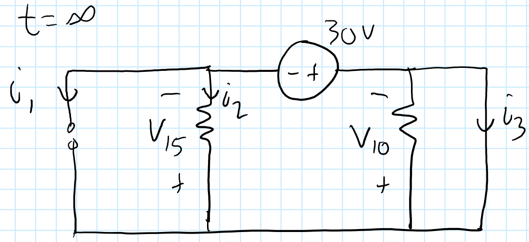
$$i_3(t) = \frac{-1}{L}\int_0^t V_{10}(t)\,dt + i_3(0^+)$$

$$= \frac{-1}{L}\int_0^t \left[A_1 e^{-10^4 t} - (A_1 + 60)e^{-4\times10^4 t}\right]dt + \underbrace{i_3(0^+)}_{-2}$$

$$= \frac{-1}{L}\left[\frac{A_1\left(e^{-10^4 t} - 1\right)}{-10^4} - \frac{(A_1+60)\left(e^{-4\times10^4 t} - 1\right)}{-4\times10^4}\right] - 2$$

$$i_3(\infty) = 2A = \frac{30V}{15\Omega}$$

$t = \infty$



$$i_3(\infty) = \frac{30V}{15\Omega} = 2A$$

$$i_3(\infty) = \frac{-1}{L}\left[\frac{A_1\left(e^{-\infty} - 1\right)}{-10^4} - \frac{(A_1+60)\left(e^{-\infty} - 1\right)}{-4\times10^4}\right] - 2$$

$$i_3(\infty) = \frac{-1}{L}\left[\frac{A_1(e^{-\infty}-1)}{-10^4} - \frac{(A_1+60)(e^{-\infty}-1)}{-4\times10^4}\right] - 2$$

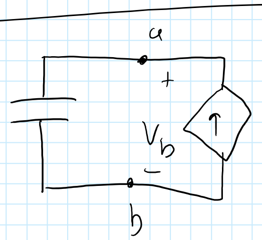
$$i_3(\infty) = \frac{-1}{L}\left[\frac{A_1}{10^4} - \frac{A_1+60}{4\times10^4}\right] - 2 = 2$$

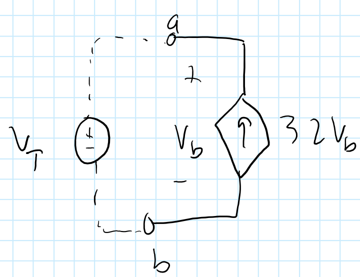
$$\frac{-1}{0.75\times10^{-3}}\left[\frac{A_1}{10^4} - \frac{A_1+60}{4\times10^4}\right] - 2 = 2$$

$$A_1 = -20$$

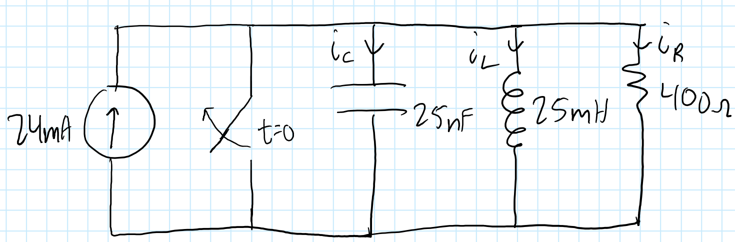
$$i_3(t) = \frac{-1}{0.75\times10^{-3}}\left[\frac{-20\,(e^{-10^4 t}-1)}{-10^4} - \frac{(-20+60)(e^{-4\times10^4 t}-1)}{-4\times10^4}\right] - 2$$

$$\boxed{i_3(t) = -\frac{8}{3}e^{-10^4 t} - \frac{4}{3}e^{-4\times10^4 t} + 2 \quad \text{Amps},\ t > 0}$$





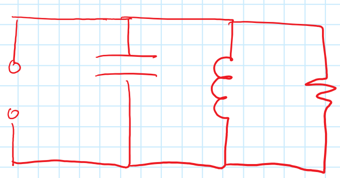
$$R_{TH} = \frac{V_T}{i_T} = \frac{V_b}{32V_b} = \frac{1}{32}\,\Omega$$



$i_C(t) = ?$, $t \geq 0$

Solu:

Step 1:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(400)(25nF)} = 5\times10^4$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25mH)(25nF)}} = 4\times10^4$$

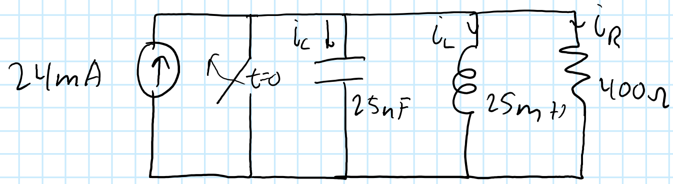
$\alpha > \omega_0$ overdamped

$i_{L_n}(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Finding Current in Parallel RLC

Monday, April 10, 2017 8:24 AM

Example 8.6



$i_L(t) = ?\quad t > 0$

Solution:

step 1) $\alpha = \frac{1}{2RC} = 5\times10^4$ $\qquad \alpha > \omega_0$ overdamped

$\omega_0 = \frac{1}{\sqrt{LC}} = 4\times10^4$

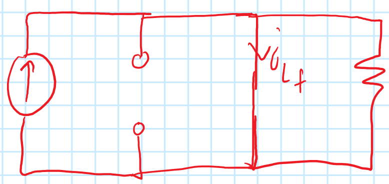
$s_1 = -20{,}000$

$s_2 = -80{,}000$

$i_{L_n}(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$i_{L_n}(t) = A_1 e^{-20000t} + A_2 e^{-80000t}$

step 2)

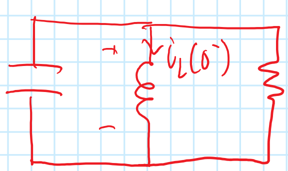


$i_{L_F} = 24mA$

Step 3) $i_L(t) = i_{L_n} + i_{L_F}$

$= 24 + A_1 e^{-20000t} + A_2 e^{-80000t}$

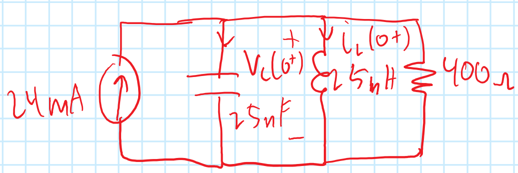
Step 4) $t = 0^-$



$i_L(0^-) = 0$

$V_c(0^-) = 0$

$t = 0^+$



$i_L(0^+) = i_L(0^-) = i_L(0) = 0$

$V_L(0^+) = V_c(0^+) = V_c(0^-) = V_L(0) = 0$

$i_L(0) = 0 = 24 + A_1 + A_2$

$A_1 + A_2 = -24 \qquad (1)$

$A_1 + A_2 = -24 \qquad (1)$

$V_L = L\dfrac{di_L}{dt} \qquad \left.\dfrac{di_L}{dt}\right|_{t=0} = \dfrac{V_L}{L} = 0$

$$\frac{di_L}{dt} = \frac{d}{dt}\left(24 + A_1 e^{-20000t} + A_2 e^{-80000t}\right)$$

$$= \left.\left(-20{,}000 A_1 e^{-20000t} - 80000 A_2 e^{-80000t}\right)\right|_{t=0} = -20000A_1 - 80000A_2$$

$20000A_1 + 80000A_2 = 0 \qquad (2)$

Solve eq (1) and (2)

$A_1 = -32$ mA

$A_2 = 8$ mA

$$\boxed{i_L(t) = 24 - 32e^{-20000t} + 8e^{-80000t} \text{ mA},\ t > 0}$$

Intro to Ch 12

Tuesday, April 11, 2017 8:03 AM

1. Read Ch. 12
2. Do Hw #6
3. Hw #6 due Wed, 4/26

Hw#6 - Chapter 12: 3, 8, 12, 14, 20, 26, 27, 29, 30, 34, 40, 46, 55

1.) Definition of the Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

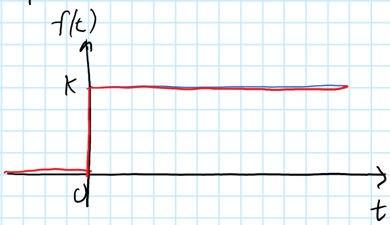
if $f(t) = 1$

$$\int_0^\infty e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_0^\infty = \frac{-1}{s}[0-1] = \frac{1}{s}$$

$$s = \underset{\text{damping factor}}{\alpha} + j\underset{\text{resonant frequency}}{\omega}$$

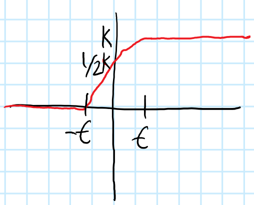
$$\mathcal{L}\{f(t)\} = F(s)$$

2.) Step Function

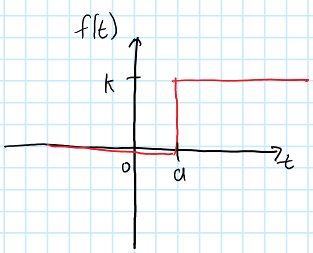


$Ku(t) = 0, \; t < 0$

$Ku(t) = K, \; t > 0$

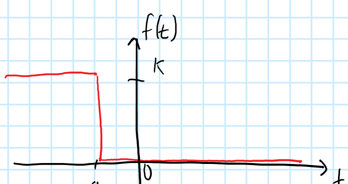


$Ku(0) = 0.5K$



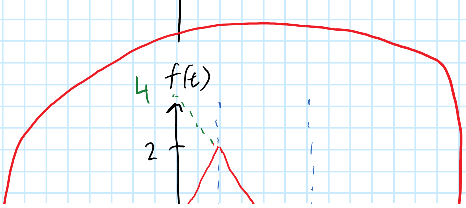
$Ku(t-a) = 0, \; t < a$

$Ku(t-a) = K, \; t > a$



$Ku(a-t) = 0, \; t < a$

$Ku(a-t) = K, \; t > a$

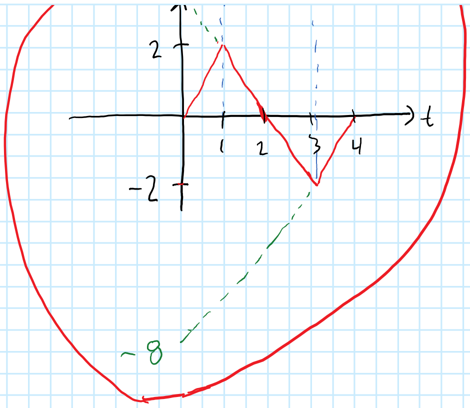


$f(t) = mt + b$

$[0, 1]$

$f(t) = 2t$

$[1, 3]$



represents

$[0,1]$

$f(t)=2t$

① $m = \dfrac{2-0}{1-0} = 2$

② $f(0) = 0 = 2(0) + b$

$b = 0$

$\therefore f(t) = 2t$

$[1,3]$

① $m = \dfrac{-2-2}{3-1} = \dfrac{-4}{2} = -2 = m$

② $f(0) = 4 = -2 \cdot 0 + b$

$b = 4$

$\therefore f(t) = -2t + 4$

$[3,4]$

① $m = \dfrac{0-(-2)}{4-3} = 2 = m$

② $f(0) = -8 = 2(0) + b$

$b = -8$

$\therefore f(t) = 2t - 8$

or $f(4) = 0 = 2(4) + b$

$0 = 8 + b$

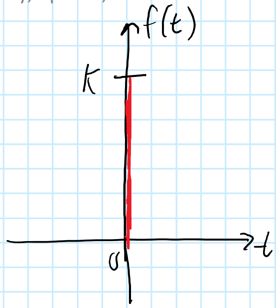
$b = -8$

$f(t) = 2t - 8$

$$\boxed{f(t) = 2t[u(t)-u(t-1)] + (-2t+4)[u(t-1)-u(t-3)] + (2t-8)[u(t-3)-u(t-4)]}$$

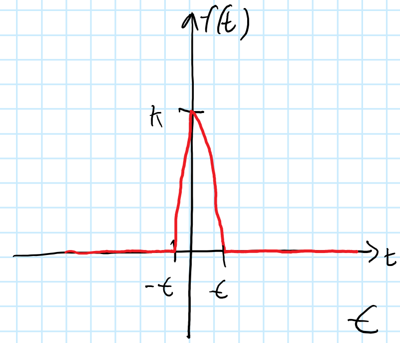
Intro to Laplace Transform

Wednesday, April 12, 2017 7:59 AM

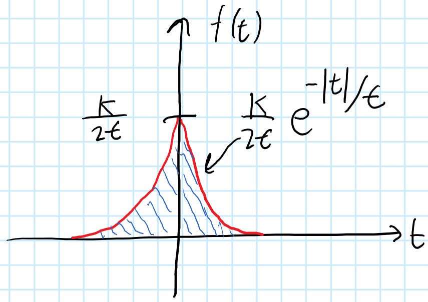


$\delta(t) = k, \quad t = 0$

$\delta(t) = 0, \quad t \neq 0$



ϵ is infintesimal



$$\int_{-\infty}^{+\infty} k\delta(t)\,dt = k$$

$$\text{Area} = \int_{-\infty}^{0} \frac{k}{2\epsilon} e^{t/\epsilon}\,dt + \int_{0}^{\infty} \frac{k}{2\epsilon} e^{-t/\epsilon}\,dt$$

$$= \frac{k}{2\epsilon} \cdot \frac{e^{t/\epsilon}}{1/\epsilon}\Bigg|_{-\infty}^{0} + \frac{k}{2\epsilon} \cdot \frac{e^{-t/\epsilon}}{-1/\epsilon}\Bigg|_{0}^{\infty}$$

$$= \frac{k}{2\epsilon}\left(\frac{e^{0}}{1/\epsilon} - \frac{e^{-\infty}}{1/\epsilon}\right) + \frac{k}{2\epsilon}\left(\frac{e^{-\infty}}{-1/\epsilon} - \frac{e^{0}}{-1/\epsilon}\right)$$

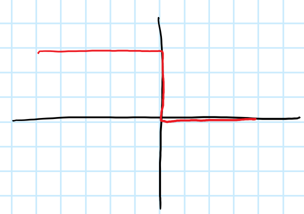
$$= \frac{k}{2\epsilon}\left(\frac{1}{1/\epsilon} - 0\right) + \frac{k}{2\epsilon}\left(0 + \frac{1}{1/\epsilon}\right)$$

$$= \frac{k\cancel{\epsilon}}{2\cancel{\epsilon}} + \frac{k\cancel{\epsilon}}{2\cancel{\epsilon}} = \frac{k}{2} + \frac{k}{2} = k$$

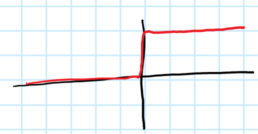
$$\mathcal{L}\{\delta(t)\} = 1$$

$$u(t)\begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$u(-t)$



$u(t)$



$$\mathcal{L}\{u(t)\} = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

$$= \int_{0^-}^{\infty} 1 \cdot e^{-st}\,dt$$

$$= -\frac{1}{s}\int_{0^-}^{\infty} 1 \cdot e^{-st}\,d(-st)$$

$$= -\frac{1}{s}\left[e^{-st}\right]_{0^-}^{\infty} = -\frac{1}{s}\left[e^{-\infty} - e^{0^-}\right] = -\frac{1}{s}\left[0 - 1\right] = \frac{1}{s}$$

$f(t) = e^{-at}$

$s\,[e \quad]_{0^-} \qquad s[\qquad] \qquad s[0-1] \qquad s$

$f(t) = e^{-at}$ $\quad \mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-at}\} = \int_0^\infty e^{-at}\cdot e^{-st}\,dt = \int_0^\infty e^{-(s+a)t}\,dt$

$$= \frac{1}{s+a}$$

$f(t) = \sin\omega t$ $\quad \mathcal{L}\{f(t)\} = \mathcal{L}\{\sin(\omega t)\} = \int_0^\infty (\sin\omega t)\cdot e^{-st}\,dt$

$$= \int_0^\infty \frac{e^{j\omega t} - e^{-j\omega t}}{2j}\, e^{-st}\,dt$$

$$= \int_0^\infty \frac{e^{-(s-j\omega)t} - e^{-(s+j\omega)t}}{2j}\,dt$$

$$= \frac{1}{2j}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right)$$

$$= \frac{1}{2j}\cdot\frac{(s+j\omega)-(s-j\omega)}{(s-j\omega)(s+j\omega)} = \frac{1}{2j}\cdot\frac{s+j\omega-s+j\omega}{s^2+\omega^2} = \frac{1}{\cancel{2j}}\cdot\frac{\cancel{2}j\omega}{s^2+\omega^2}$$

$$= \frac{\omega}{s^2+\omega^2}$$

Using Laplace Transforms

Thursday, April 13, 2017 8:11 AM

Table 12.1 – Laplace transforms Table 12.2 $\Longrightarrow$ P.440

$\cos \omega t \longrightarrow \frac{s}{s^2+\omega^2}$

$te^{-at} \longrightarrow \frac{1}{(s+a)^2}$

$e^{-at}\sin\omega t \longrightarrow \frac{\omega}{(s+a)^2+\omega^2}$

$e^{-at}\cos\omega t \longrightarrow \frac{s+a}{(s+a)^2+\omega^2}$

Operational Transforms

$\mathcal{L}\{f(t)\} = F(s)$

$\mathcal{L}\{K f(t)\} = KF(s)$

Addition + Subtraction

$\mathcal{L}\{f_1(t)+f_2(t)-f_3(t)\} = F_1(s)+F_2(s)-F_3(s)$

Differentiation

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \int_0^\infty \frac{df(t)}{dt}e^{-st}dt = f(t)e^{-st}\Big|_0^\infty - \int_0^\infty f(t)(-se^{-st})dt$$

$$= (0-f(0)) + s\underbrace{\int_0^\infty f(t)e^{-st}dt}_{F(s)} = \boxed{sF(s)-f(0)}$$

Integration

$$\mathcal{L}\left\{\int_0^t f(x)dx\right\} = \int_0^\infty\left[\int_0^t f(x)dx\right]e^{-st}dt$$

Let $u = \int_0^t f(x)dx$ $du = f(t)dt$

$dv = e^{-st}dt$ $v = \frac{-e^{-st}}{s}$

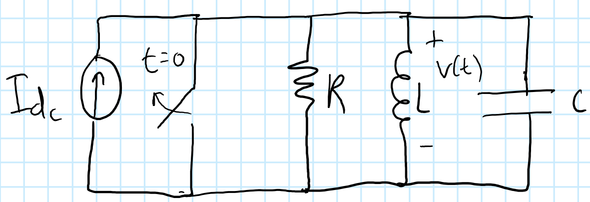
$$= uv - \int v\,du = \frac{-e^{-st}}{s}\int_0^t f(x)dx \Big|_0^\infty + \int_0^\infty \frac{e^{-st}}{s} f(t)dt$$

$$= \frac{1}{s}\int_0^\infty e^{-st} f(t)dt = \boxed{\frac{F(s)}{s}}$$

2nd order differentiation - P. 437

$$\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2F(s) - s\,f(0) - \frac{df(0)}{dt}$$

Applying the Laplace Transform



Time domain:

$$i_R + i_L + i_C = I_{dc}$$

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)dt + \frac{C\,dv(t)}{dt} = I_{dc}u(t)$$

Frequency domain (Laplace Transform):

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - V(0)] = I_{dc}\left(\frac{1}{s}\right)$$

$V(0) = 0\text{ V}$

$$V(s)\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{I_{dc}}{s}$$

$$V(s) = \frac{I_{dc}/s}{s^2 + \frac{1}{RC}(s) + \frac{1}{LC}}$$

Partial Fraction Method

Monday, April 17, 2017 8:05 AM

$$F(s)=\frac{96s^2+1652s+5760}{s^3+14s^2+48s}=\frac{96(s^2+17s+60)}{s(s^2+14s+48)}$$

$$s=\frac{-14\pm\sqrt{14^2-4(1)(48)}}{2}=-7\pm\frac{\sqrt{4}}{2}=-7\pm 1$$

$s_1=-6 \Rightarrow s_1+6=0$

$s_2=-8 \Rightarrow s_2+8=0$

$$s=\frac{-17\pm\sqrt{17^2-4(1)(60)}}{2}$$

$$s=\frac{-17\pm\sqrt{49}}{2}$$

$$s=\frac{-17\pm 7}{2}$$

$s_1=-5$

$s_2=-12$

$$=\frac{96(s+5)(s+12)}{s(s+6)(s+8)}$$

$$=\frac{K_1}{s}+\frac{K_2}{s+8}+\frac{K_3}{s+6}$$

$$s\cdot\left.\frac{96(s+5)(s+12)}{s(s+6)(s+8)}\right|_{s=0}=K_1+\left.\frac{sK_2}{s+8}\right|_{s=0}+\left.\frac{sK_3}{s+6}\right|_{s=0}$$

$$K_1=\frac{96(0+5)(0+12)}{(0+6)(0+8)}=\frac{96(60)}{48}=120$$

$$(s+8)\left.\frac{96(s+5)(s+12)}{s(s+6)(s+8)}\right|_{s=-8}=\left.\frac{(s+8)K_1}{s}\right|_{s=-8}+\left.\frac{(s+8)K_2}{s+8}\right|_{s=-8}+\left.\frac{(s+8)K_3}{s+6}\right|_{s=-8}$$

$$\left.\frac{96(s+5)(s+12)}{s(s+6)}\right|_{s=-8}=0+K_2+0$$

$$\frac{96(-8+5)(-8+12)}{-8(-8+6)}=K_2 \qquad K_2=\frac{96(-12)}{16}=-72$$

$$(s+6)\left.\frac{96(s+5)(s+12)}{s(s+6)(s+8)}\right|_{s=-6}=\left.\frac{(s+6)K_1}{s}\right|_{s=-6}+\left.\frac{(s+6)K_2}{s+8}\right|_{s=-6}+\left.\frac{(s+6)K_3}{s+6}\right|_{s=-6}$$

$$\left.\frac{96(s+5)(s+12)}{s(s+8)}\right|_{s=-6}=0+0+K_3$$

$$\frac{96(-6+5)(-6+12)}{-6(-6+8)}=\frac{96(-6)}{-12}=48=K_3$$

$$\therefore\ \frac{120}{\ }-\frac{72}{\ }+\frac{48}{\ }$$

$$\therefore \quad \frac{120}{s} - \frac{72}{s+8} + \frac{48}{s+6}$$

$$\mathcal{L}^{-1}\left\{\frac{96s^2 + 1652s + 5760}{s^3 + 14s^2 + 48s}\right\} = \boxed{120\,u(t) - 72e^{-8t}u(t) + 48e^{-6t}u(t)}$$

$$= \left(120 - 72e^{-8t} + 48e^{-6t}\right)u(t)$$

Inverse Laplace Transform of Imaginary Numbers

Wednesday, April 19, 2017 8:02 AM

$$F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$$

$$s_{1,2} = \frac{-6 \pm \sqrt{6^2-4(1)(25)}}{2(1)} = \frac{-6 \pm \sqrt{-64}}{2} = \frac{-6 \pm j8}{2} = -3 \pm j4$$

$$= \frac{100(s+3)}{(s+6)(s+3-j4)(s+3+j4)}$$

$$= \frac{K_1}{s+6} + \frac{K_2}{(s+3-j4)} + \frac{K_3}{(s+3+j4)}$$

$$K_1 = \left.\frac{100(s+3)}{s^2+6s+25}\right|_{s=-6} = \frac{100(-3)}{25} = -12$$

$$\left.\frac{(s+3-j4)[100(s+3)]}{(s+6)(s+3-j4)(s+3+j4)}\right|_{s=-3+j4} = \underbrace{\left.\frac{(s+3-j4)K_1}{s+6}\right|_{s=-3+j4}}_{=0}$$

$$+ \left.\frac{\cancel{(s+3-j4)}K_2}{\cancel{s+3-j4}}\right|_{s=-3+j4}$$

$$+ \underbrace{\left.\frac{(s+3-j4)K_3}{s+3+j4}\right|_{s=-3+j4}}_{=0}$$

$$K_2 = \left.\frac{100(s+3)}{(s+6)(s+3+j4)}\right|_{s=-3+j4} = \frac{100(-3+j4+3)}{(-3+j4+6)(-3+j4+3+j4)} = \frac{\overset{50}{\cancel{100}}\,j4}{(3+j4)(j\cancel{8})} = \frac{50}{3+j4}$$

$$= \frac{50(3-j4)}{(3+j4)(3-j4)} = \frac{150-j200}{9+16} = 6-j8 \rightarrow \text{convert to polar form}$$

$$a+jb = \sqrt{a^2+b^2}\ \tan^{-1}\angle\frac{b}{a}$$

$$= c\angle\theta = c\cos\theta + j\sin\theta = a+jb$$

$$c\angle\theta = ce^{j\theta}$$

$$c_1e^{j\theta_1} \cdot c_2e^{j\theta_2} = c_1 \cdot c_2 e^{j(\theta_1+\theta_2)} = c_1\angle\theta_1 \cdot c_2\angle\theta_2 = c_1 \cdot c_2$$

$$= \sqrt{6^2+(-8)^2}\ \tan^{-1}\left(\frac{-8}{6}\right)$$

$$\boxed{=10e^{-j53.13^\circ}}$$

$$K_3 = \left.\frac{100(s+3)}{(s+6)(s+3-j4)}\right|_{s=-3-j4} = \frac{100(-3-j4+3)}{(-3-j4+6)(-3-j4+3-j4)} = 6+j8 = \boxed{10e^{j53.13^\circ}}$$

$$= \frac{-12}{s+6} + \frac{10e^{-j53.13^\circ}}{s+3-j4} + \frac{10e^{j53.13^\circ}}{s+3+j4} = \frac{-12}{s+6} + \frac{10\angle -53.13^\circ}{s+3-j4} + \frac{10\angle 53.13^\circ}{s+3+j4}$$

$$= \frac{-12}{s+6} + \frac{10e^{-j53.13^\circ}}{s+3-j4} + \frac{10e^{j53.13^\circ}}{s+3+j4} = \frac{-12}{s+6} + \frac{10\angle -53.13^\circ}{s+3-j4} + \frac{10\angle 53.13^\circ}{s+3+j4}$$

$$\mathcal{L}^{-1}\left\{\frac{100(s+3)}{(s+6)(s^2+6s+25)}\right\} = \mathcal{L}^{-1}\left\{\frac{-12}{s+6} + \frac{10\angle -53.13^\circ}{s+3-j4} + \frac{10\angle 53.13^\circ}{s+3+j4}\right\}$$

$$= \left(-12e^{-6t} + 10\angle -53.13^\circ \cdot e^{-(3-j4)t} + 10\angle 53.13^\circ e^{-(3+j4)t}\right) u(t)$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$

$$= \left(-12e^{-6t} + 10e^{-j53.13^\circ} \cdot e^{-(3-j4)t} + 10e^{j53.13^\circ} \cdot e^{-(3+j4)t}\right) u(t)$$

$$= \left(-12e^{-6t} + 10e^{-3t} \cdot e^{j(4t-53.13^\circ)} + 10e^{-3t} \cdot e^{-j(4t-53.13^\circ)}\right.$$

Note: $e^a \cdot e^b \cdot e^c = e^{(a+b+c)}$

$e^a \cdot e^{(b+c)} = e^c \cdot e^{(a+b)}$

$$= \left(-12e^{-6t} + \frac{2\cdot 10e^{-3t}\left(e^{j(4t-53.13^\circ)} + e^{-j(4t-53.13^\circ)}\right)}{2}\right) u(t)$$

$$= \left(-12e^{-6t} + 20e^{-3t}\cos(4t-53.13^\circ)\right) u(t)$$

phase

Laplace Real Repeated Roots

Monday, April 24, 2017 8:04 AM

Repeated Real Roots of $F(s)$

$$\frac{100(s+25)}{s(s+5)^3} = \frac{K_1}{s} + \frac{K_2}{(s+5)^3} + \frac{K_3}{(s+5)^2} + \frac{K_4}{(s+5)}$$

$$K_1 = \left.\frac{100(s+25)}{(s+5)^3}\right|_{s=0} = \frac{100(0+25)}{(0+5)^3} = \frac{2500}{125} = 20$$

To find K_2, we multiply both sides by $(s+5)^3$ and the evaluate both sides at -5:

$$\left.\frac{100(s+25)}{s}\right|_{s=-5} = \left.\frac{K_1(s+5)^3}{s}\right|_{s=-5} + K_2 + K_3(s+5)\Big|_{s=-5} + K_4(s+5)^2\Big|_{s=-5}$$

$$\frac{100(-5+25)}{-5} = 0 + K_2 + 0 + 0 \qquad K_2 = \frac{2000}{-5} = -400$$

To find K_3, we first must multiply both sides by $(s+5)^3$. Next we differentiate both sides once with respect to s and then evaluate at $s=-5$.

$$\frac{d}{ds}\left[\frac{100(s+25)}{s}\right]\Bigg|_{s=-5} = \frac{d}{ds}\left[\frac{K_1(s+5)^3}{s}\right]_{s=-5} + \frac{d}{ds}\Big[K_2\Big]_{s=-5} + \frac{d}{ds}\Big[K_3(s+5)\Big]_{s=-5}$$

$$+\frac{d}{ds}\Big[K_4(s+5)^2\Big]_{s=-5}$$

$$100\left[\frac{s-(s+25)}{s^2}\right]_{s=-5} = 0 + 0 + K_3 + 0$$

$$K_3 = 100\left[\frac{-5-(-5+25)}{(-5)^2}\right] = 100\left[\frac{-25}{25}\right] = -100$$

To find K_4, we first multiply both sides by $(s+5)^3$. Next, we differentiate both sides twice with respect to "s" and then evaluate both sides at $s=-5$. After simplifying the first derivative, the second deriv becomes:

$$100\frac{d}{ds}\left[\frac{-25}{s^2}\right]_{s=-5} = K_1\frac{d}{ds}\left[\frac{(s+5)^2(25-5)}{s^2}\right]_{s=-5} + 0 + \frac{d}{ds}\Big[K_3\Big]_{s=-5} + \frac{d}{ds}\Big[2K_4(s+5)\Big]_{s=-5}$$

$$-40 = 2K_4$$

$$K_4 = -20$$

$$= \frac{20}{s} - \frac{400}{(s+5)^3} - \frac{100}{(s+5)^2} - \frac{20}{(s+5)}$$

$$\mathcal{L}^{-1}\left\{\frac{100(s+15)}{s(s+5)^3}\right\} = \underbrace{\left[20 - 200t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}\right]}_{\text{Time domain result}} u(t)$$

$$F(s) = \frac{s^4 + 13s^3 + 66s^2 + 200s + 300}{s^2 + 9s + 20}$$

$$\begin{array}{r l}
 & s^2 + 4s + 10 + \frac{30s+100}{s^2+9s+20} \\
s^2+9s+20 \,\big) & s^4 + 13s^3 + 66s^2 + 200s + 300 \\
 & -(s^4 + 9s^3 + 20s^2) \\
\hline
 & 4s^3 + 46s^2 + 200s \\
 & -(4s^3 + 36s^2 + 80s) \\
\hline
 & 10s^2 + 120s + 300 \\
 & -(10s^2 + 90s + 200) \\
\hline
 & 30s + 100
\end{array}$$

$$F(s) = s^2 + 4s + 10 + \frac{30s+100}{s^2+9s+20}$$

$$= s^2 + 4s + 10 + \frac{30s+100}{(s+5)(s+4)}$$

$$= s^2 + 4s + 10 + \frac{K_1}{s+4} + \frac{K_2}{s+5}$$

$$\frac{30s + 100}{(s+4)(s+5)} = \frac{K_1}{s+4} + \frac{K_2}{s+5}$$

$$\left.\frac{(30s+100)}{(s+5)}\right|_{s=-4} = \left.K_1 + \frac{K_2(s+4)}{(s+5)}\right|_{s=-4}$$

$$\frac{-20}{1} = K_1 \qquad K_1 = -20$$

$$K_2 = \left.\frac{30s+100}{s+4}\right|_{s=-5} = \frac{-50}{-1} \qquad K_2 = 50$$

$$= s^2 + 4s + 10 - \frac{20}{s+4} + \frac{50}{s+5}$$

$$\mathcal{L}^{-1}\left[\frac{s^4 + 13s^3 + 66s^2 + 200s + 300}{s^2 + 9s + 20}\right] = \mathcal{L}^{-1}\left[s^2 + 4s + 10 - \frac{20}{s+4} + \frac{50}{s+5}\right]$$

$$= \left[\frac{d^2\delta(t)}{dt^2} + \frac{4\,d\,\delta(t)}{dt} + 10\delta(t) - \left(20e^{-4t} - 50e^{-5t}\right)\right]u(t)$$

Tables 12.1, 12.2, 12.3

Applying Laplace Transform

Tuesday, April 25, 2017 8:04 AM

Polar and Zeros of $F(s)$

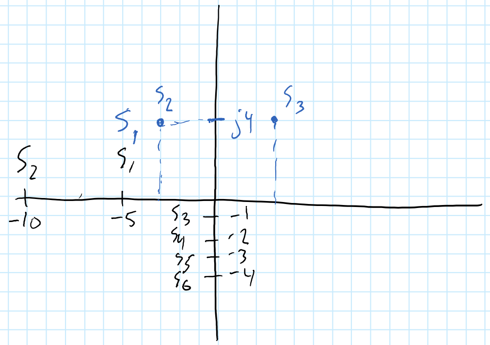
$$F(s) = \frac{4(s+5)(s+10)}{(s+1)(s+2)(s+3)(s+4)}$$

Numerator: $(s+5)(s+10)$

$s_1 = -5$, $s_2 = -10 \Rightarrow$ Zeros

Denominator: $(s+1)(s+2)(s+3)(s+4)$

$s_3 = -1$, $s_4 = -2$, $s_5 = -3$, $s_6 = -4 \Rightarrow$ Poles



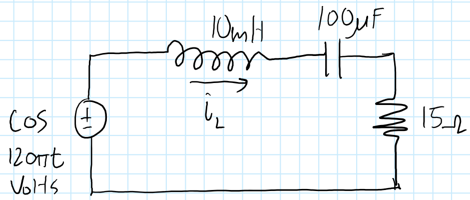
S-plane

$$F(s) = \frac{9(s+5)}{(s+3-j4)(s-3-j4)}$$

Zero: $s_1 = -5$

Poles: $s_2 = -3 + j4$

$s_3 = 3 + j4$



Use Laplace Transform to determine the complete response of $i_L(t)$

Solu: KVL: $(15\,\Omega)\, i_L(t) + (10 \times 10^{-3} H)\dfrac{di_L(t)}{dt} + \left(\dfrac{1}{100\times10^{-6}} F\right)\displaystyle\int_0^t i_L(t)dt$

$- \cos 120\pi t = 0$

$$15 i_L(t) + 0.01 \frac{di_L(t)}{dt} + 10^4 \int_0^t i_L(t)dt = \cos 120\pi t$$

Using Table 12.1 and 12.2

$$15 I_L(s) + 0.01 s I_L(s) + 10^4 \frac{I_L(s)}{s} = \frac{s}{s^2 + (120\pi)^2}$$

$$I_L(s)\left[15 + 0.01 s + \frac{10^4}{s}\right] = \frac{s}{s^2 + (120\pi)^2}$$

$$I_L(s) = \frac{\dfrac{s}{s^2+(120\pi)^2}}{15 + 0.01 s + \dfrac{10^4}{s}} = \frac{\dfrac{s^2}{s^2+(120\pi)^2}}{15s + 0.01s^2 + 10^4} = \frac{\dfrac{100s^3}{s^2+(120\pi)^2}}{1500s + s^2 + 10^6} = \frac{100s^2}{(s^2+1500s+10^6)(s^2+(120\pi)^2)}$$

$$I_L(s) = \frac{100s^2}{(s+750+j661.44)(s+750-j661.44)(s-j120\pi)(s+j120\pi)}$$

$$s_{1,2} = \frac{-1500 \pm \sqrt{1500^2 - 4(1)(10^6)}}{}$$

$$I_L(s) = \frac{100s}{\underbrace{(s+750+j661.44)(s+750-j661.44)}(s-j120\pi)(s+j120\pi)}$$

$$s_{1,2} = \frac{-1500 \pm \sqrt{1500^2 - 4(1)(10^6)}}{2(1)}$$

$$s_{1,2} = -750 \pm j661.44$$

$$s_1 = -750 - j661.44$$

$$s_2 = -750 + j661.44$$

$$= \frac{K_1}{(s+750+j661.44)} + \frac{K_1^*}{(s+750-j661.44)} + \frac{K_2}{(s-j120\pi)} + \frac{K_2^*}{(s+j120\pi)}$$

$$K_1 = \left.\frac{100s^2}{(s+750s+j661.44)(s-j120\pi)(s+j120\pi)}\right|_{s=-750-j661.44} = 0.07357\angle 97.89^\circ$$

$$K_2 = \left.\frac{100s^2}{(s+750-j661.44)(s+750+j661.44)(s+j120\pi)}\right|_{s=j120\pi} = 0.018345\angle 56.61^\circ$$

$$= \frac{0.07357\angle 97.89^\circ}{(s+750+j661.44)} + \frac{0.07357\angle -97.89^\circ}{(s+750-j661.44)} + \frac{0.018345\angle 56.61^\circ}{s-j120\pi} + \frac{0.018345\angle -56.61^\circ}{s+j120\pi}$$

$$\mathcal{L}^{-1}\left[I_L(s)\right] = 147.14e^{-750t}\cos(661.44t - 97.89^\circ) + 36.69\cos(120\pi t + 56.61^\circ)\ \text{mA}$$

HW #7

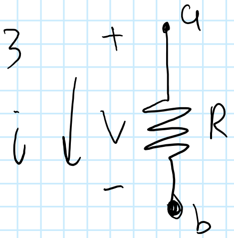
Wednesday, April 26, 2017 8:04 AM

1. Read Ch 17
2. Do HW #7 - Chapter 13: 4, 7, 8, 9, 10, 11, 17, 21, 23, 25, 27, 57

Due Monday 5/08

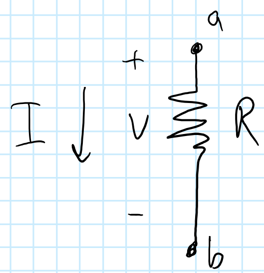
Time Domain

Ch. 13



$V = Ri$

Frequency Domain

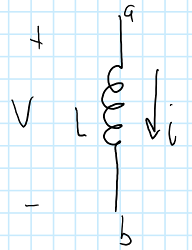


$V = RI$

Frequency Domain Models for Elements

Thursday, April 27, 2017 8:00 AM

An inductor in the s-domain

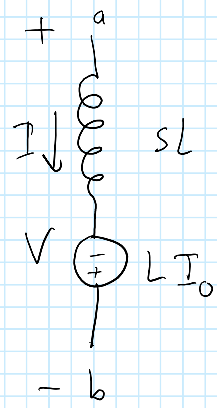


$V = L\frac{di}{dt}$

Table 12.2

$\mathcal{L}\left\{L\frac{di}{dt}\right\} = L\left[sI - I_0\right]$

$= sLI - LI_0 = V$

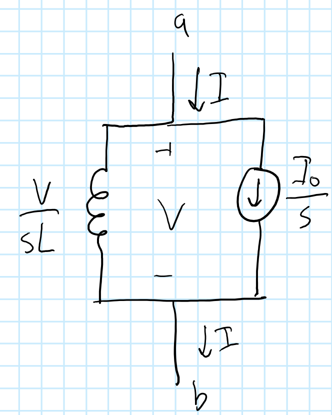


$V = sLI - LI_0$

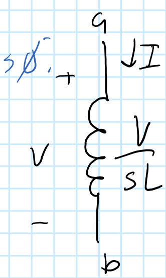
$V + LI_0 = sLI$

$I = \frac{V + LI_0}{sL} = \frac{V}{sL} + \frac{I_0}{s}$

impedence(ind) = sL



if initial conditions is ∅,



A capacitor in the s-domain

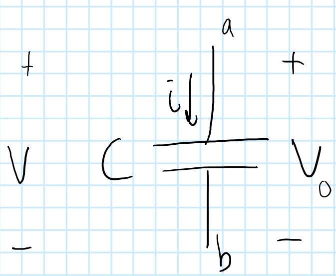
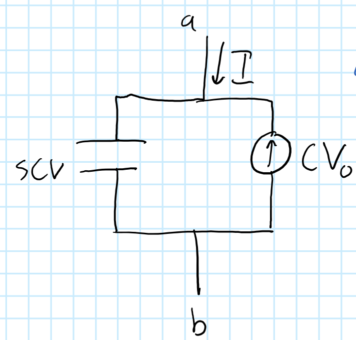


Table 12.2

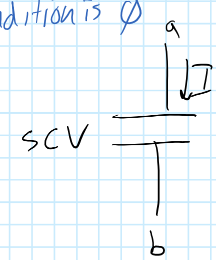
$I = C\left[sV - V(0^-)\right]$

$= sCV - CV_0$

impedence(cap) = $\frac{1}{sC}$



if initial condition is ∅

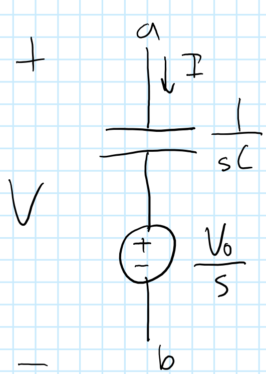


$i = C\frac{dV}{dt}$

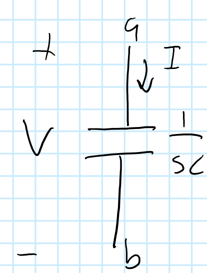
$I = sCV - CV_0$

$sCV = I + CV_0$

$V = \frac{I + CV_0}{sC} = \frac{I}{sC} + \frac{V_0}{s}$

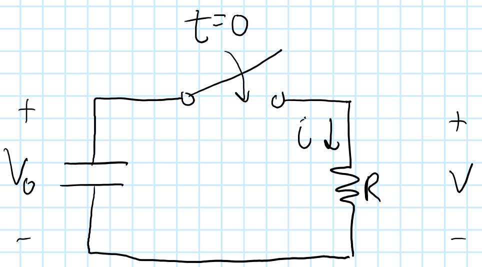


if initial conditions = ∅

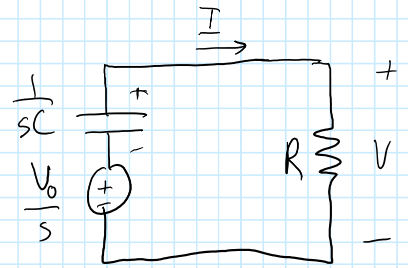


Natural Response of an RC circuit

Natural Response of an RC circuit



RC circuit in time domain



RC circuit in frequency domain

KVL:

$$\frac{V_0}{s} - \frac{1}{sC} I - RI = 0$$

$$\frac{1}{sCI} + RI = \frac{V_0}{s} \qquad I\left(\frac{1}{sC} + R\right) = \frac{V_0}{s}$$

$$I = \frac{\frac{V_0}{s}}{\left(\frac{1}{sC} + R\right)} \cdot \frac{s}{s} = \frac{V_0}{\frac{1}{C} + sR} \cdot \frac{\frac{1}{R}}{\frac{1}{R}} = \frac{V_0/R}{\frac{1}{RC} + s}$$

Using Table 12.1: $i(t) = \mathcal{L}^{-1}\{I\}$

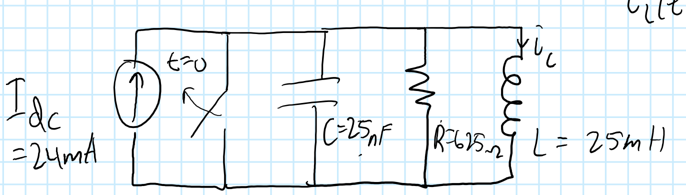
$$i = \frac{V_0}{R} e^{-t/RC}$$

$$V = Ri = V_0 e^{-t/RC}$$

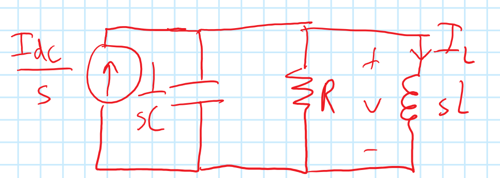
Step Response of a parallel RLC circuit

$i_L(t) = ?$

Solu: frequency domain



Time domain



KCL: $\frac{I_{dc}}{s} - VsC - \frac{V}{R} - \frac{V}{sL} = 0$

$$\frac{I_{dc}}{s} = sV_C + \frac{V}{R} + \frac{V}{sL}$$

$$V\left(s + \frac{1}{R} + \frac{1}{}\right) = I_{dc}$$

$$V\left(sC + \frac{1}{R} + \frac{1}{sL}\right) = \frac{I_{dc}}{s}$$

$$V = \frac{I_{dc}/s}{sC + \frac{1}{R} + \frac{1}{sL}} \cdot \frac{s}{s} = \frac{I_{dc}}{s^2 C + \frac{s}{R} + \frac{1}{L}} \cdot \frac{\frac{1}{C}}{\frac{1}{C}} = \frac{\frac{I_{dc}}{C}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$I_L = \frac{V}{sL} = \frac{\frac{I_{dc}}{C}}{sL\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)} = \frac{I_{dc}/LC}{s\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)} = \frac{384 \times 10^5}{s\left(s^2 + 64{,}000s + 16 \times 10^8\right)}$$

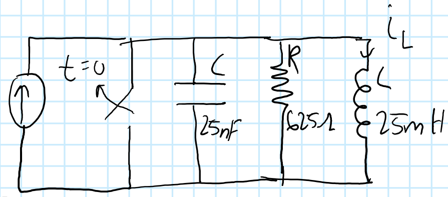
$$I_L = \frac{384 \times 10^5}{s(s + 32000 - j24000)(s + 320000 + j24000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s + 32000 - j24000)} + \frac{K_2^*}{(s + 32000 + j24000)}$$

Examples of S-domain Circuits

Tuesday, May 2, 2017 8:03 AM

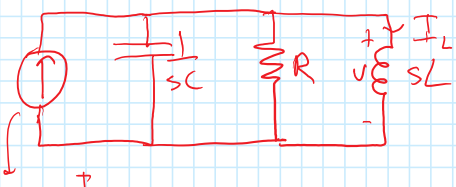
Example: 8.7 in book



$I_g = I_m \cos \omega t$

Sdu in s domain : $I_m = 24\,mA$

$\omega = 40{,}000$ rad/sec



$I_g = \frac{s I_m}{s^2+\omega^2}$

$$V = \frac{(I_g/C)s}{s^2 + (1/RC)s + \frac{1}{LC}} = \frac{\left(\frac{sI_m}{s^2+\omega^2}\right)\cdot \frac{s}{C}}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}} = \frac{(I_m/C)s^2}{(s^2+\omega^2)(s^2+\frac{1}{RC}s+\frac{1}{LC})}$$

$$I_L = \frac{V}{sL} = \frac{(I_m/C)s^2}{(s^2+\omega^2)(s^2+\frac{1}{RC}s+\frac{1}{LC})sL} = \frac{(I_m/LC)s}{(s^2+\omega^2)(s^2+\frac{1}{RC}s+\frac{1}{LC})}$$

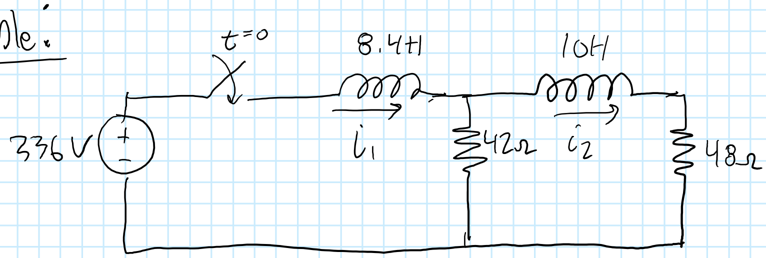
$$= \frac{(384\times10^5)s}{(s^2+16\times10^8)(s^2+64000s+16\times10^8)}$$

$$= \frac{K_1}{s+j40000} + \frac{K_1^*}{s-j40000} + \frac{K_2}{s+32000-j24000} + \frac{K_2^*}{s+32000+j24000}$$

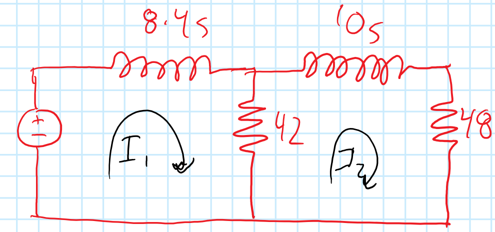
$$= \frac{7.5\times10^3\angle-90^\circ}{s-j40000} + \frac{7.5\times10^3\angle 90^\circ}{s+j40000} + \frac{12.5\times10^{-3}\angle 90^\circ}{s+32000-j24000} + \frac{12.5\times10^{-3}\angle-90^\circ}{s+32000+j24000}$$

$$i_L(t) = \mathcal{L}^{-1}\{I_L\} = \left[15\cos(40000t-90^\circ) + 25e^{-32000t}\cos(24000t+90^\circ)\right] mA$$

Example:



Sdu:



S domain

Mesh 1: $\frac{336}{s} - 8.4sI_1 - 42(I_1 - I_2) = 0$

$\frac{336}{s} = (42 + 8.4s)I_1 - 42I_2$ (1)

Mesh 2: $-42(I_2 - I_1) - 10s(I_2) - 48(I_2) = 0$

Mesh 2: $-42(I_2 - I_1) - 10s(I_2) - 48(I_2) = 0$

$0 = -42I_1 + (90+10s)I_2 \qquad (2)$

$(42+8.4s)I_1 - 42I_2 = \frac{336}{s}$

$-42I_1 + (90+10s)I_2 = 0$

$I_1 = \dfrac{\begin{vmatrix} 336/s & -42 \\ 0 & 90+10s \end{vmatrix}}{\begin{vmatrix} 42+8.4s & -42 \\ -42 & 90+10s \end{vmatrix}}$

$= \dfrac{\frac{336(90+10s)}{s}}{84(s^2+14s+24)} = \dfrac{336(90+10s)}{84s(s^2+14s+24)}$

$= \dfrac{3360(s+9)}{84s(s^2+14s+24)}$

$= \dfrac{40(s+9)}{s(s^2+14s+24)} = \dfrac{40(s+9)}{s(s+2)(s+12)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+2} + \dfrac{K_3}{s+12} = \dfrac{15}{s} - \dfrac{14}{s+2} - \dfrac{1}{s+12}$

Kramer's Rule

$aI_1 + bI_2 = e$

$cI_1 + dI_2 = 0$

$I_1 = \dfrac{\begin{vmatrix} e & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$I_2 = \dfrac{\begin{vmatrix} a & e \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$I_2 = \dfrac{\begin{vmatrix} 42+8.4s & 336/s \\ -42 & 0 \end{vmatrix}}{\begin{vmatrix} 42+8.4s & -42 \\ -42 & 90+10s \end{vmatrix}}$

$= \dfrac{168}{s(s+2)(s+12)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+2} + \dfrac{K_3}{s+12}$

$= \dfrac{7}{s} - \dfrac{8.4}{s+2} + \dfrac{1.4}{s+12}$

$i_1(t) = \mathcal{L}^{-1}\{I_1\}$

$= \boxed{(15 - 14e^{-2t} - e^{-12t})u(t) \text{ Amps}}$

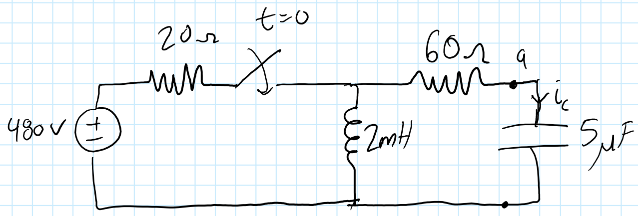
$i_2(t) = \mathcal{L}^{-1}\{I_2\}$

$= \boxed{(7 - 8.4e^{-2t} + 1.4e^{-12t})u(t) \text{ Amps}}$

Transfer Function

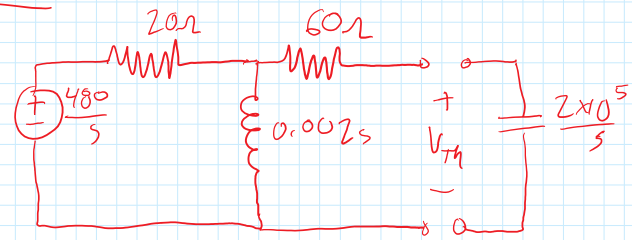
Wednesday, May 3, 2017 8:04 AM

Example:



Use Thevenin equivalent to find V_{Th} and Z_{th} at a and b, in s domain.

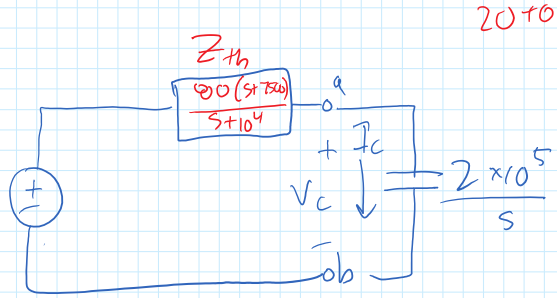
Sol'n:



$$V_{TH} = \frac{480}{s}\left(\frac{0.002s}{20+0.002s}\right) = \frac{480}{s+10^4}$$

$$Z_{th} = 60 + 20 // 0.002s = 60 + \frac{20(0.002s)}{20+0.002s} = \frac{80(s+7500)}{s+10^4}$$

$$V_{th} = \frac{480}{s+10^4}$$



$$I_c = \frac{V_{TH}}{Z_{th} + \frac{2\times10^5}{s}}$$

$$V_c = I_c\left(\frac{2\times10^5}{s}\right)$$

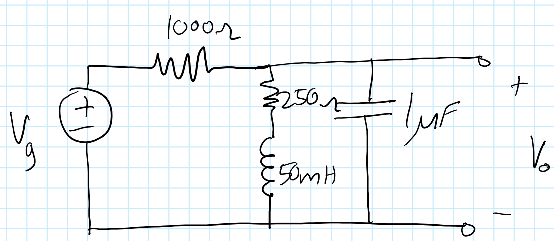
The transfer function

$$H(s) = \frac{Y(s)}{X(s)}$$

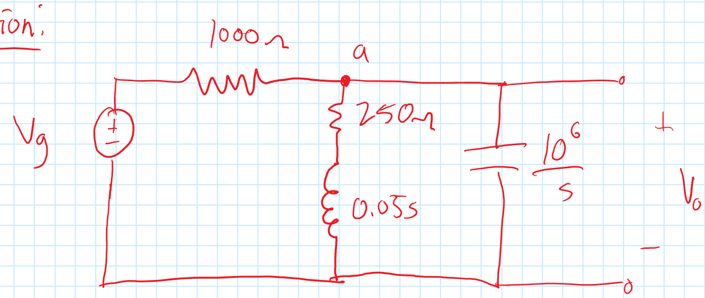
$Y(s)$ ← output

$X(s)$ ← input

Example 13.1



Solution:



$$H(s) = \frac{V_o(s)}{V_g(s)}$$

KCL at a: $I_{leaving}$ = "+" ; $I_{entering}$ = "−"

$$\frac{V_o - V_g}{1000\Omega} + \frac{V_o}{250\Omega + 0.05s} + \frac{V_o}{\frac{10^6}{s}} = 0$$

$$\frac{V_o}{1000} + \frac{V_o}{250+0.05s} + \frac{V_o s}{10^6} = \frac{V_g}{1000}$$

$$V_o\left(\frac{1}{1000} + \frac{}{250+0.05s} + \frac{s}{10^6}\right) = \frac{V_g}{1000}$$

$$\frac{V_o}{V_g} = \frac{\frac{1}{1000}}{\left(\frac{1}{1000} + \frac{1}{250+0.05s} + \frac{s}{10^6}\right)} = \frac{1000(s)\ \ldots}{s^2+6000s\ \ldots}$$

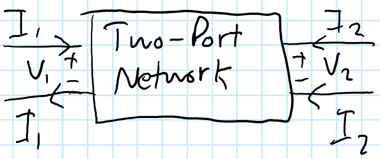
Hw #8 Chapter 18: 2, 4, 6, 7, 9, 10, 12, 13, 18, 19, 21, 29, 31, 33, 36, 37

Due Thursday, 5/11

Two-Port Circuits

Thursday, May 4, 2017 8:03 AM

Four Common Two-Port Network Parameters



1. Admittance Parameters

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

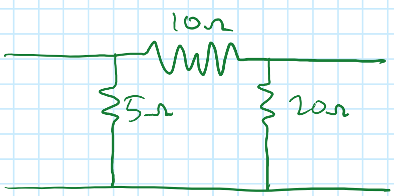
$y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2 = 0}$ short circuit input admittance

$y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1 = 0}$ short circuit output admittance

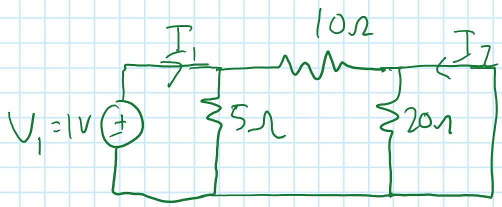
$y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1 = 0}$ short circuit transfer admittance

$y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2 = 0}$ short circuit transfer admittance

Example 1: Find the y-parameter for the network shown below



short output to find y_{11} + y_{21}



$V_2 = 0$

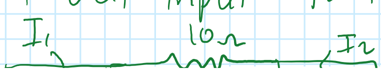
$$I_1 = \frac{V_1}{5\Omega // 10\Omega} = 0.3 V_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{0.3 V_1}{V_1} = \boxed{0.3 \mho}$$

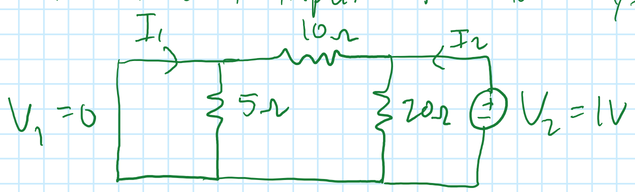
$$I_2 = -I_1 \left(\frac{5}{15}\right) = -0.1 V_1 \text{ Amps}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-0.1 V_1}{V_1} = \boxed{-0.1 \mho}$$

Short out input to find y_{22} + y_{12}



Short out input to find y_{22} + y_{12}



$y_{22} = \frac{I_2}{V_2}$ $\qquad I_2 = \frac{V_2}{20\Omega // 10\Omega} = \frac{30V_2}{200} = 0.15V_2$

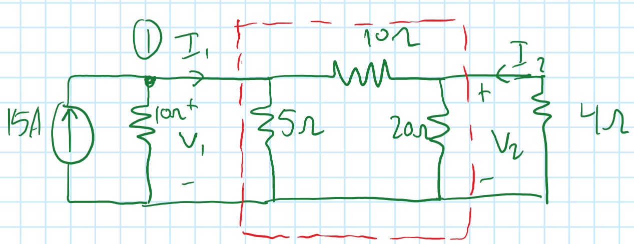
$y_{22} = \frac{0.15V_2}{V_2} = \boxed{0.15 \mho}$ or 0.15 S

$I_1 = -I_2\left(\frac{20}{30}\right) = -0.15V_2\left(\frac{20}{30}\right) = -0.1V_2$

$y_{12} = \frac{I_1}{V_2} = \frac{-0.1V_2}{V_2} = \boxed{-0.1 \mho}$

For first (total network); $\begin{cases} I_1 = 0.3V_1 - 0.1V_2 \\ I_2 = -0.1V_1 + 0.15V_2 \end{cases}$

Example 2: Use the result of example 1 to find I_1 and I_2 in the circuit shown below



KCL at (1)

$15 - I_1 - \frac{V_1}{10\Omega} = 0$

$15 - I_1 - 0.1V_1 = 0$

$I_1 = 15 - 0.1V_1$

ohm's law at the output

$I_2 = \frac{-V_2}{4\Omega} = -0.25V_2$

$I_1 = 15 - 0.1V_1 = \overbrace{0.3V_1 - 0.1V_2}^{\text{from ex. 1}}$

$I_1 = -0.25V_2 = \underbrace{-0.1V_1 + 0.15V_2}_{\text{from ex. 1}}$

$15 = 0.4V_1 - 0.1V_2$

$0 = -0.1V_1 + 0.4V_2$

$V_1 = 40V$

$V_2 = 10V$

$I_1 = 15 - 0.1V_1 = 15 - 0.1(40) = 11A$

$I_2 = -0.25V_2 = -0.25(10) = -2.5A$

Impedance Parameters

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ open circuit input impedance

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ open circuit output impedance

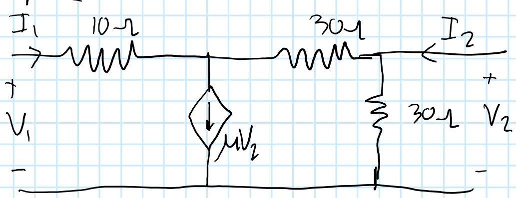
$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ open circuit transfer impedance

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ open circuit transfer impedance

Two Port Examples

Monday, May 8, 2017 8:08 AM

Example:

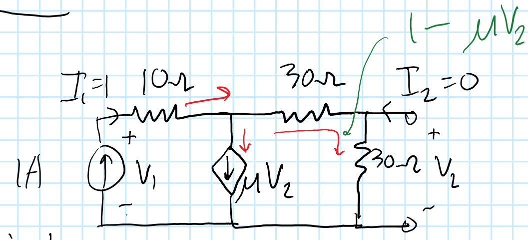


$\mu = \frac{1}{60}$ Find z parameters

Soln:

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$



Ohm's Law

$V_2 = 30\Omega(1-\mu V_2)$

$V_2 = 30 - 30\mu V_2$

$V_2 + 30\mu V_2 = 30$

$V_2(1+30\mu) = 30$

$V_2 = \frac{30}{1+30\mu} = \frac{30}{1+\frac{30}{1}\cdot\frac{1}{60}} = \frac{30}{1.5} = 20$

$Z_{21} = \frac{V_2}{I_1} = \frac{20V}{1A} = \boxed{20\Omega}$

Supermesh:

V_{rise} = "+" ; V_{drop} = "−"

$V_1 - 10\Omega(1A) - (30\Omega + 30\Omega)(1-\mu V_2) = 0$

$V_1 - 10 - 60(1-\mu V_2) = 0$

$V_1 = 10 + 60(1-\mu V_2)$

$= 10 + 60\left(1 - \frac{1}{60}\cdot 20\right)$

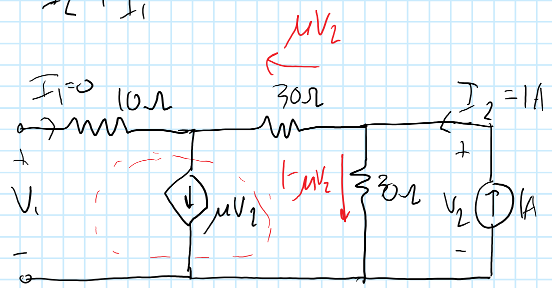
$= 10 + 60\left(2/3\right) = 10 + 40 = 50 = V_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{50V}{1A} = \boxed{50\Omega}$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$



$$V_2 = 30(1-\mu V_2)$$
$$V_2 = 30 - 30\mu V_2$$
$$V_2 + 30\mu V_2 = 30$$
$$V_2(1+30\mu) = 30$$
$$V_2 = \frac{30}{1+30\mu} = \frac{30}{1+\frac{30}{60}} = \frac{30}{1.5} = 20\,V$$
$$V_2 = 20\ V$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{20\,V}{1A} = \boxed{20\Omega}$$

Supermesh:

$$V_1 - \cancel{I_1(10\Omega)} + 30\Omega(\mu V_2) - 30\Omega(1-\mu V_2) = 0$$
$$\emptyset$$

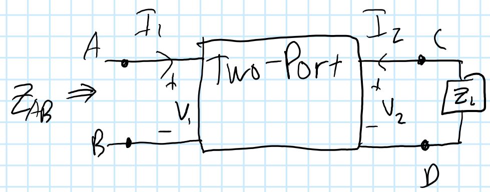
$$V_1 + 30\mu V_2 - 30 + 30\mu V_2 = 0$$
$$V_1 = 30 - 60\mu V_2 = 30 - \frac{60}{60}V_2 = 30 - 20 = 10V$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{10V}{1A} = \boxed{10\Omega}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$\Longrightarrow \boxed{\begin{aligned} V_1 &= 50I_1 + 10I_2 \\ V_2 &= 20I_1 + 20I_2 \end{aligned}}$$

Loading Effect on Two-Port Network

a.) Input Impedance Z_{AB} for the following network



$$Z_L = \frac{V_2}{\ }\quad \therefore\ V_2 = -Z_L I_2$$

$-I_2$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

also

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$-Z_L I_2 = Z_{21} I_1 + Z_{22} I_2$$

$$-Z_L I_2 - Z_{22} I_2 = Z_{21} I_1$$

$$-(Z_L + Z_{22}) I_2 = Z_{21} I_1$$

$$\frac{I_2}{I_1} = \frac{-Z_{21}}{Z_L + Z_{22}}$$

$$Z_{AB} = \frac{V_1}{I_1}$$

$$Z_{AB} = \frac{Z_{11} I_1 + Z_{12} I_2}{I_1}$$

$$= Z_{11} + Z_{12} \cdot \frac{I_2}{I_1}$$

$$= Z_{11} + Z_{12} \cdot \left(\frac{-Z_{21}}{Z_L + Z_{22}} \right)$$

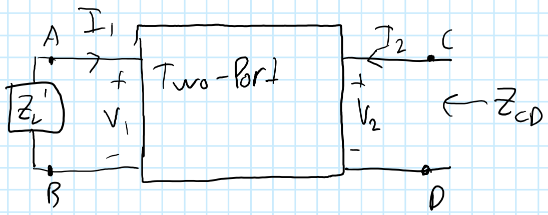
$$= Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}$$

$$\boxed{Z_{AB} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}}$$

Output Impedance

Tuesday, May 9, 2017 8:01 AM

Z_{CD}



$$Z_L' = \frac{V_1}{-I_1} = \frac{-V_1}{I_1} \Rightarrow V_1 = -Z_L' I_1$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$-Z_L' I_1 = Z_{11}I_1 + Z_{12}I_2$$

$$-Z_L' I_1 - Z_{11}I_1 = Z_{12}I_2$$

$$-(Z_L' + Z_{11})I_1 = Z_{12}I_2$$

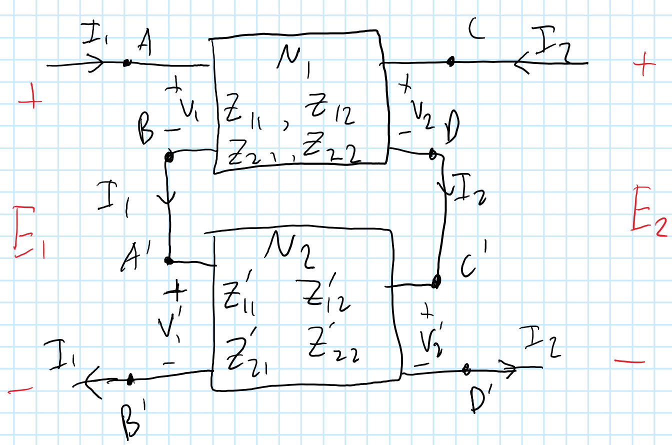
$$\frac{I_1}{I_2} = \frac{-Z_{12}}{Z_L' + Z_{11}}$$

$$Z_{CD} = \frac{V_2}{I_2} \qquad V_2 = Z_{22}I_2 + Z_{21}I_1$$

$$Z_{CD} = \frac{Z_{22}I_2 + Z_{21}I_1}{I_2} = Z_{22} + Z_{21}\frac{I_1}{I_2}$$

$$\boxed{Z_{CD} = Z_{22} - \frac{Z_{21}Z_{12}}{Z_{11} + Z_L'}}$$

Series Connections of Two-Port Network



Using KVL:

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} V_1' \\ V_2' \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11}+Z'_{11}, & Z_{12}+Z'_{12} \\ Z_{21}+Z'_{21}, & Z_{22}+Z'_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Relation Between Y and Z parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}}{y_{11}y_{22} - y_{12}y_{21}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \frac{y_{22}}{y_{11}y_{22} - y_{12}y_{21}}$$

$$z_{12} = \frac{-y_{12}}{y_{11}y_{22} - y_{12}y_{21}}$$

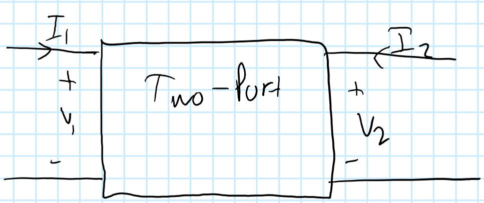
$$z_{21} = \frac{-y_{21}}{y_{11}y_{22} - y_{12}y_{21}}$$

$$z_{22} = \frac{y_{11}}{y_{11}y_{22} - y_{12}y_{21}}$$

a parameters

$$V_1 = a_{11} V_2 - a_{12} I_2$$
$$I_1 = a_{21} V_2 - a_{22} I_2$$

$$\Longrightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



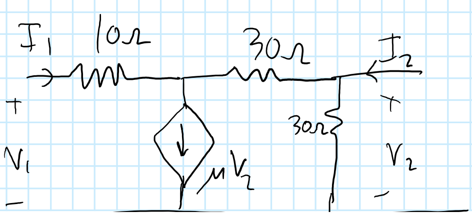
$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$a_{12} = \left.-\frac{V_1}{I_2}\right|_{V_2=0}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_1=0}$$

$$a_{22} = \left.\frac{-I_1}{V_1}\right|_{V_2=0}$$

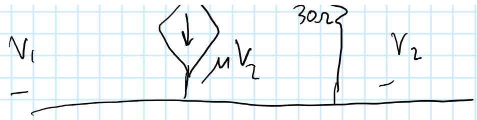
Example: Determine a parameters for the following circuit



Solution:

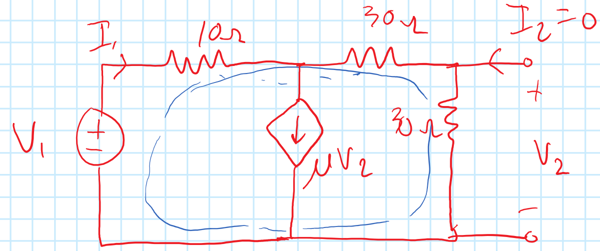
$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$a_{21} = \frac{I_1}{}$$



$$\mu = \frac{1}{60}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$



Supermesh:

$$V_1 - 10\Omega\,(I_1) - 30\Omega\,(I_1 - \mu V_2) - 30\Omega\,(I_1 - \mu V_2) = 0$$

$$V_1 - 10 I_1 - 60(I_1 - \mu V_2) = 0 \qquad (1)$$

$$V_2 = 30(I_1 - \mu V_2)$$

$$V_1 = 30 I_1 - 30\mu V_2$$

$$V_2 + 30\mu V_2 = 30 I_1$$

$$I_1 = \frac{(1+30\mu)V_2}{30} \qquad (2)$$

$$a_{21} = \frac{I_1}{V_2} = \frac{\frac{1+30\mu\cancel{V_2}}{30}}{\cancel{V_2}} = \frac{1+30\cdot\frac{1}{60}}{30} = \frac{1.5}{30} = \boxed{0.05\text{ Amps}}$$

substitute (2) into (1)

$$V_1 - 70\,\frac{(1+30\mu)V_2}{30} + 60\mu V_2 = 0$$

$$V_1 - \frac{70(1+30\mu) - 60\mu + 30}{30} V_2 = 0$$

$$V_1 - \frac{7\left(1+30\cdot\frac{1}{60}\right) - 180\cdot\frac{1}{60}}{3} V_2 = 0$$

$$V_1 \quad \frac{-7 + \frac{210}{60} - 3}{3} V_2 = 0$$

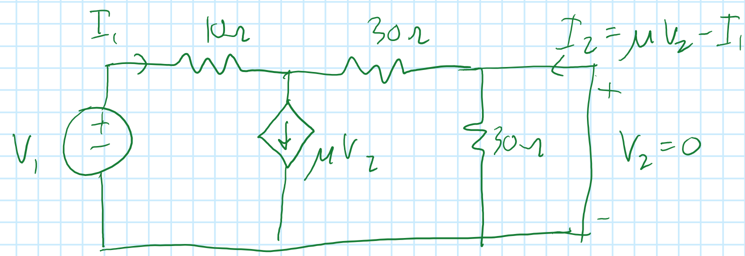
$$V_1 - 2.5 V_2 = 0 \qquad V_1 = 2.5 V_2 \qquad \frac{V_1}{} = 2.5$$

$V_1 - 2.5V_2 = 0$ $\quad V_1 = 2.5V_2$ $\quad \frac{V_1}{V_2} = 2.5$

$a_{11} = 2.5$

$a_{12} = \frac{V_1}{I_2}\Big|_{V_2=0} = \boxed{-40}$

$a_{22} = \frac{-I_1}{I_2}\Big|_{V_2=0} = \frac{-I_1}{\mu V_2 - I_1} = \frac{-I_1}{-I_1} = \boxed{1}$



Supermesh

$V_1 - 10i_1 + 30i_2 = 0$

$\therefore\ i_1 = -i_2$

$V_1 + 10I_2 + 30I_2 = 0$

$V_1 + 10I_2 + 30I_2 = 0$

$V_1 + 40I_2 = 0$

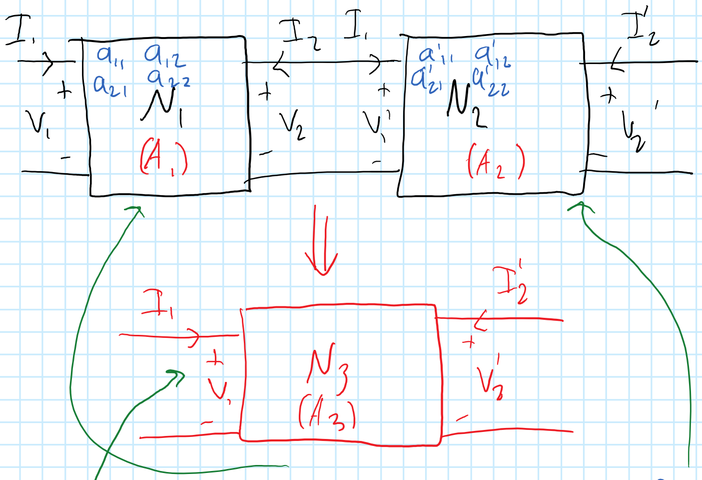
$V_1 = -40I_2$

$\frac{V_1}{I_2} = -40$

Cascade Connections

Wednesday, May 10, 2017 8:04 AM

Cascade Connections



Let $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = A_1 \begin{bmatrix} V_1 \\ -I_2 \end{bmatrix}$, $\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = A_2 \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = A_3 \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$

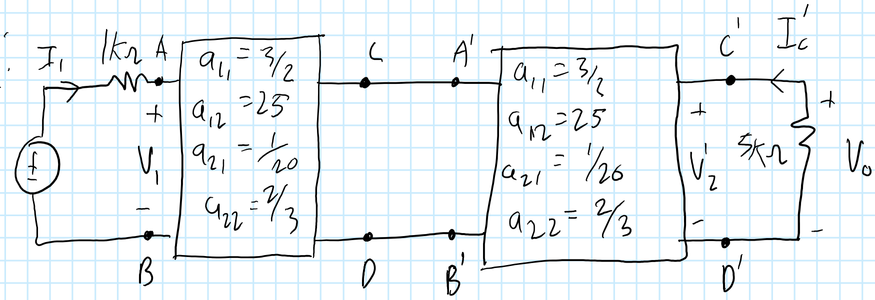
$\because$ $V_2 = V_1'$

$I_2 = -I_1'$

$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = A_1 \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = A_1 \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = A_1 \cdot A_2 \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = A_3 \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$ where $A_3 = A_1 \cdot A_2$

$$A_3 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a_{11}' & a_{12}' \\ a_{21}' & a_{22}' \end{bmatrix}$$

Example:



A Two-stage amplifier is shown above in terms of "a" parameters

Find the voltage gain of the amplifier

Solu: $A_3 = A_1 \cdot A_2 = \begin{bmatrix} 3/2 & 25 \\ 1/20 & 2/3 \end{bmatrix} \cdot \begin{bmatrix} 3/2 & 25 \\ 1/20 & 2/3 \end{bmatrix} = \begin{bmatrix} 3/2 \cdot 3/2 + 25 \cdot \frac{1}{20} & 3/2 \cdot 25 + 25 \cdot 2/3 \\ 1/20 \cdot 3/2 + 2/3 \cdot \frac{1}{20} & 1/20 \cdot 25 + 2/3 \cdot 2/3 \end{bmatrix}$

Solu: $A_3 = A_1 \cdot A_2 = \begin{bmatrix} 3/2 & 25 \\ 1/20 & 2/3 \end{bmatrix} \cdot \begin{bmatrix} 3/2 & 25 \\ 1/20 & 2/3 \end{bmatrix} = \begin{bmatrix} 3/2 \cdot 3/2 + 25 \cdot \frac{1}{20} & 3/2 \cdot 25 + 25 \cdot 2/3 \\ 1/20 \cdot 3/2 + 2/3 \cdot \frac{1}{20} & 1/20 \cdot 25 + 2/3 \cdot 2/3 \end{bmatrix}$

$$= \begin{bmatrix} 9/4 + 25/20 & 75/2 + 50/3 \\ 3/40 + 2/60 & 25/20 + 4/9 \end{bmatrix}$$

$$= \begin{bmatrix} 3.5 & 54.1667 \\ 0.10833 & 1.6944 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 3.5 & 54.1667 \\ 0.10833 & 1.6944 \end{bmatrix} \begin{bmatrix} V_0 \\ -I_2' \end{bmatrix}$$

gain: $\frac{V_0}{V_S} = ?$

KVL: $V_S - 1000 I_1 - V_1 = 0 \implies V_S = 1000 I_1 + V_1$

$V_0 = -5000 I_2' \implies$

$$\frac{V_0}{V_S} = \frac{-5000 I_2'}{1000 I_1 + V_1} = \frac{-5000 I_2'}{1000[0.10833 \cdot V_0 - 1.6944 I_2'] + [3.5 \cdot V_0 - 54.1667 I_2']}$$

$$\frac{V_0}{V_S} = \frac{-5000 I_2'}{108.33 V_0 - 1694.4 I_2' + 3.5 V_0 - 54.1667 I_2'}$$

$$= \frac{-5000 I_2'}{111.83 V_0 - 1748.57 I_2'}$$

$$= \frac{-5000 I_2'}{111.83(-5000 I_2') - 1748.57 I_2'}$$

$$\boxed{\frac{V_0}{V_S} = 0.00891 = \text{gain}}$$

Hybrid Parameters $\longrightarrow$ H parameters

$V_1 = h_{11} I_1 + h_{12} V_2$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$ short circuit input impedance

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$ short circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$ open circuit reverse voltage gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$ open circuit output admittance

$$\left.\begin{array}{l} h_{11} \approx h_{ie} \\ h_{12} \approx h_{re} \\ h_{21} \approx h_{fe} \\ h_{22} \approx h_{oe} \end{array}\right\} \text{for E315}$$