

Intro

Monday, August 22, 2016 9:00 AM

Norm Galassi

Office: SKSU 105 10-10:50 MW Mon also 11-11:50

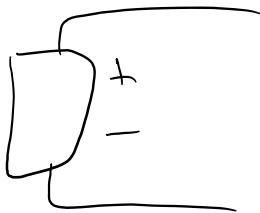
<http://nrgalassi.org/EECE211>
must be caps

use gmail

charge $q_e = 1.602 \times 10^{-19}$ Coul

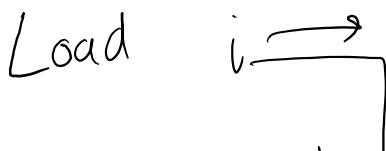
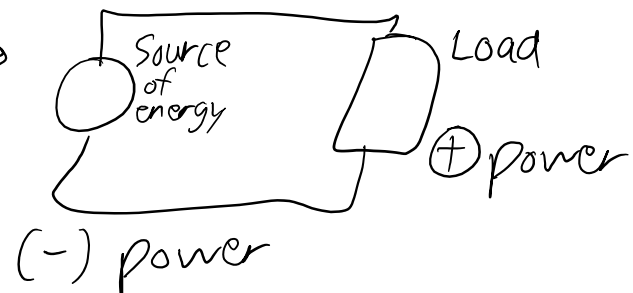
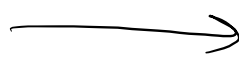
⊗ Current $= \frac{dq}{dt}$ $\frac{1 \text{ coul}}{1 \text{ sec}} = 1 \text{ amp}$ $\begin{matrix} \rightarrow + \\ \rightarrow - \end{matrix}$

⊗ Voltage $= \frac{dw}{dq}$

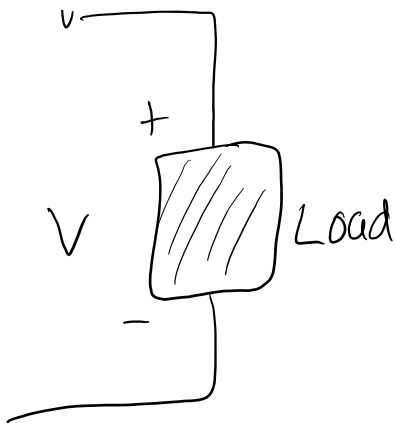


⊗ Power $= \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = V \cdot i$

Passive sign convention

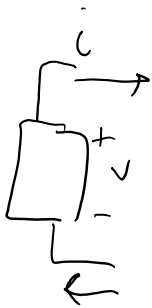


Load

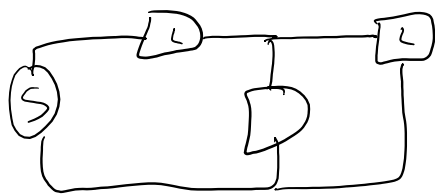


$$P = (+) V i$$

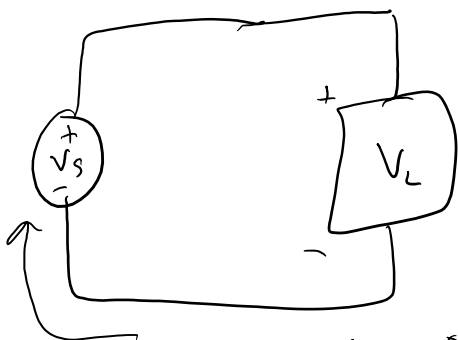
Source



$$\sum i = 0 \quad \text{in} = \text{out}$$



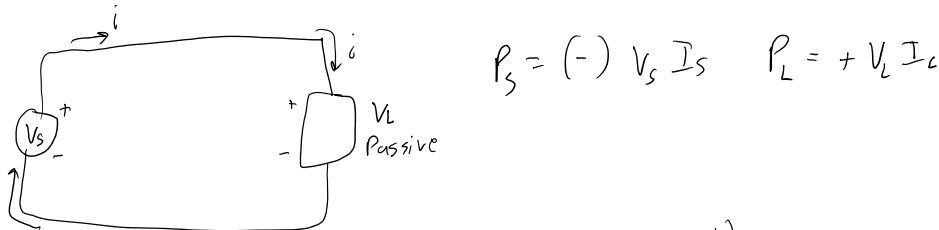
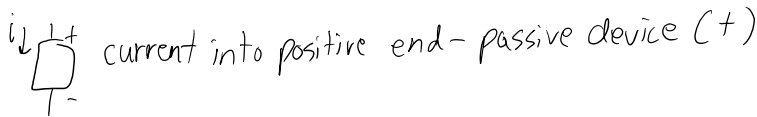
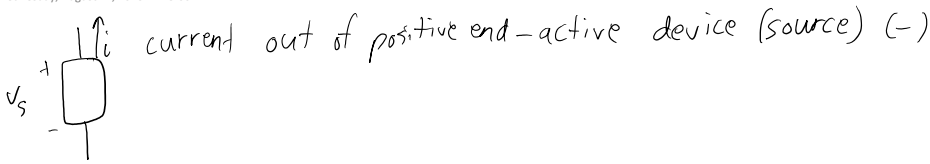
$$\text{source} = \sum \text{loads}$$



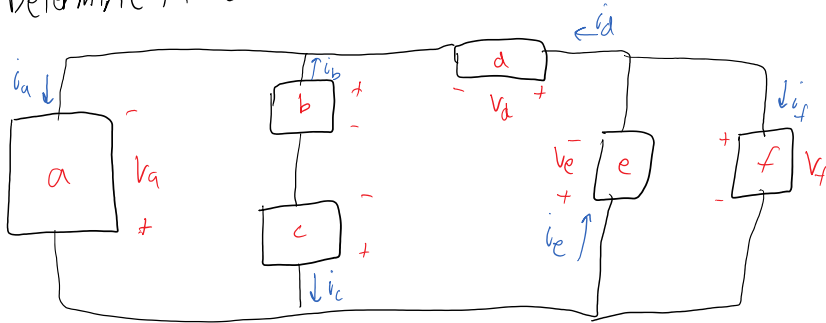
$$-V_s + V_L = 0 \quad \sum v = 0$$

Passive Sign Convention

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Ex: Determine if circuit is valid (Power in = Power out)

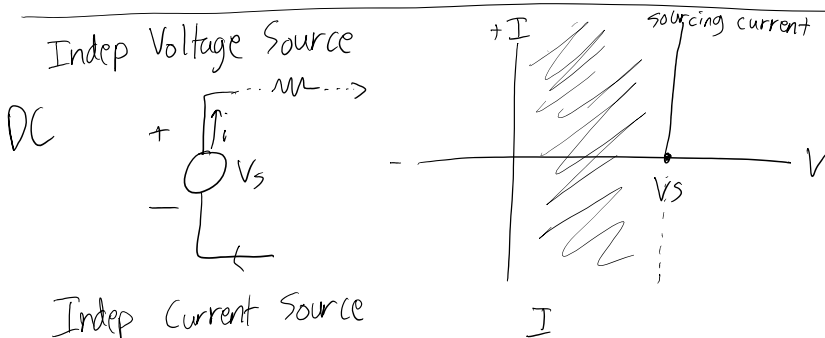


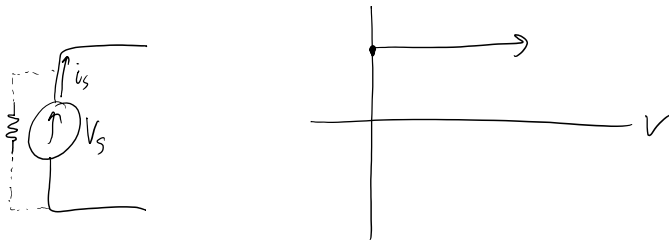
element	V	I	Type of Element	Calculation	what it really is → element
a	-3kV	-250μA	active	$(-)(-3 \times 10^3) \cdot (250 \times 10^{-6}) = -0.75W$	active
b	4kV	-400μA	active	$(-)(4 \times 10^3) \cdot (-400 \times 10^{-6}) = +1.6W$	passive
c	1kV	400μA	active	$(-)(1 \times 10^3) \cdot (400 \times 10^{-6}) = -0.4W$	active
d	1kV	150μA	passive	$(+)(1 \times 10^3) \cdot (150 \times 10^{-6}) = +0.15W$	passive
e	-4kV	200μA	passive	$(+)(-4 \times 10^3) \cdot (200 \times 10^{-6}) = -0.8W$	active
f	4kV	50μA	passive	$(+)(4 \times 10^3) \cdot (50 \times 10^{-6}) = +0.2W$	passive

$$-0.75 - 0.4 - 0.8 + 1.6 + 0.15 + 0.2 = 0$$

-1.95
 $+ 1.95$

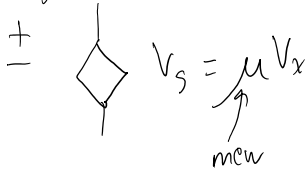
Valid circuit ✓
(If not = 0, invalid circuit)



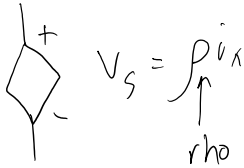


dependent source Probably not on exam

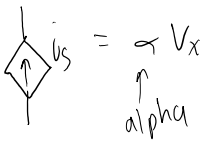
Voltage Controlled Voltage Source (VCVS)



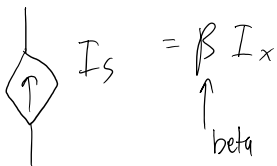
Current Controlled Voltage Source (CCVS)



Voltage Controlled Current Source (VCCS)



Current Controlled Current Source (CCCS)

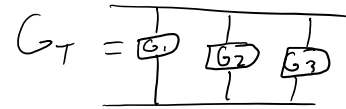
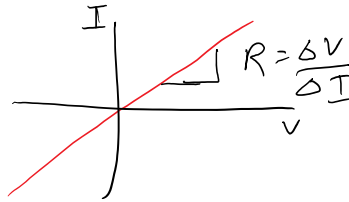


All of these are usually in semiconductors/solid state

Kirchoff's Laws

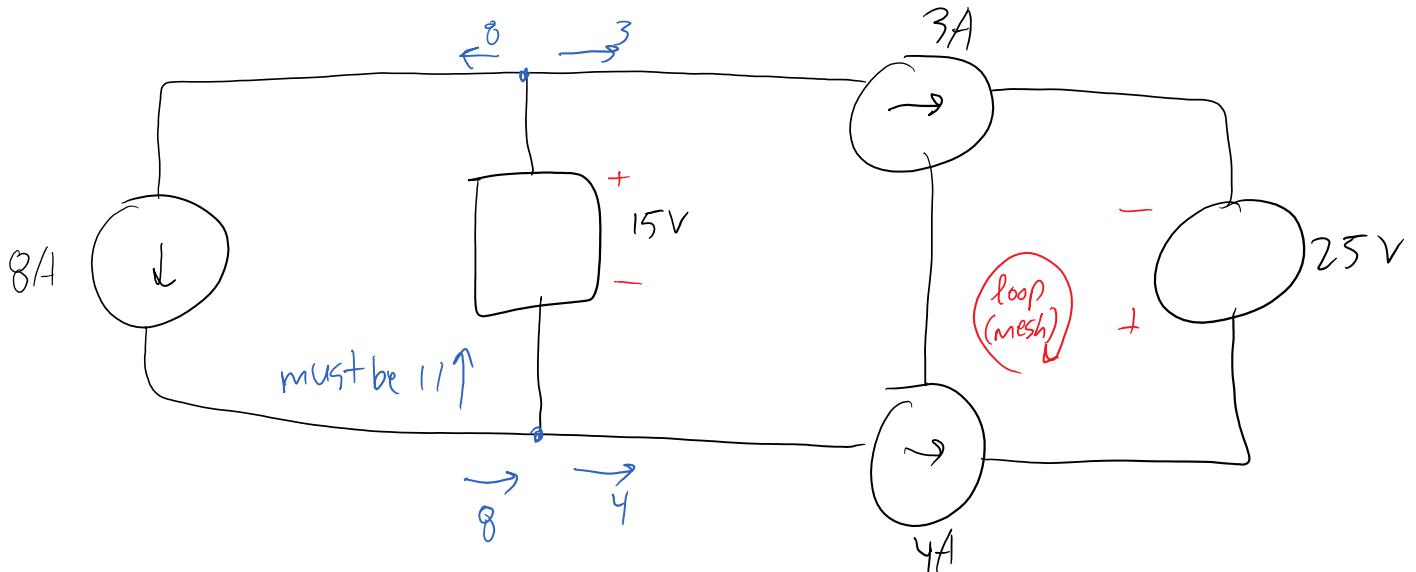
Friday, August 26, 2016 9:00 AM

$G = \text{conductance} = \frac{1}{R}$



$R_T = \frac{1}{G_T}$

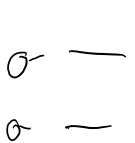
$P_R = (+) VI = I^2R = \frac{V^2}{R}$



15 & 3 invalid circuit

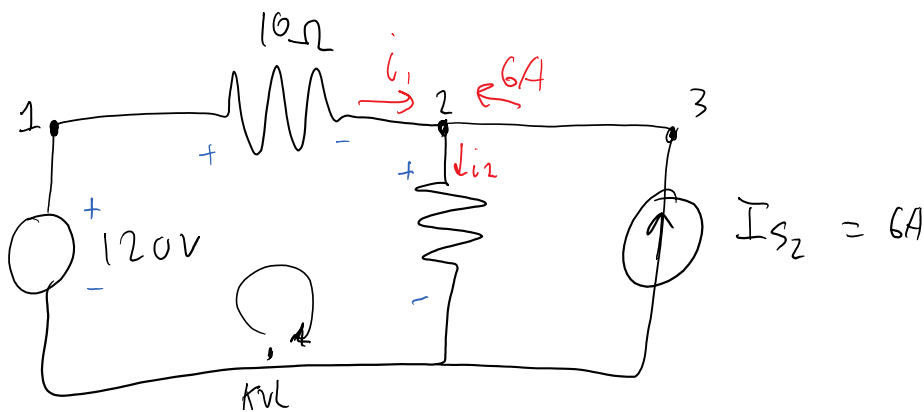


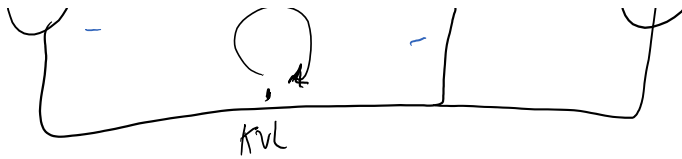
short circuit
 $R = 0$



open circuit
 $R = \infty$

Kirchoff's Voltage Law (loops)
 $\Sigma \text{Voltage around a loop} = 0$
Kirchoff's Current Law (nodes)
 $\Sigma \text{current entering node} = 0$





$$\text{Node 3} = 6A$$

$$\text{KCL} \rightarrow \text{Node 2}$$

$$I_1 - I_2 + I_3 = 0$$

$$I_1 - I_2 = -6$$

$$10I_1 + 50I_2 = 120$$

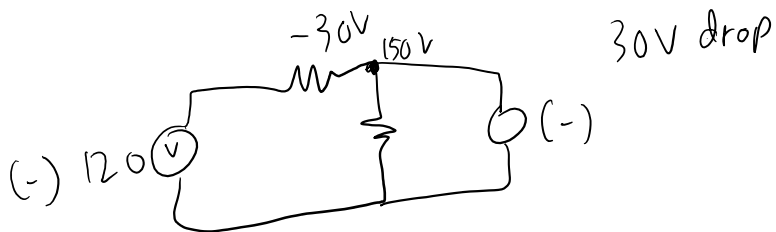
KVL - loops

$$-120V + i_1 10\Omega + i_2 50\Omega = 0$$

$$\left| \begin{array}{cc|c} 1 & -1 & I_1 \\ 10 & 50 & I_2 \end{array} \right| \begin{array}{c} -6 \\ 120 \end{array}$$

$$\begin{array}{l} I_1 = -3 \\ I_2 = 3 \end{array}$$

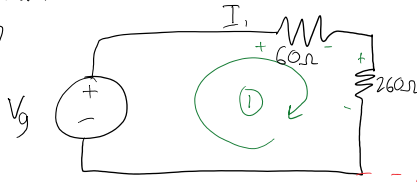
$$\text{So } i_1 = -3 \leftarrow i_1$$



$$\begin{aligned} &-(120V)(-3A) + (-3A)(-30V) + (3A)(150V) - 150(6) \\ &+ 360W + 90W + 450W - 900W = 0 \checkmark \end{aligned}$$

Dependent Source Examples

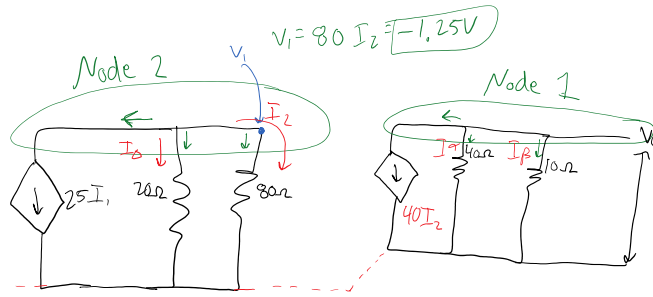
p2.37



Indep Source

Find V_1 and V_g when $V_0 = +5V$

$$\begin{aligned}
 -V_g + 60I_1 + 260I_1 &= 0 \\
 -V_g + 320I_1 &= 0 \\
 -V_g &= -320I_1 \\
 V_g &= 320I_1 = \boxed{1V}
 \end{aligned}$$



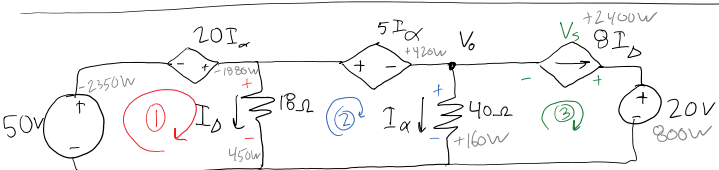
$$V_1 = 80 I_2 = -1.25V$$

$$\begin{aligned}
 I_\alpha &= \frac{5}{40} = 0.125 \\
 I_\beta &= \frac{5}{10} = 0.5
 \end{aligned}$$

Hint: Begin at right hand side (i.e. V_0)

$$\begin{aligned}
 \text{Node 2:} \\
 -25I_1 - I_D - I_2 &= 0 \\
 -25I_1 - I_D + 0.015625 &= 0 \\
 I_D &= \frac{-1.25}{20} = -0.0625 \\
 -25I_1 + 0.0625 + 0.015625 &= 0 \\
 -25I_1 + 0.078125 &= 0 \\
 -25I_1 &= -0.078125 \\
 I_1 &= \boxed{0.003125}
 \end{aligned}$$

$$\begin{aligned}
 \text{Node 1:} \\
 -40I_2 - I_\alpha - I_\beta &= 0 \\
 -40I_2 - 0.125 - 0.5 &= 0 \\
 -40I_2 &= +0.625A \\
 I_2 &= \boxed{-0.015625A}
 \end{aligned}$$



Calculate I_D and V_0

$$\begin{aligned}
 20V + V_0 &= V_0 \\
 V_0 &= 80V \\
 \text{or} \\
 V_0 &= 40I_\alpha = 80V
 \end{aligned}$$

Loop 1

$$\begin{aligned}
 -50V - 20I_\alpha + 18I_D &= 0 \\
 -20I_\alpha + 18I_D &= 50
 \end{aligned}$$

Loop 2

$$\begin{aligned}
 -18I_D + 5I_\alpha + 40I_\alpha &= 0 \\
 45I_\alpha - 18I_D &= 0
 \end{aligned}$$

Loop 3

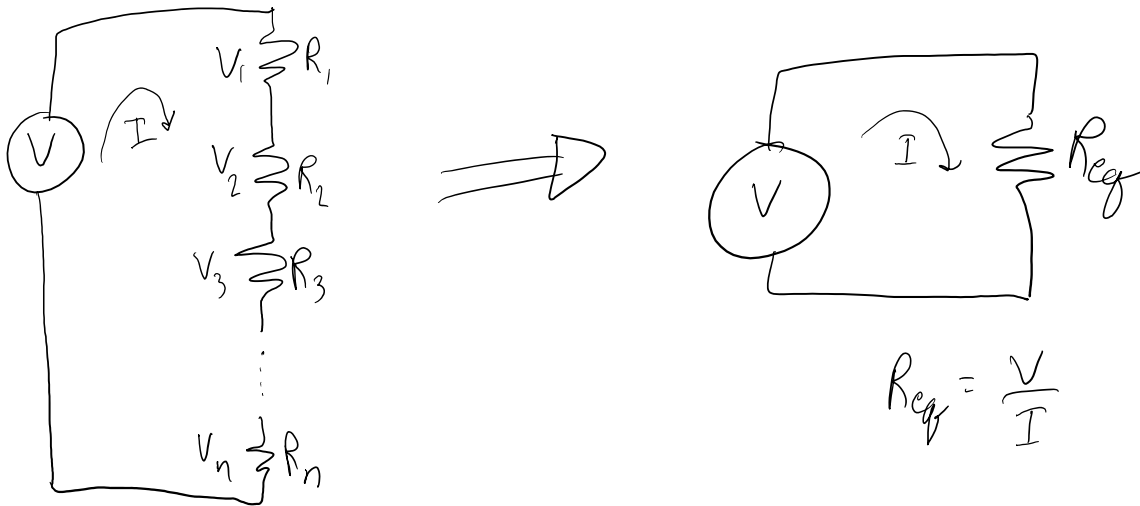
$$\begin{aligned}
 -40I_\alpha - V_0 + 20 &= 0 \\
 -40I_\alpha - V_0 &= -20
 \end{aligned}$$

I_α	I_D	V_0	I_α	V_0
-20	18	0	50	0
45	-18	0	0	0
-40	0	-1	-20	-20

$$\begin{aligned}
 I_\alpha &= 2 \\
 I_D &= 5 \\
 V_0 &= -60
 \end{aligned}$$

Series and Parallel Circuit

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$$R_{eq} = \frac{V}{I}$$

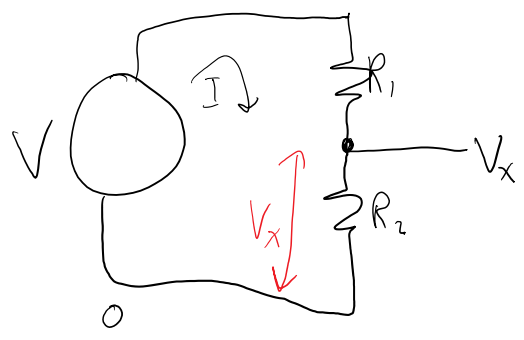
$$V = V_1 + V_2 + \dots + V_n$$

$$IR_1 + IR_2 + \dots + IR_n$$

$$V = I(R_1 + R_2 + \dots + R_n)$$

$$R_{eq} = \frac{V}{I} \quad \boxed{R_{eq} = R_1 + R_2 + \dots + R_n}$$

Voltage Divider



$$I = \frac{V}{R_{eq}}$$

$$\frac{V_x}{V} = \frac{IR_2}{I(R_1 + R_2)} = \boxed{V_x = \frac{VR_2}{R_T}}$$

Parallel Resistors

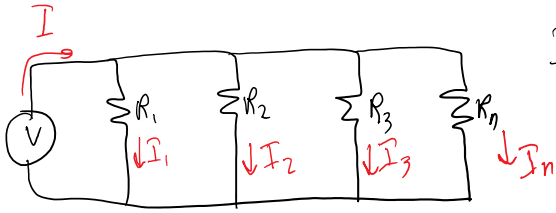
T

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

parallel resistors

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

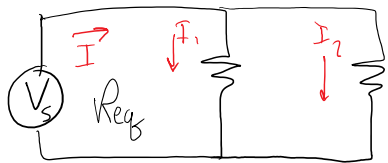
$$I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \dots + \frac{V}{R_n}$$



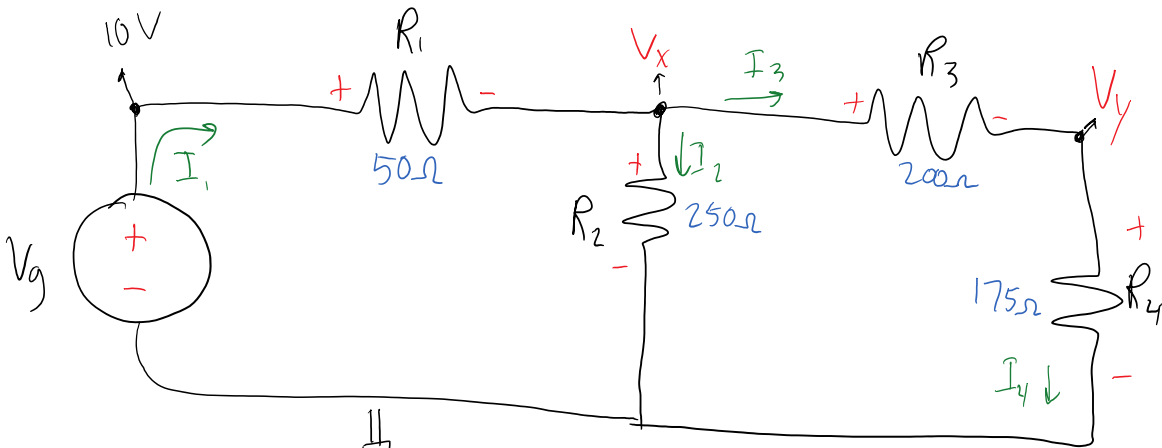
$$R_{eq} = \frac{V}{I} = \frac{V}{\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \dots + \frac{V}{R_n}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}}$$

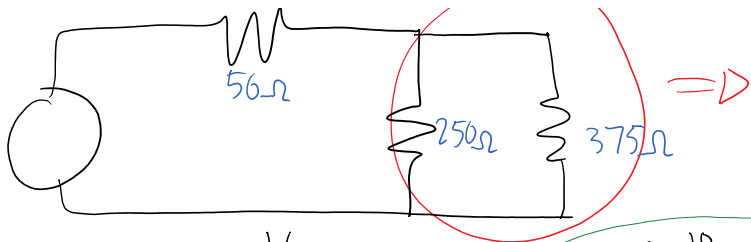
For 2 Resistors in Parallel (||): $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \frac{R_1 R_2}{R_1 + R_2}$

Resistance of each resistor \rightarrow
 $R_{eq} = \frac{R}{N} = \frac{1000}{20} = 50$
 # of resistors \rightarrow



$$\frac{I_1}{I_2} = \frac{V_s / R_1}{\frac{V_s (R_1 + R_2)}{R_1 R_2}} = \frac{1}{R_1} \times \frac{R_1 R_2}{R_1 + R_2} = \frac{I_1}{I} = \frac{R_2}{R_1 + R_2}$$

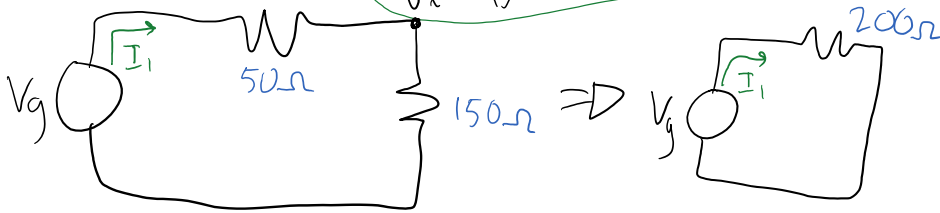




$$\frac{250 \times 375}{250 + 375} = 150 \Omega$$



$$V_x = \frac{150}{150 + 375} \cdot 10V = \frac{3}{4} \cdot 10 \quad V_x = 7.5V$$



$$I_1 = \frac{10}{200} = 0.05A$$

$$I_1 = 0.05A$$

$$I_2 = 0.03A$$

$$I_3 = 0.02A$$

$$I_4 = 0.02A$$

voltage divider $\rightarrow V_x = 7.5V$

$$V_y = 3.5V$$

$$I_2 = \frac{7.5V}{250\Omega} = 0.03A$$

$$I_1 = I_2 + I_3$$

$$0.05 = 0.03 + I_3 \quad I_3 = 0.02A$$

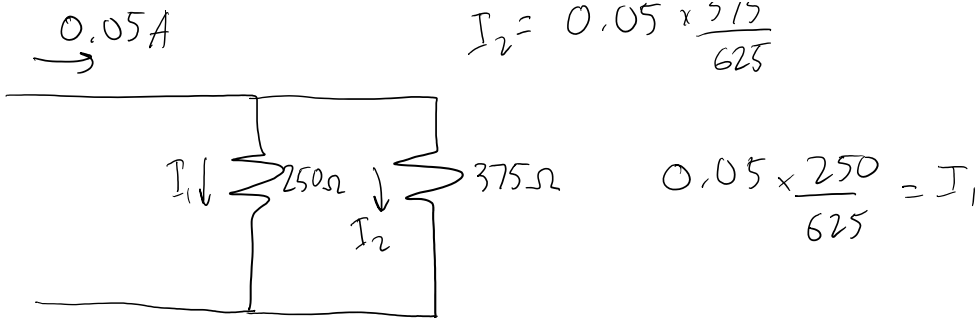
$$I_3 = I_4 \rightarrow \text{resistors } R_3 \text{ and } R_4 \text{ are in series}$$

$$V_y = \frac{R_4}{R_3 + R_4} (V_x)$$

$$V_y = \frac{175 \cdot (7.5V)}{200 + 175} = 3.5V$$

$$\rightarrow 0.05A$$

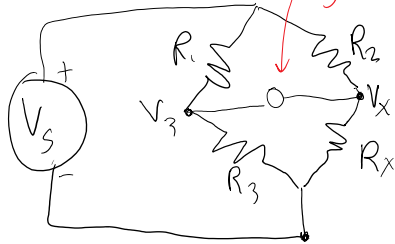
$$I_2 = 0.05 \times \frac{375}{625}$$



Delta and Wye Conversions

Friday, September 2, 2016 8:59 AM

Wheatstone Bridge *not on exam*



galvanometer

• R_3 is a variable resistor

• no current thru galvanometer when no voltage on galvanometer \Rightarrow balanced



$$V_3 = \frac{V_S (R_3)}{R_1 + R_3}$$

$$V_x = \frac{V_S (R_x)}{R_2 + R_x}$$

$$V_3 = V_x \quad \text{so} \quad \frac{V_S (R_3)}{R_1 + R_3} = \frac{V_S (R_x)}{R_2 + R_x}$$

$$R_3 R_2 + \cancel{R_3 R_x} = R_1 R_x + \cancel{R_3 R_x}$$

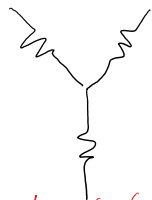
$$R_3 R_2 = R_1 R_x$$

$$R_x = \frac{R_3 R_2}{R_1}$$

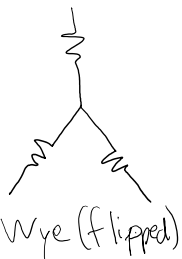
Delta to Wye & Wye to Delta Transformations - In HW, probably not on exam



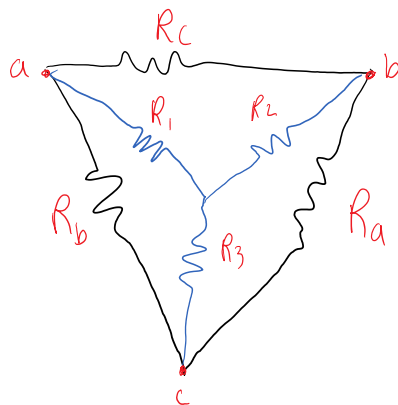
Delta Configuration



Wye Configuration



Wye (flipped)



Delta to Wye

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

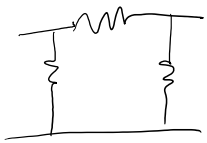
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

Wye to Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

Delta seen as Pi Network (usually)





Wye seen as T Network (usually)

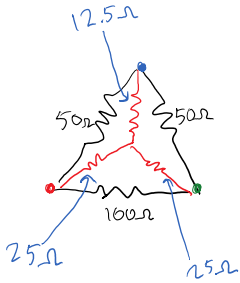
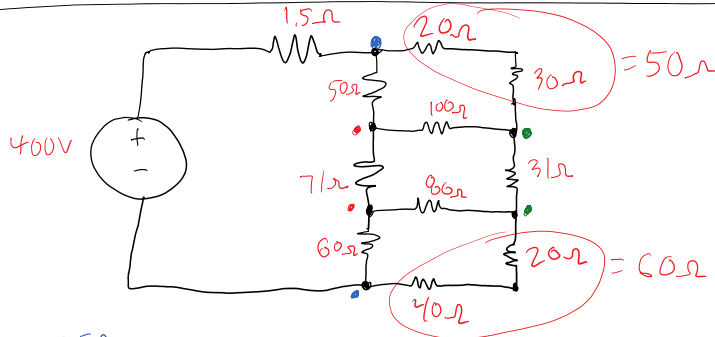


$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

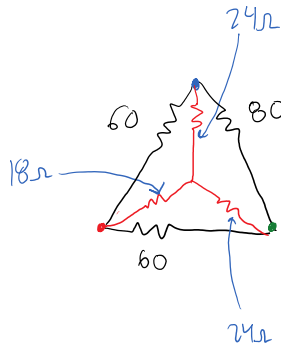
Ex:



$$\frac{50 \cdot 50}{200} = 12.5\Omega$$

$$\frac{50 \cdot 100}{200} = 25\Omega$$

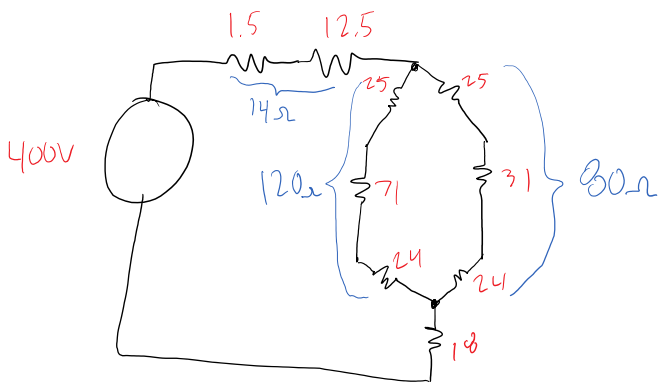
$$\frac{50 \cdot 100}{200} = 25\Omega$$



$$\frac{60 \cdot 80}{200} = 24$$

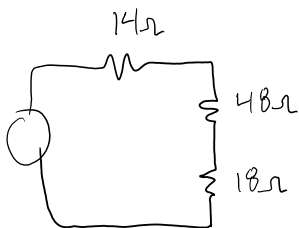
$$\frac{60 \cdot 80}{200} = 24$$

$$\frac{60 \cdot 60}{200} = 18$$

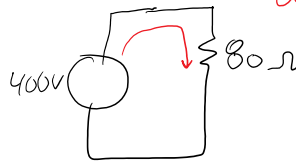


$$\frac{120 \cdot 80}{200} = 48\Omega$$

Find Power in 31Ω Resistor, Find Load

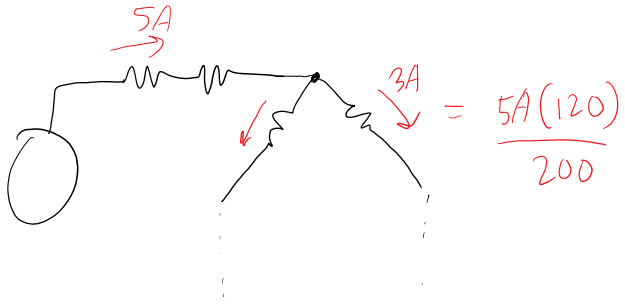


$$I = \frac{400V}{80\Omega} = 5A$$



$$\text{Load} = 80\Omega$$

(Load = 80Ω)



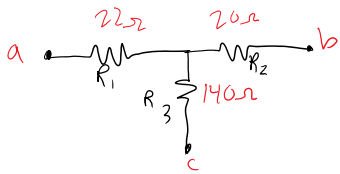
$$I_p = \frac{R_{Not Path}}{R_{Sum of Path}} \cdot I_T$$

$$P = I^2 R$$

$$P = 3^2(31)$$

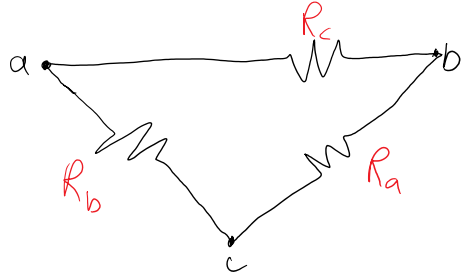
$$P = 9(31) = 279\Omega$$

Wye to Delta



$$R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$(22)(20) + 20(140) + 140(22) = 6320$$



$$R_a = \frac{6320}{R_1} = \frac{6320}{22} = 287\Omega$$

$$R_b = \frac{6320}{R_2} = \frac{6320}{20} = 316\Omega$$

$$R_c = \frac{6320}{R_3} = \frac{6320}{140} = 45\Omega$$

Node Voltage Method

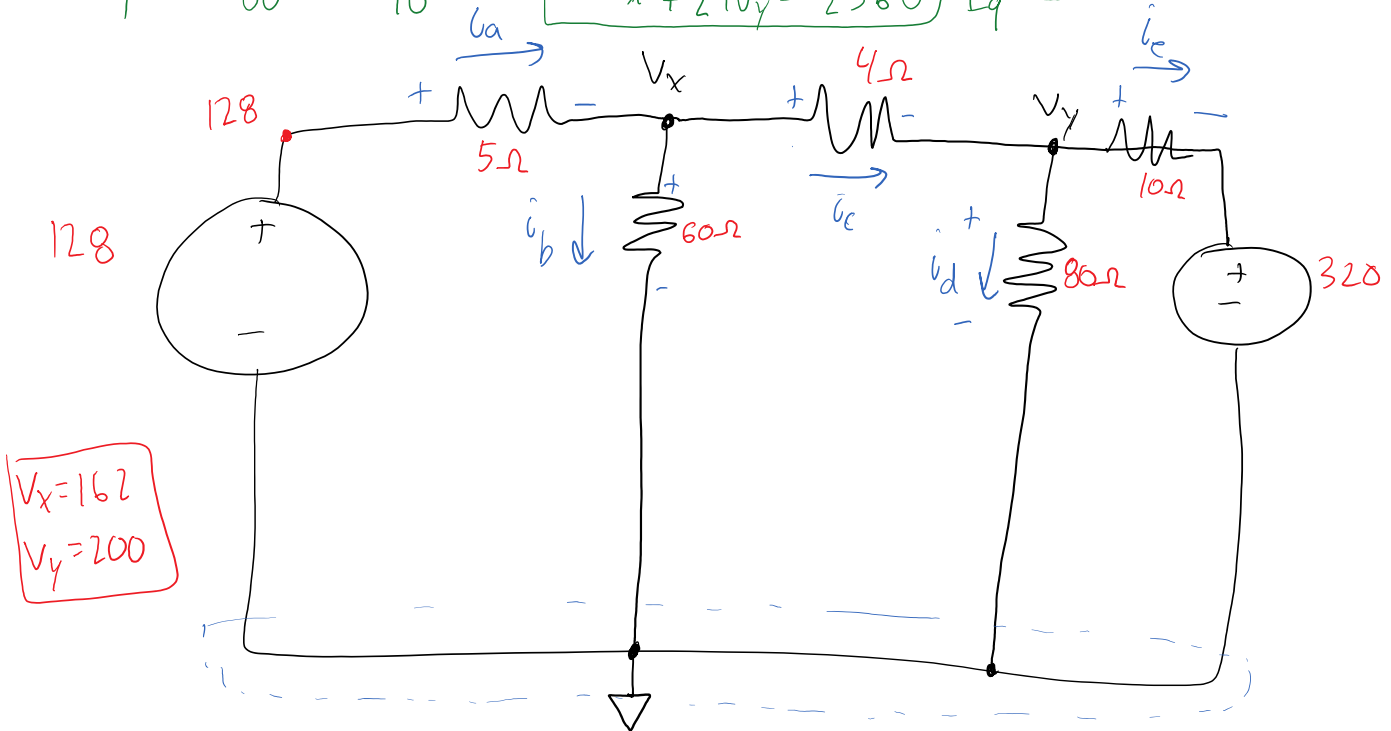
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$$i_a = \frac{128 - V_x}{5\Omega} \quad i_b = \frac{V_x - 0}{60\Omega} \quad i_c = \frac{V_x - V_y}{4\Omega} \quad i_d = \frac{V_y - 0}{80\Omega} \quad i_e = \frac{V_y - 320}{10\Omega}$$

$$\frac{V_x - V_y}{4} = \frac{V_y}{80} + \frac{V_y - 320}{10}$$

$$20V_x - 20V_y = V_y + 8V_y - 2560$$

$$\boxed{-20V_x + 29V_y = 2560} \text{ Eq\#2}$$



$$\boxed{V_x = 162}$$

$$\boxed{V_y = 200}$$

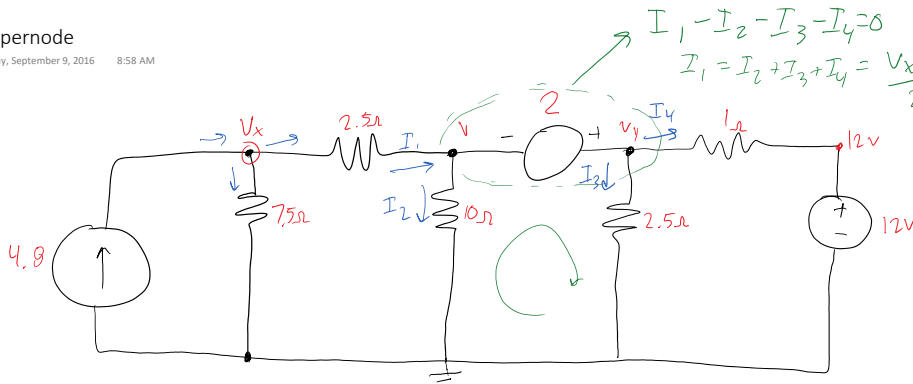
$$i_a = i_b + i_c \quad \frac{128 - V_x}{5} = \frac{V_x}{60} + \frac{V_x - V_y}{4} \quad 1536 - 12V_x = V_x + 15V_x - 15V_y$$

$$\text{Eq\#1: } \boxed{28V_x - 15V_y = 1536}$$

- ① Identify a reference node in the circuit and assign a value of zero to that point in the circuit.
- ② Identify independent nodes ← any node where the voltage is unknown
- ③ Assign current directions to all "branches" in the circuit, Write KCL for all independent nodes using Ohm's law to express the currents in terms of the voltages or voltage differences
- ④ Solve the system of equations
- ⑤ Use the node voltages to find the desired electrical quantities

Supernode

Friday, September 9, 2016 8:58 AM



$$I_1 - I_2 - I_3 - I_4 = 0$$

$$I_1 = I_2 + I_3 + I_4 = \frac{V_x - V}{2.5} = \frac{V}{10} + \frac{V_y}{2.5} + \frac{V_y - 12}{1} \Rightarrow -5V + 4V_x - 4V_y = -120$$

$$4.8 = \frac{V_x}{7.5} + \frac{V_x - V}{2.5} \Rightarrow -3V + 4V_x = 36$$

Loop containing supernode

$$-V - 2 + V_y = 0 \Rightarrow -V + V_y = 2$$

$$V = 8$$

$$V_x = 15$$

$$V_y = 10$$

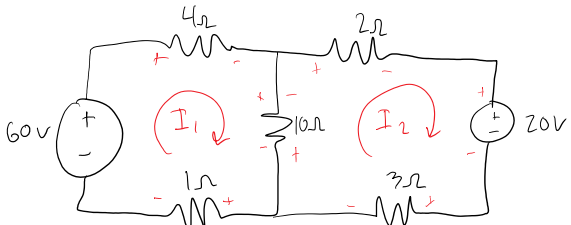
Whenever two non-reference nodes are directly connected by an independent or dependent voltage source, it is convenient to apply the supernode concept.

A supernode is a closed surface and Kirchoff's current law applies to closed surfaces as well.

- 1) Define supernode
- 2) Write KCL for supernode
- 3) write KCL for all other nodes
- 4) write KVL for supernode

Mesh Current Method

A generalized technique only valid for Planar Circuits that can be drawn on a flat piece of paper.

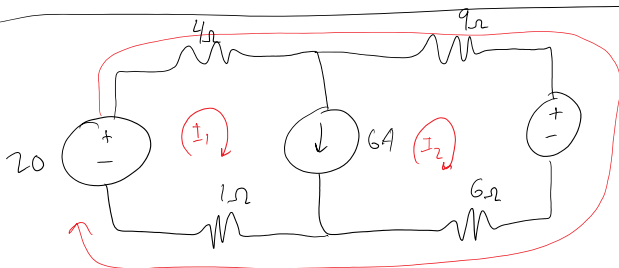


$$\text{Mesh 1: } -60 + 4I_1 + 10(I_1 - I_2) + 1I_1 = 0 \Rightarrow 15I_1 - 10I_2 = 60$$

$$\text{Mesh 2: } 10(I_2 - I_1) + 2I_2 + 20 + 3I_2 = 0 \Rightarrow -10I_1 + 15I_2 = 20$$

$$I_1 = 5.6 \text{ A} \quad I_2 = 2.4 \text{ A}$$

- 1) Define currents flowing in meshes of a planar circuit
- 2) write the KVL around the meshes using ohm's law to express the voltages across the resistors in terms of the mesh currents.
- 3) Solve the system of equations
- 4) Use the mesh currents to find desired electrical quantities.



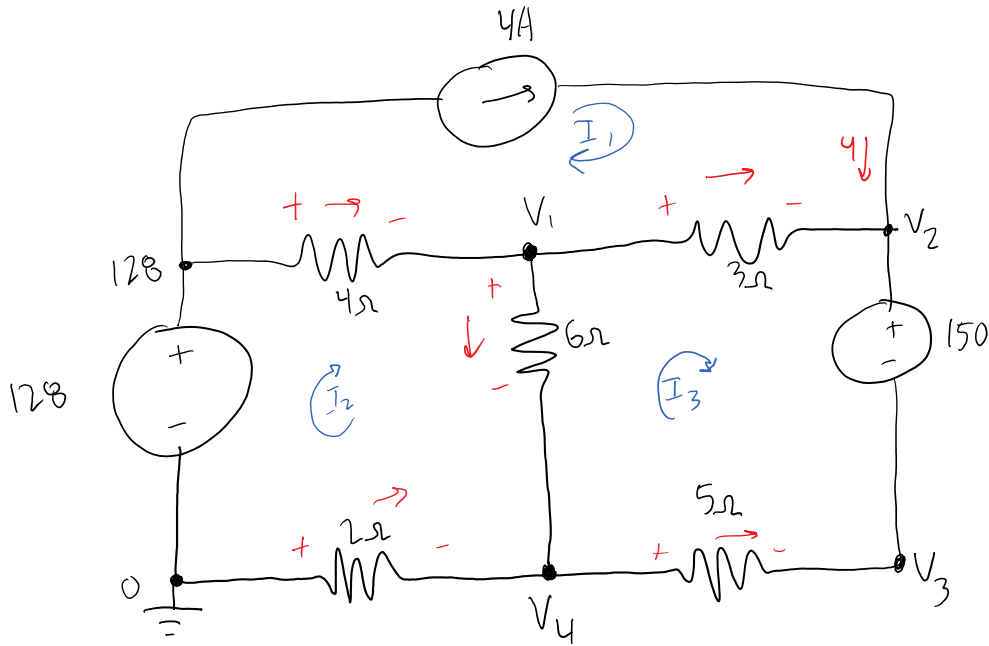
Super Mesh

$$6A = I_1 - I_2$$

- 1) Pick a closed path that does not include the current source
- 2) Write mesh KVL for meshes not including current source
- 3) Write equation for currents shared with current source
- 4) solve equations
- 5) use the currents to find all other electrical quantities

Node Voltage vs Mesh Current

Monday, September 12, 2016 9:00 AM



Node Voltage

$$\frac{128 - V_1}{4} = \frac{V_1 - V_4}{6} + \frac{V_1 - V_2}{3}$$

$$384 - 3V_1 = 2V_1 - 2V_4 + 4V_1 - 4V_2$$

$$\textcircled{1} \quad 9V_1 - 4V_2 - 2V_4 = 384$$

Supernode

$$4 + \frac{V_1 - V_2}{3} + \frac{V_4 - V_3}{5} = 0$$

$$60 + 5V_1 - 5V_2 + 3V_4 - 3V_3 = 0$$

$$\textcircled{2} \quad 5V_1 - 5V_2 + 3V_4 - 3V_3 = -60$$

$$\textcircled{3} \quad V_2 - V_3 = 150$$

Node V4

$$\frac{0 - V_4}{2} + \frac{V_1 - V_4}{6} = \frac{V_4 - V_3}{5}$$

$$-15V_4 + 5V_1 - 5V_4 - 6V_4 + 6V_3 = 0$$

$$\textcircled{4} \quad 5V_1 + 6V_3 - 26V_4 = 0$$

Mesh

$$\textcircled{2} \quad -128 + 4(I_2 - I_1) + 6(I_2 - I_3) + 2I_2 = 0$$

$$12I_2 - 6I_3 - 16 - 128 = 0$$

$$\boxed{12I_2 - 6I_3 = 144}$$

$$\textcircled{3} \quad 6(I_3 - I_2) + 3(I_1 - I_3) + 150 + 5I_3 = 0$$

$$-6I_2 + 14I_3 - 12 + 150 = 0$$

$$\boxed{-6I_2 + 14I_3 = -138}$$

$$I_1 = 4A$$

$$I_2 = 9A$$

$$I_3 = -6A$$

$$-15V_4 + 5V_1 - 3V_4 - 6V_4 + 6V_3 - \dots$$

$$(4) \quad 5V_1 + 6V_3 - 26V_4 = 0$$

$$V_1 = 108 \text{ V}$$

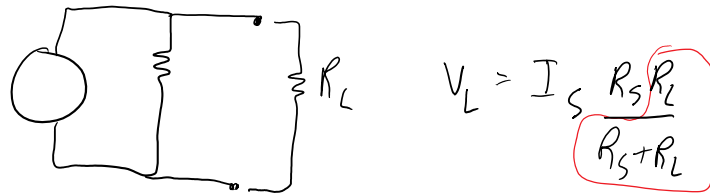
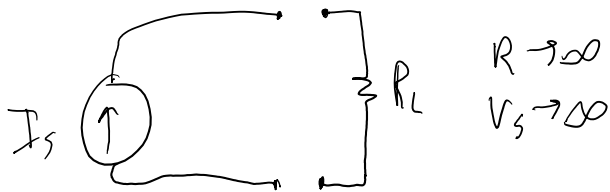
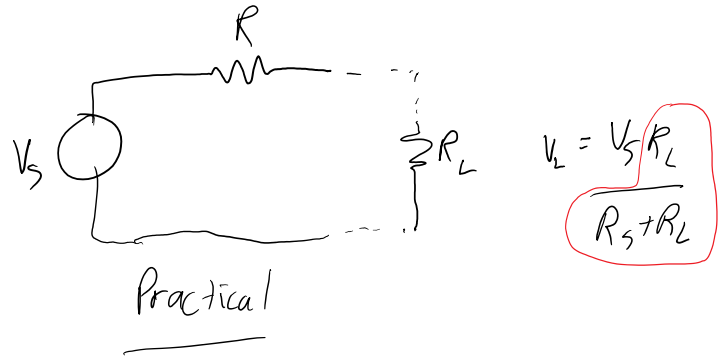
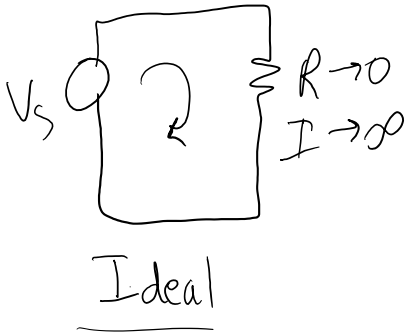
$$V_2 = 138 \text{ V}$$

$$V_3 = -12 \text{ V}$$

$$V_4 = 18 \text{ V}$$

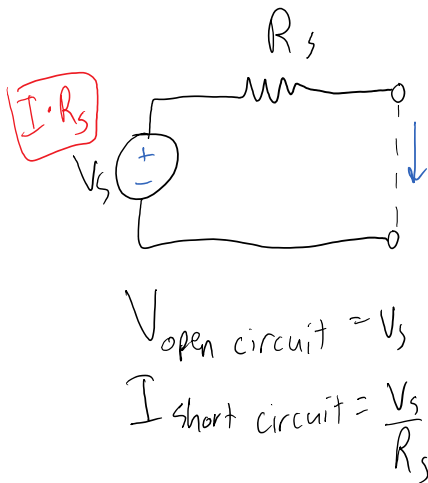
Source Transformations

Wednesday, September 14, 2016 9:00 AM

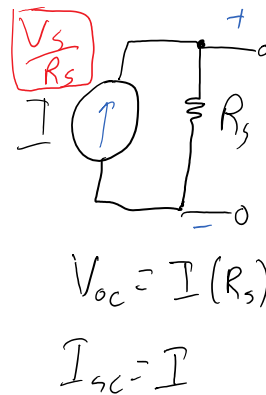


$I_f V_L = V_L$

$V_s = I_s R_s$

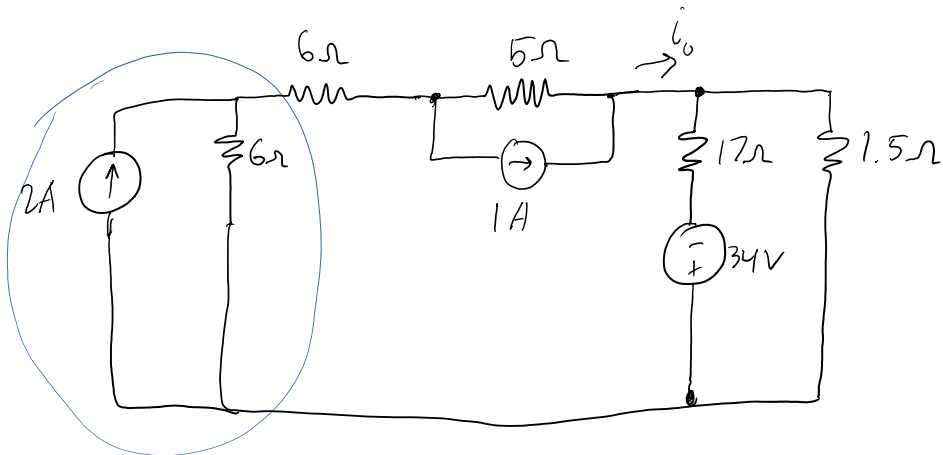


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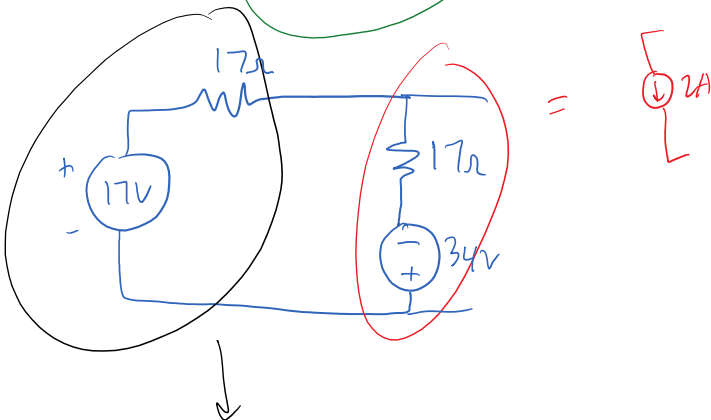
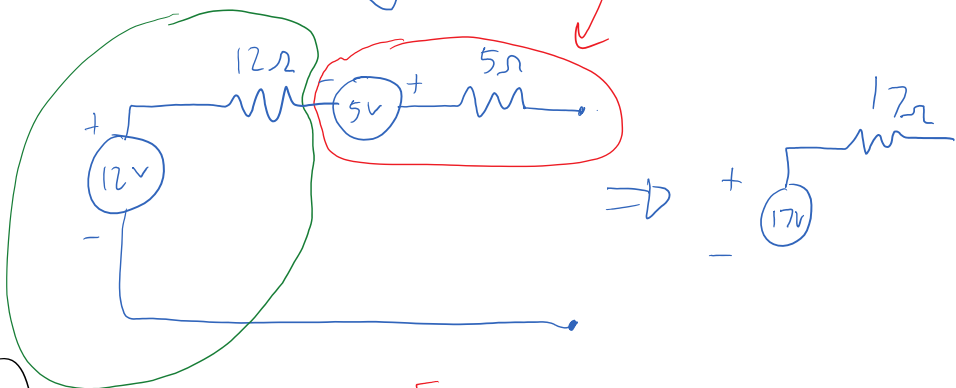
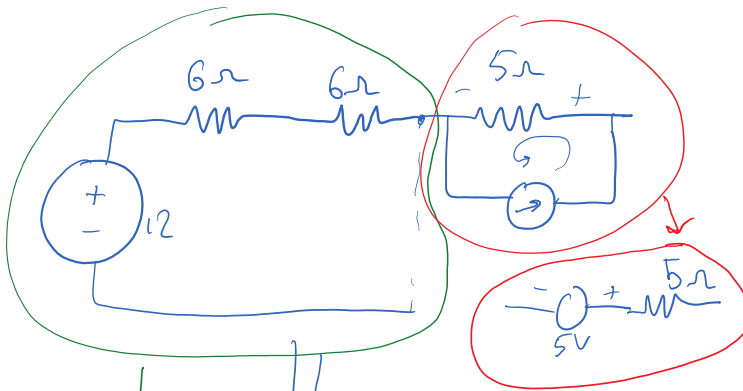


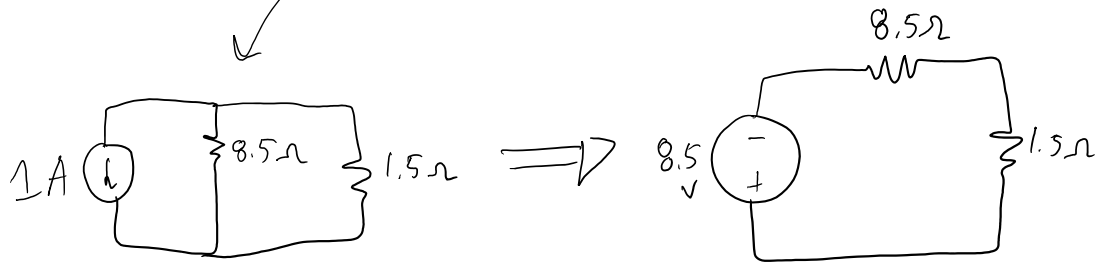
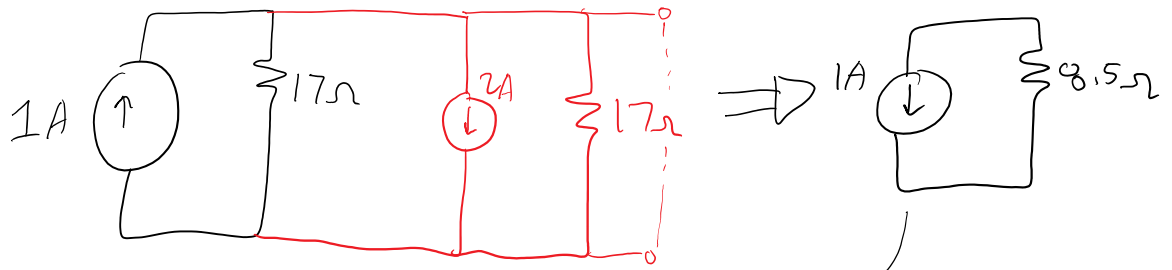
Method of Source Transformation

A practical voltage source can be replaced by a practical current source and a practical current source can be replaced by a practical voltage source as long as $V_s = I_s (R_s)$ and $I_s = \frac{V_s}{R_s}$



$I_{sc} = 2A$
 $V_{oc} = 12V \Rightarrow$
 $R_s = 6\Omega$

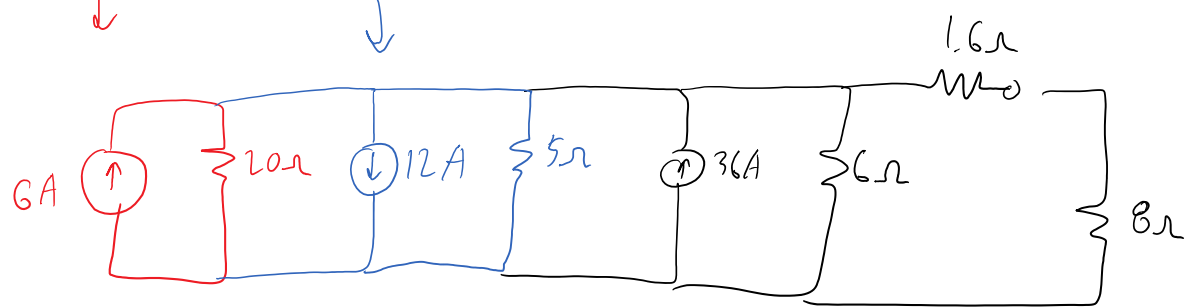
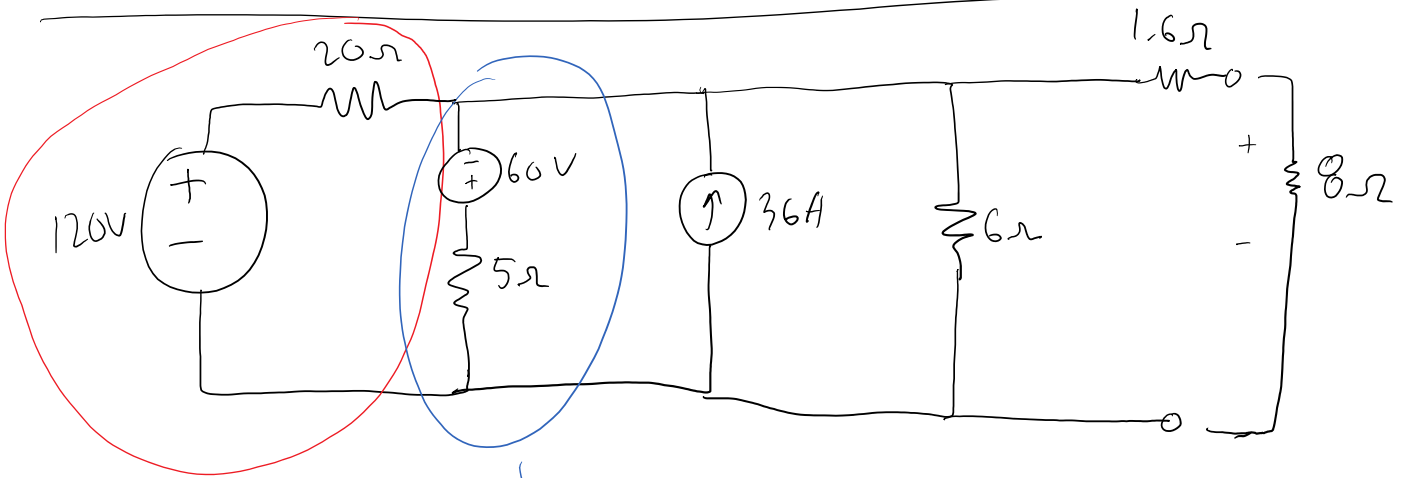


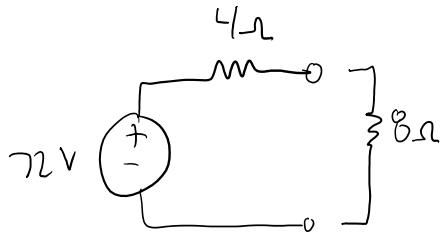
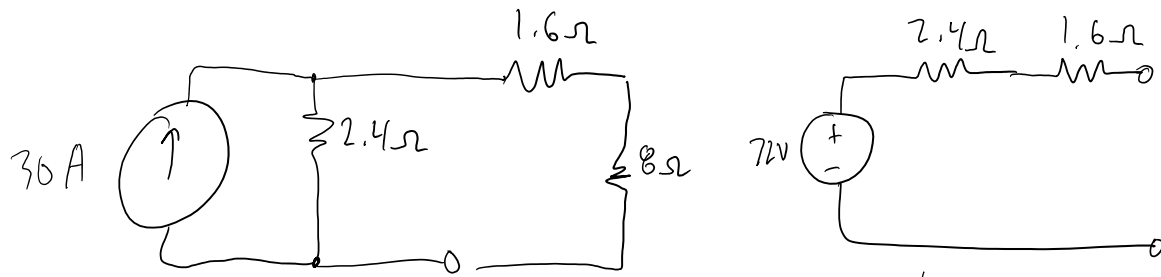


$$I_0 = \frac{8.5V}{10\Omega} = 0.85A$$

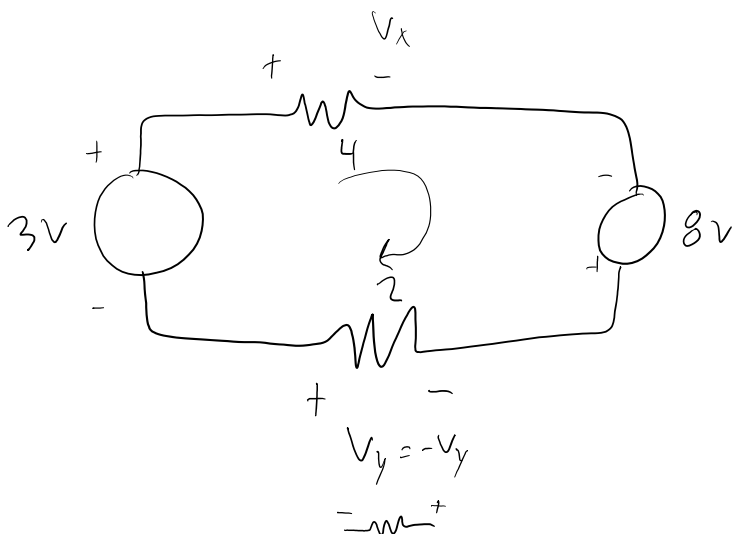
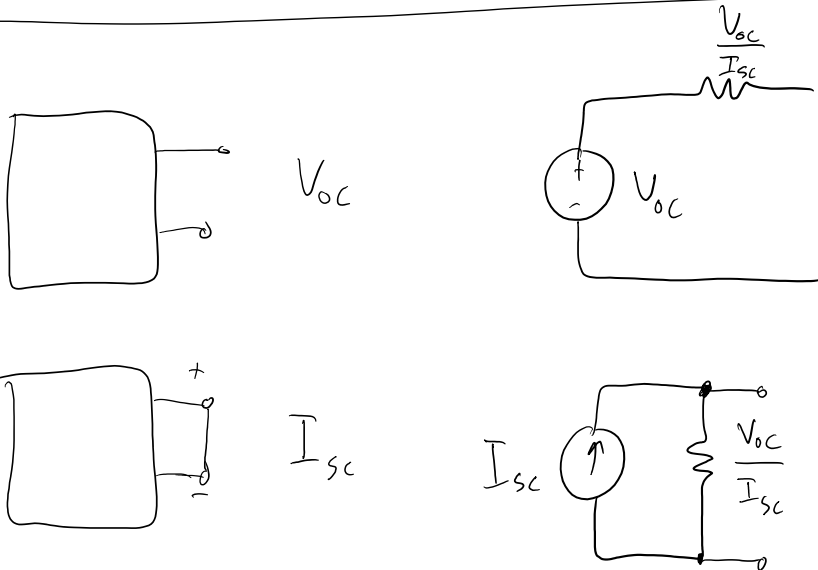
$$V_0 = \frac{8.5(1.5)}{10} = 1.275V$$

$$V_0 = \frac{R_1(R_2)}{R_1+R_2}$$

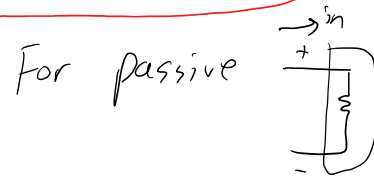




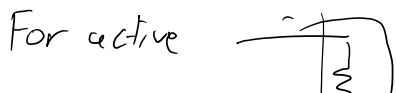
$$V_o = \frac{72(8)}{12} = 48$$

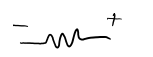


$$-3 + 4I - 8 + 2I$$

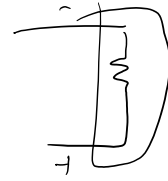


in on +, out of -



$$v_y = -v_x$$


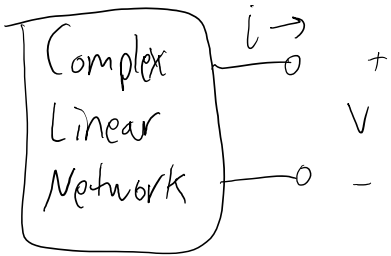
For active



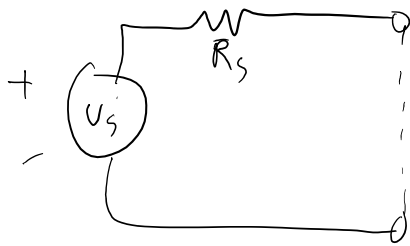
in on -, out of +

Thevenin and Norton

Friday, September 16, 2016 9:02 AM



Thevenin Form

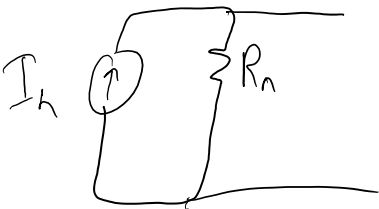


$$V_{oc} = V_s$$

$$I_{sc} = \frac{V_s}{R_s}$$

Equivalent if I_{sc} are equal and V_{oc} are equal $R_n = R_s$ and $V_{oc} = V_n$ or $I_{sc} = I_n$

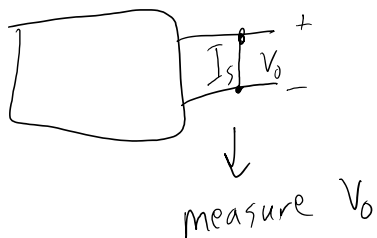
Norton Form



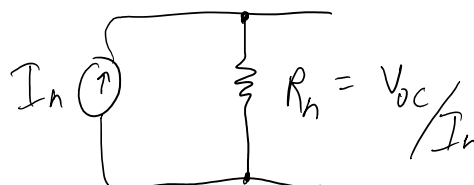
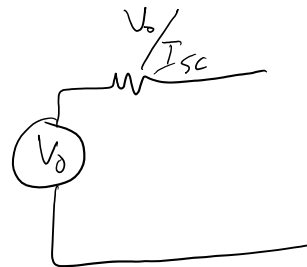
$$V_{oc} = I_n R_n$$

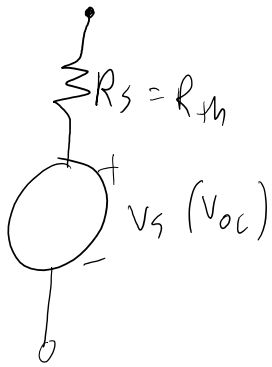
$$I_{sc} = I_n$$

Any complex, linear, one port, network can be replaced with an equivalent circuit consisting of a source and a resistor such that the characteristics at the terminal are the same.

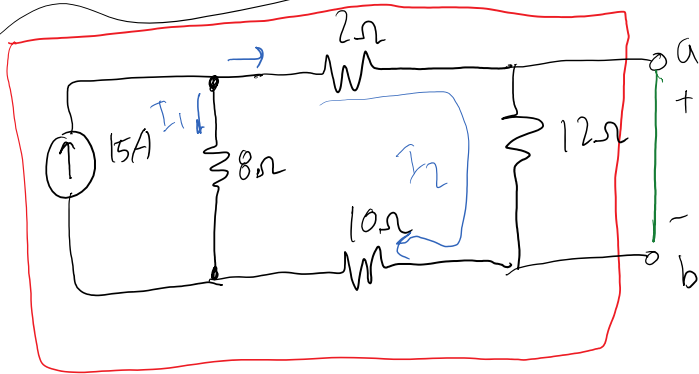
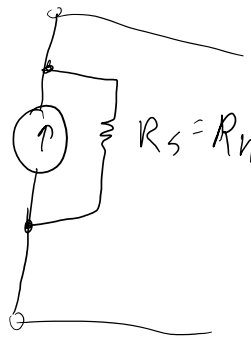


Thevenin Form =

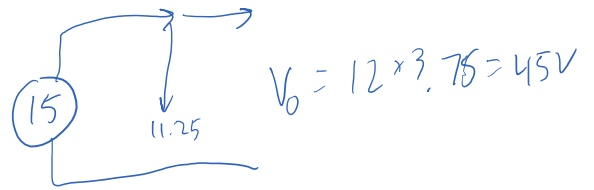




$$I_{th} \quad V_{oc} / I_{th}$$



$$V_{oc} = V_{th}$$



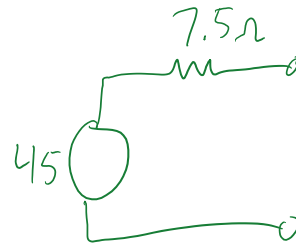
$$I_1 = \frac{15 \times 24}{32} = 11.25 \text{ A}$$

$$I_2 = \frac{15 \times 8}{32} = 3.75 \text{ A}$$

$$I_{sc} = \frac{15 \times 8}{20} = 6 \text{ A}$$

$$R_N = \frac{45 \text{ V}}{6 \text{ A}} = 7.5 \Omega$$

Norton



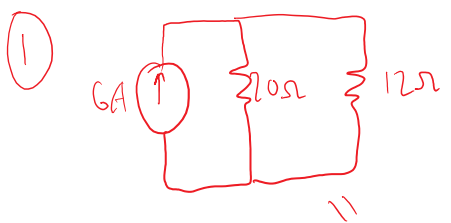
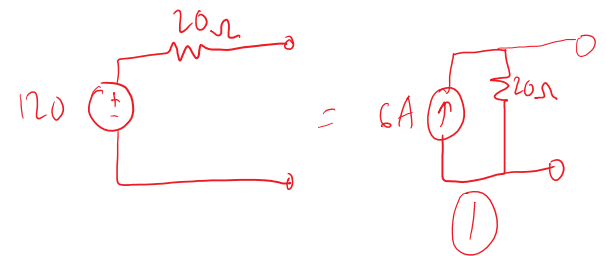
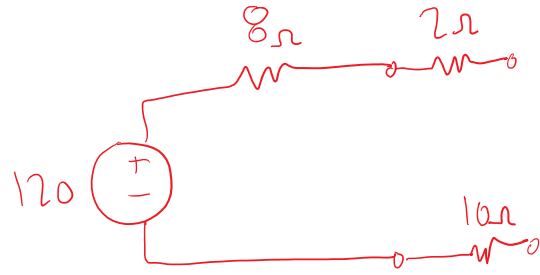
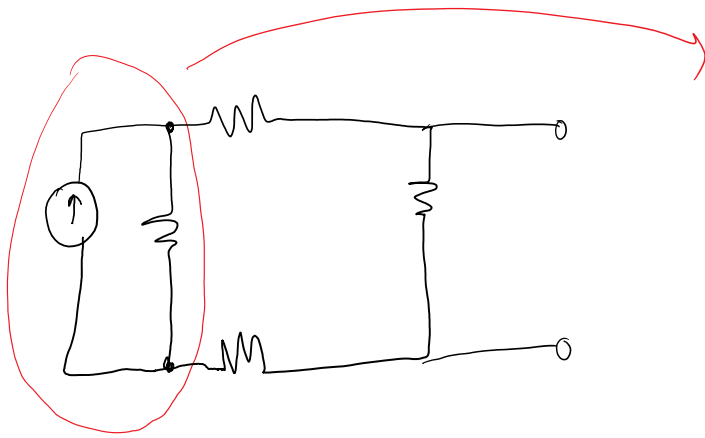
$$V_{oc} = 45$$

$$I_s = 6 \text{ A}$$

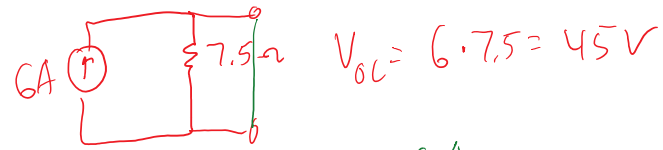


$$V_{oc} = 45$$

$$I_{sc} = 6 \text{ A}$$

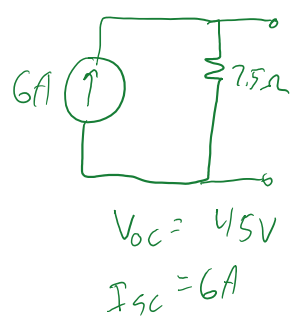
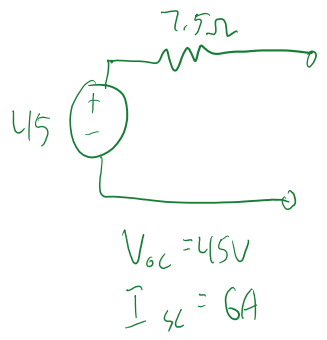


$$\frac{12 \times 10}{12 + 10} = \frac{240}{3.2} = 7.5 \Omega$$



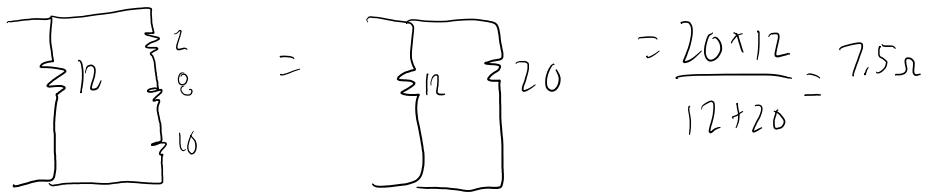
$$I_N = I_{sc} = 6A$$

$$R_N = \frac{45}{6} = 7.5 \Omega$$



"Easy Way"

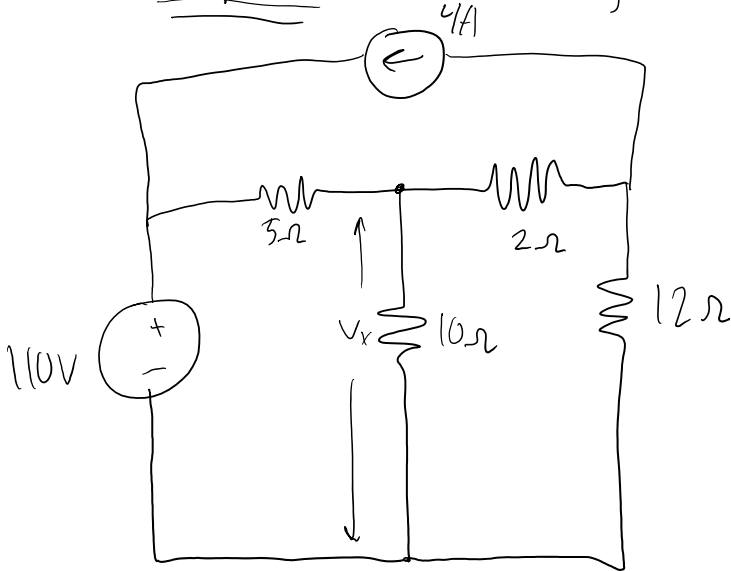
- 1) short all voltage sources or/and open all current sources
- 2) compare the equivalent resistance from the point of the output "port"



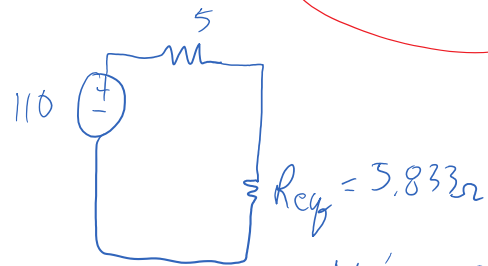
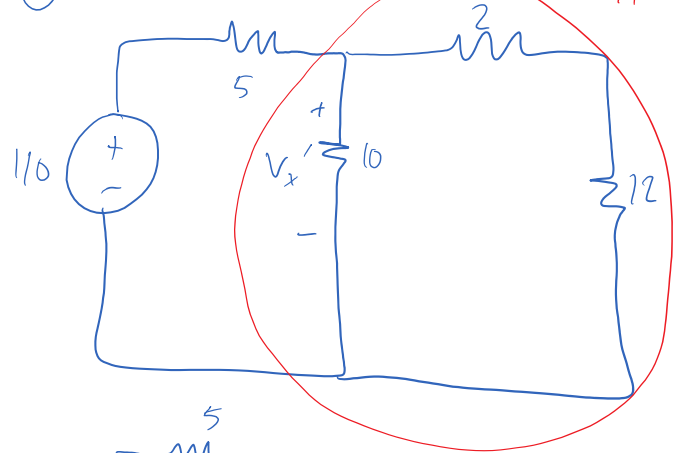
Superposition

Monday, September 19, 2016 9:02 AM

Given a linear network, any circuit voltage or current may be calculated as the algebraic sum of the individual voltages or currents caused by two or more independent sources acting alone.



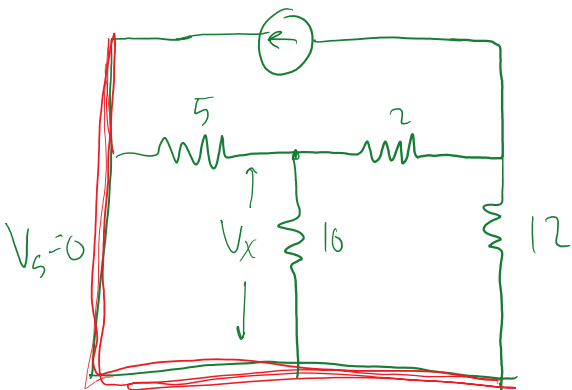
① Zero out current source 14 || 10



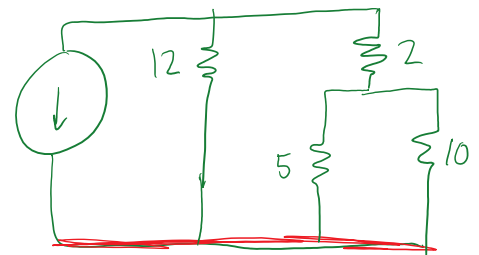
$$V_x' = \frac{5.833}{5 + 5.833} \cdot 110$$

$$V_x' = 59.231 \text{ V}$$

② Zero out Voltage source, Activate current source



↓
Common wire





$$V_X = V_X' + V_X''$$

$$V_X = 50V$$

$$I_1 = \frac{4(12)}{12 + 5.33}$$

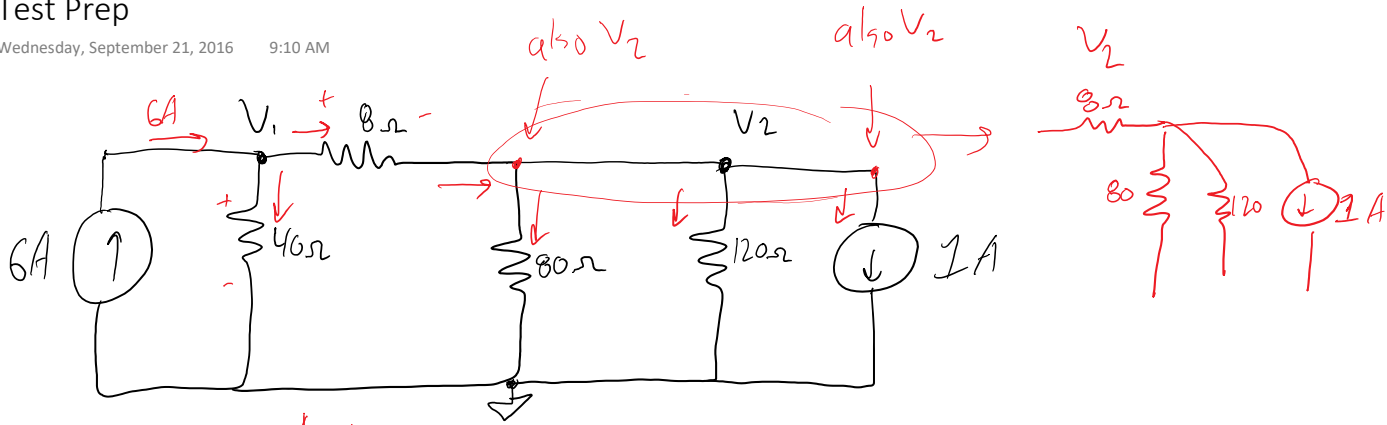
$$I_1 = 2.769A$$

$$V_X = I_1(3.33\Omega)$$

(Direction of resistor flipped) $V_X'' = -9.231V$

Max Power

If $R_{th} = R_{Load}$, max power has been transferred



Node Voltage Analysis

$$\underline{V_1}: 6 = \frac{V_1 - 0}{40} + \frac{V_1 - V_2}{8} \quad 240 = V_1 + 5V_1 - 5V_2 \quad \textcircled{1} \quad 6V_1 - 5V_2 = 240$$

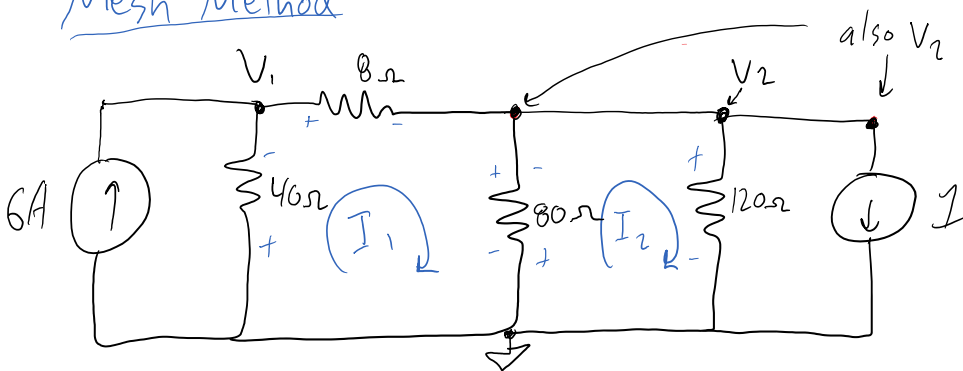
$$\underline{V_2}: \frac{V_1 - V_2}{8} = \frac{V_2 - 0}{80} + \frac{V_2 - 0}{120} + 1$$

$$15V_1 - 15V_2 = 1.5V_2 + V_2 + 120$$

$$\textcircled{2} \quad 15V_1 - 17.5V_2 = 120$$

$$\begin{aligned} V_1 &= 120V \\ V_2 &= 96V \end{aligned}$$

Mesh Method



Mesh 1:

$$40(I_1 - 6) + 8I_1 + 80(I_1 - I_2) = 0$$

$$40I_1 - 240 + 8I_1 + 80I_1 - 80I_2 = 0$$

$$\textcircled{1} \quad 128I_1 - 80I_2 = 240$$

Mesh 2:

$$80(I_2 - I_1) + 120(I_2 - 1) = 0$$

$$80I_2 - 80I_1 + 120I_2 - 120 = 0$$

$$\textcircled{2} \quad -80I_1 + 200I_2 = 120$$

$$I_1 = 3A$$

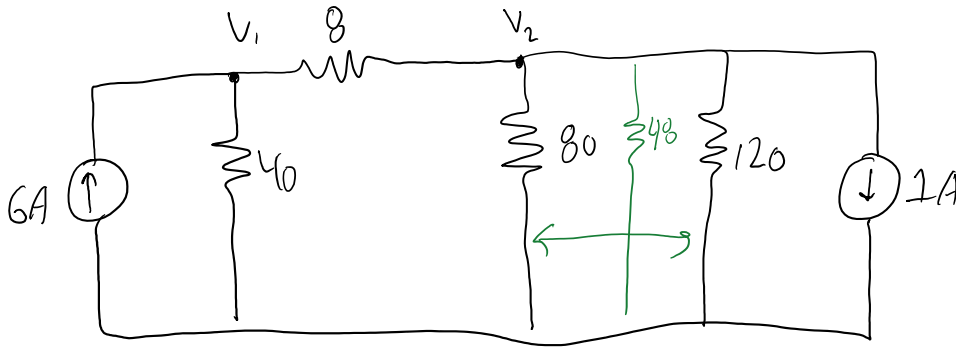
$$I_2 = 1.8A$$

$$V_1 = 40(6 - I_1) \quad V_1 = 40(3) \quad \boxed{V_1 = 120V} \checkmark$$

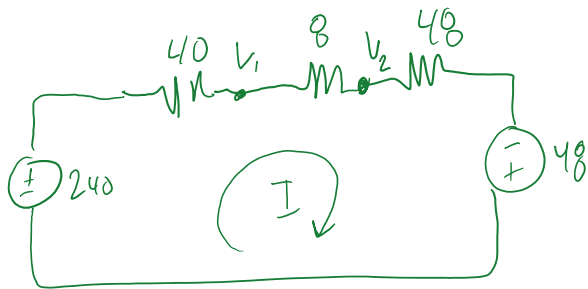
$$V_2 = V_1 - 8(I_1) \quad V_2 = 120 - 24 \quad \boxed{V_2 = 96} \checkmark$$

Test Prep (cont)

Friday, September 23, 2016 9:01 AM



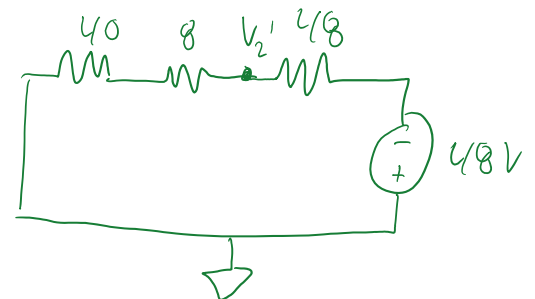
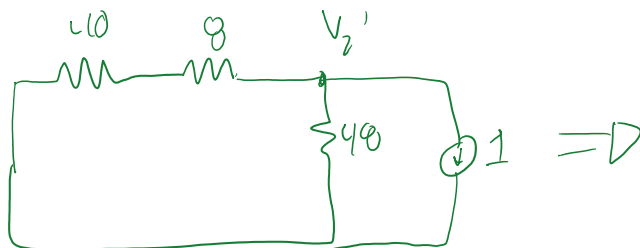
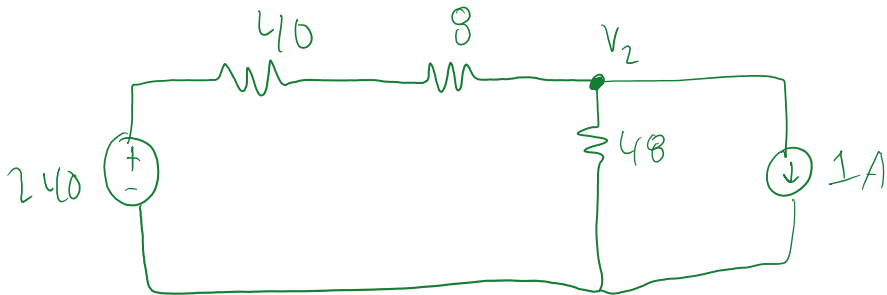
$$R_N = \frac{80 \cdot 120}{80 + 120} = 48$$



$$-240 + 40I + 8I + 48I - 48 = 0$$

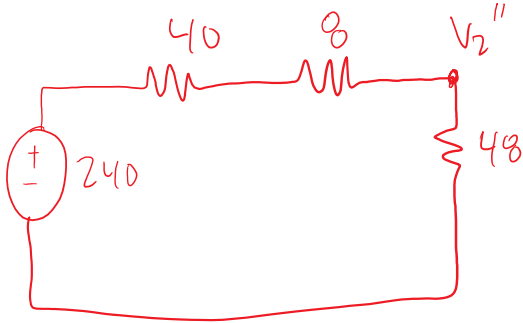
$$96I = 288 \quad I = 3A$$

$$\begin{aligned} 240 - 3(40) &= 120 = V_1 \\ 120 - 8(3) &= 96 = V_2 \end{aligned}$$



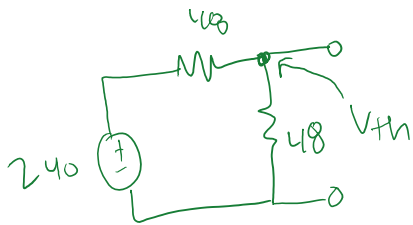
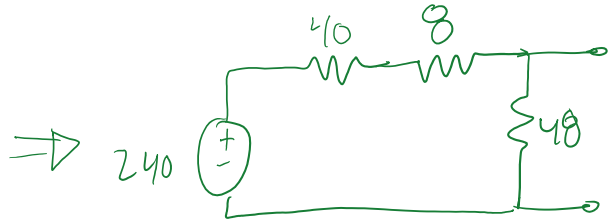
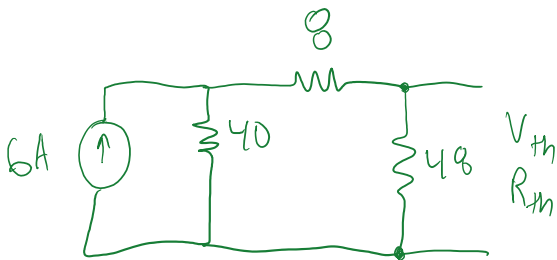
$$V_2' = -48 \left(\frac{40+8}{96} \right) = -24$$

Voltage divider



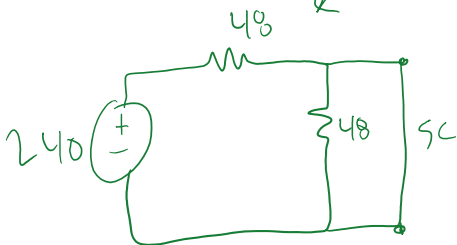
$$V_2'' = 240 \left(\frac{48}{96} \right) = 120V$$

$$V_2 = V_2' + V_2'' = -24 + 120 = 96$$



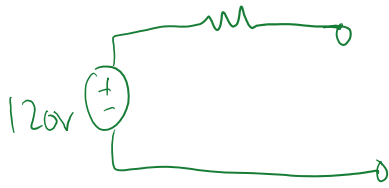
$$V_{th} = 240 \left(\frac{48}{96} \right) = 120V$$

$$R_{th} = \frac{V_{th}}{I_{sc}}$$

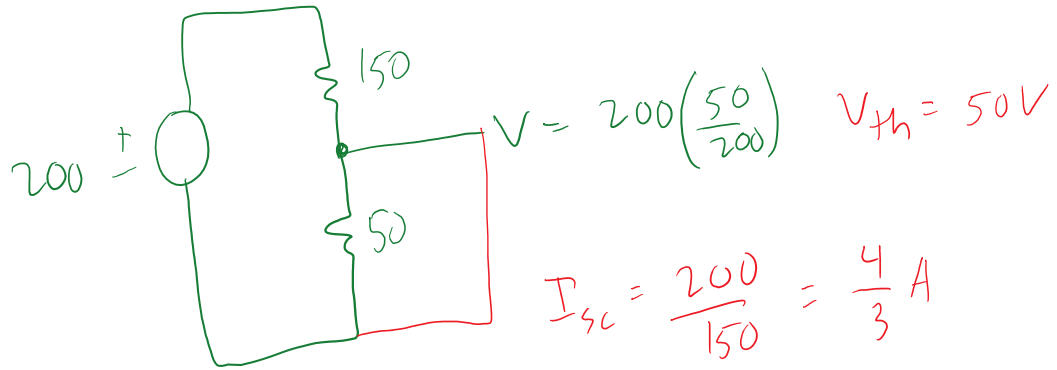


$$I_{sc} = \frac{V}{R} = \frac{240}{48} = 5A$$

$$R_{th} = \frac{120V}{5A} = 24\Omega$$

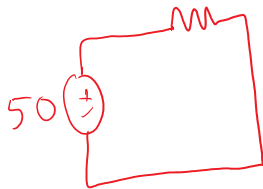


Voltage Divider



$$I_{sc} = \frac{200}{150} = \frac{4}{3} A$$

$150/4 \Omega$



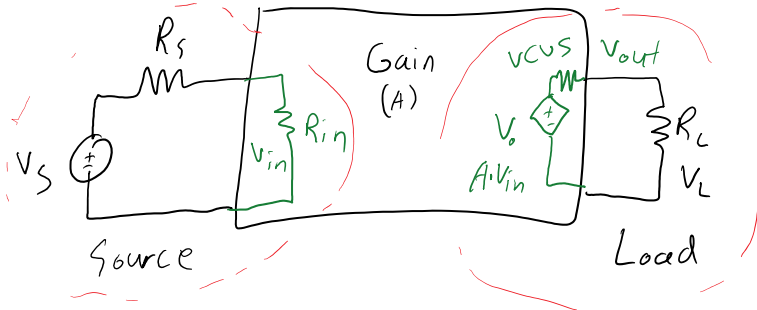
$$R_{th} = \frac{V}{I} \quad R_{th} = \frac{50}{\frac{4}{3}} = \frac{150}{4}$$

Operational Amplifiers

Wednesday, September 28, 2016 8:59 AM

Role of an amplifier

- A circuit designed to boost the power of the source voltage V_s

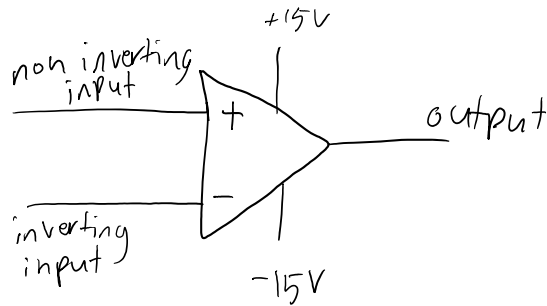


$$V_L = A \cdot V_s$$

$$V_{in} = \frac{R_{in}}{R_s + R_{in}} V_s \quad R_{in} \rightarrow \text{large}$$

- Input Resistance Large (10^6 to 10^9)
- High Gain (A) (10^5 to 10^6)

$$V_L = \frac{A \cdot V_{in} \cdot R_L}{R_L + R_o} \quad \text{so want very low output resistance } (< 1 \Omega)$$

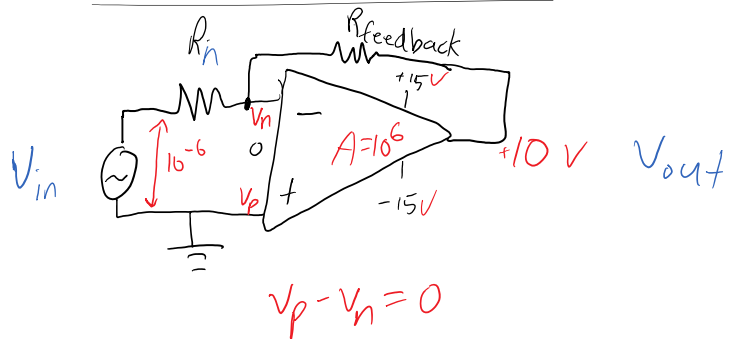


$$R_{in} \rightarrow \infty$$

$$R_{out} \sim 0$$

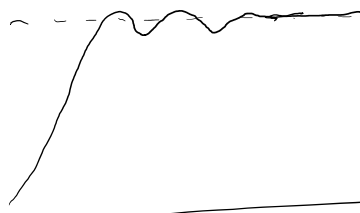
$$A \rightarrow \infty$$

Classic opamp circuit

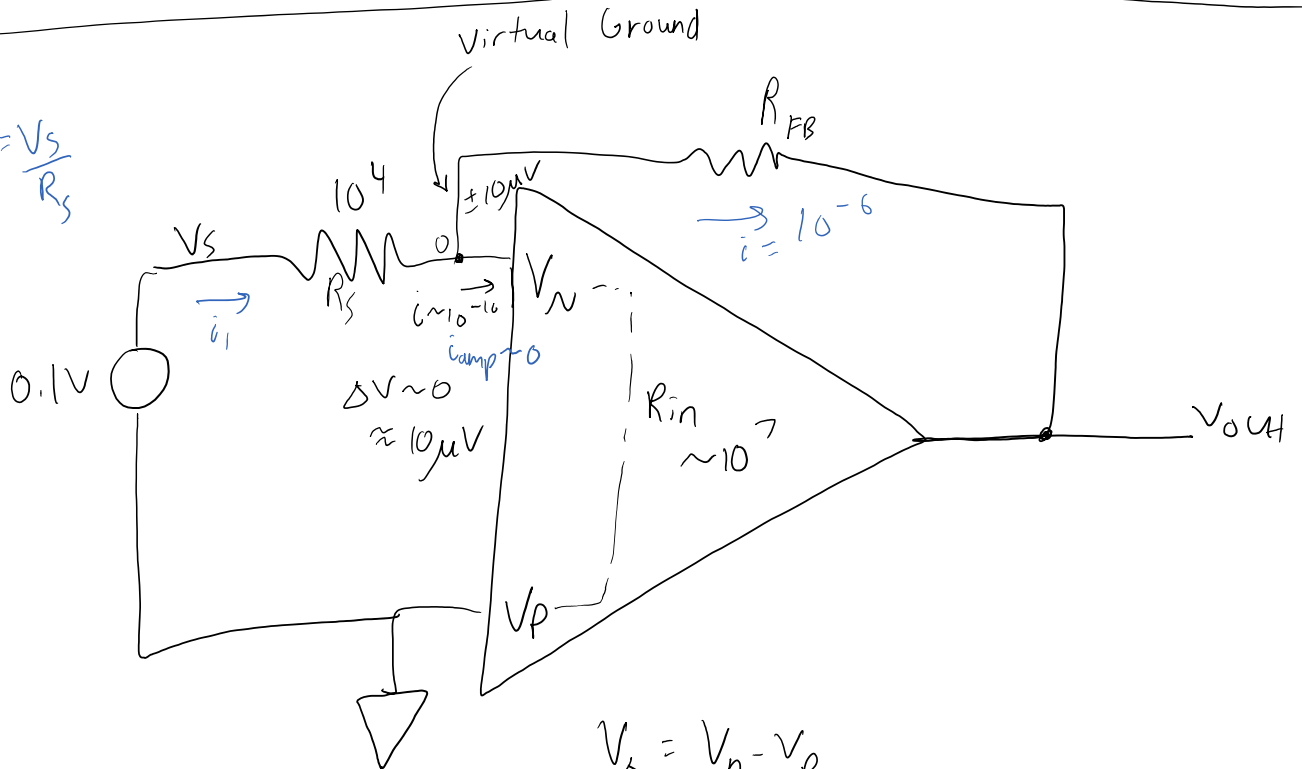


$$V_p - V_n = 0$$

$$V_{in} = \frac{V_s \cdot R_n}{R}$$



$$i_1 = \frac{V_s}{R_s}$$

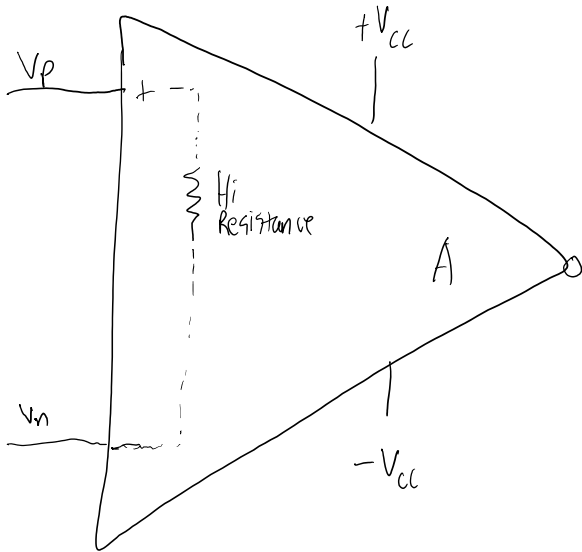


$$V_{\Delta} = V_n - V_p$$

$$V_{out} = -10^6 \times 10^{-5} = -10 \text{ V}$$

Negative Feedback

Friday, September 30, 2016 9:03 AM



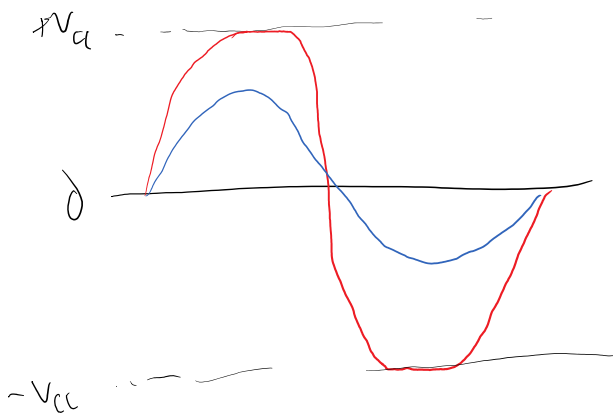
- 1) High Gain ($10^5, 10^6$)
- 2) Differential Input
- 3) High Input Resistance
- 4) Low output resistance

$$V_{out} = A(V_p - V_n)$$

5) Any differential signal in the input is multiplied by A

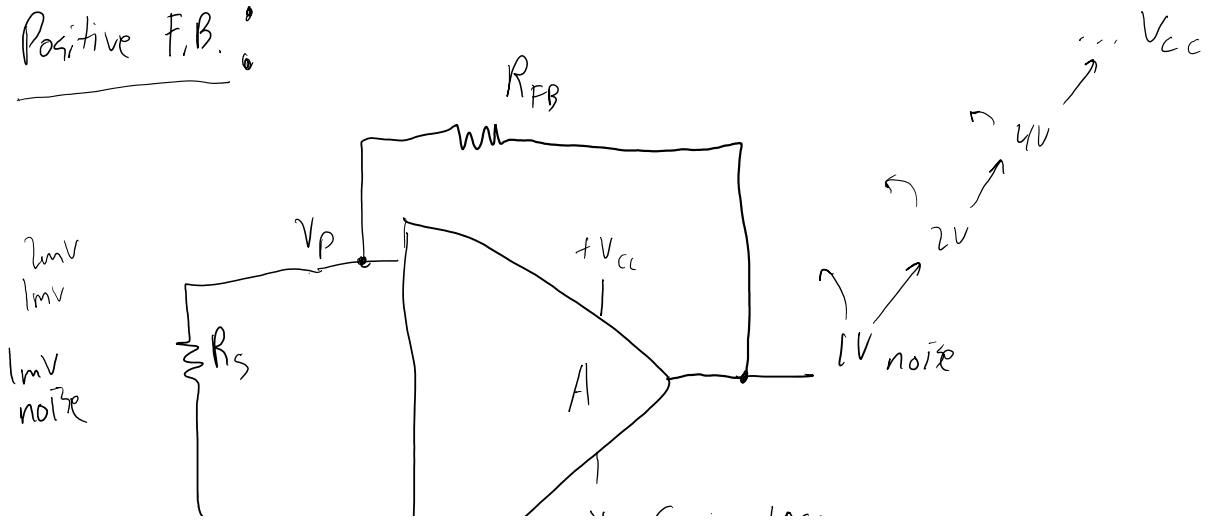
$$V_{out} = A(V_p - V_n) = V(V_n - V_p)$$

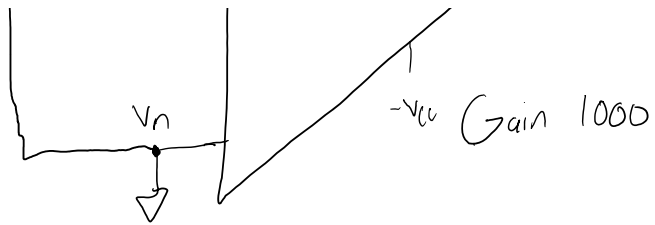
6) Has bipolar power supplies to support a bipolar output



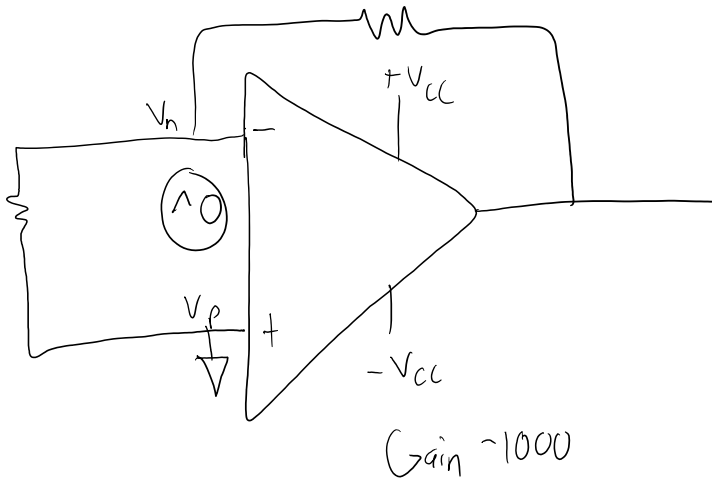
Feedback

Positive F.B.

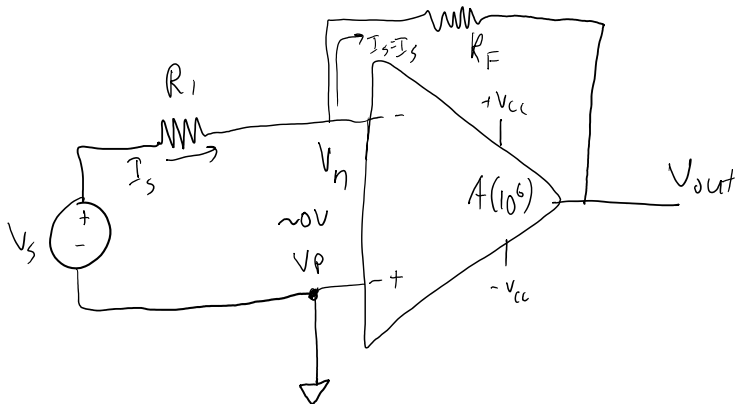




Negative FB:



Inverting Op Amp Circuit

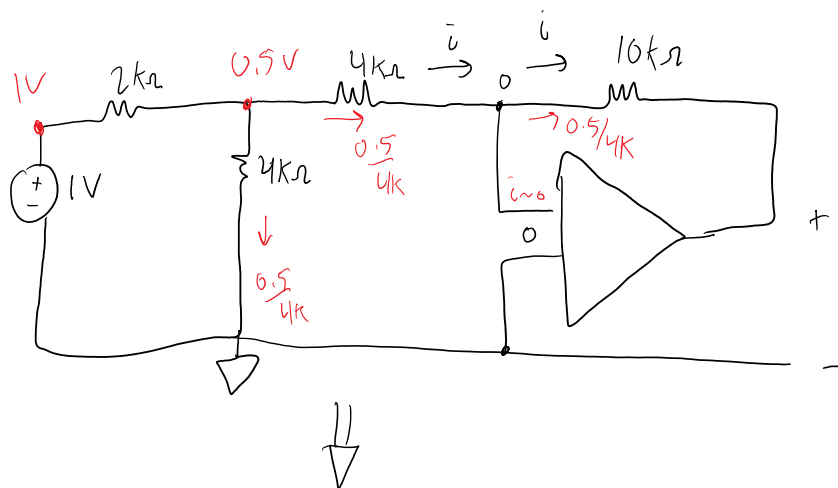


$$I_s = \frac{v_s - 0}{R_s} = \frac{v_s}{R_s}$$

$$v_o = 0 - I_s R_f = -\frac{v_s}{R_s} R_f$$

$$\text{Gain of circuit} = G = \frac{-v_o}{v_s} = \frac{-\frac{v_s R_f}{R_s}}{v_s} = \frac{R_f}{R_s}$$

$$\text{Gain} = -\frac{R_f}{R_s}$$



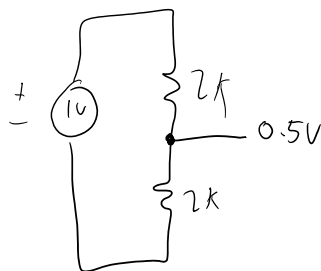
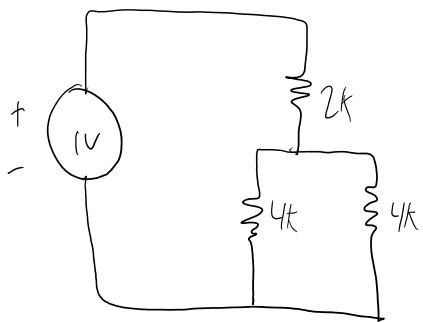
$$V_{out} = 0 - \left(\frac{0.5}{4k} \right) (10k)$$

$$i_{FB} = \frac{0.5}{4k}$$

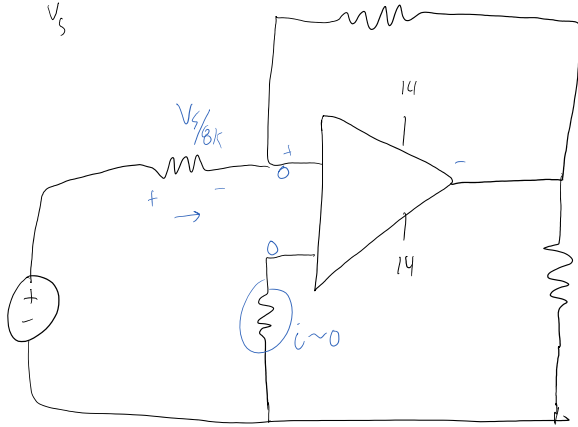
$$V_{out} = i_{FB} R_{FB}$$

$$V_{out} = -0.5 \cdot 2.5$$

$$V_{out} = -1.25V$$



1) a) $A_v = \frac{V_o}{V_s}$



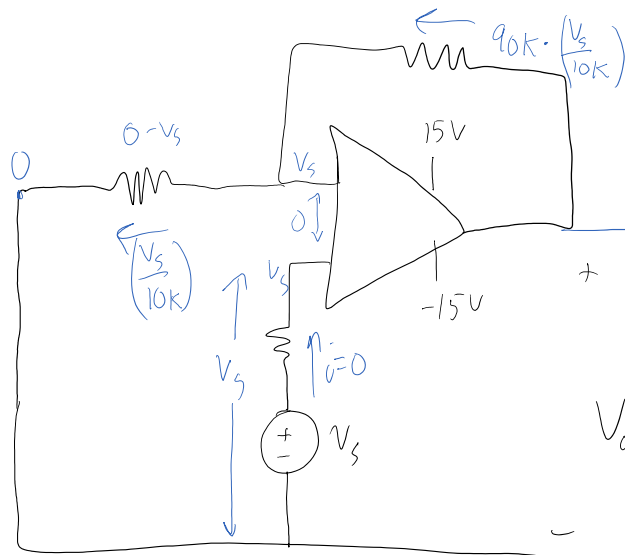
$V_{out} = 48k \left(\frac{V_s}{8k} \right) = -6V_s$

b) For -3V

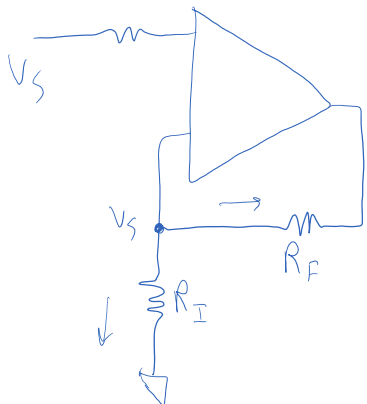
$V_{out} = -6(-3) = 18V$ -theor
14V -actual

$V_{out} = -6(1.5) = -9V$ -theor
actual

2)



$V_s \left(\frac{R_F}{R_I} + 1 \right) = V_s \left(\frac{90k}{10k} + 1 \right) = 10V_s$



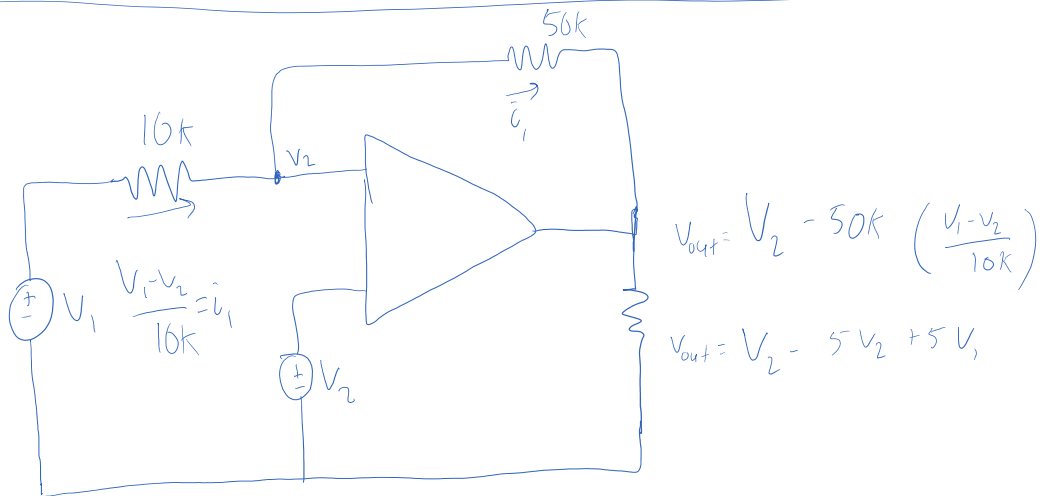
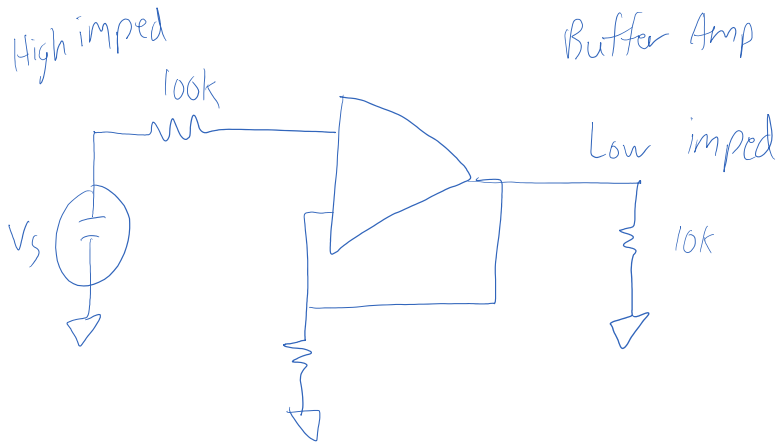
$\frac{V_s}{R_I}$

$V_{RF} = R_F \cdot \frac{V_s}{R_I}$

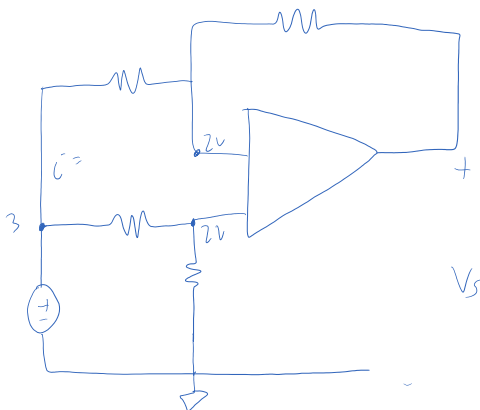
$V_{out} = V_{RF} + V_{RI}$

$$V_{out} = V_s \left(\frac{R_F}{R_I} \right) + V_s$$

$$= V_s \left(\frac{R_F}{R_I} + 1 \right)$$

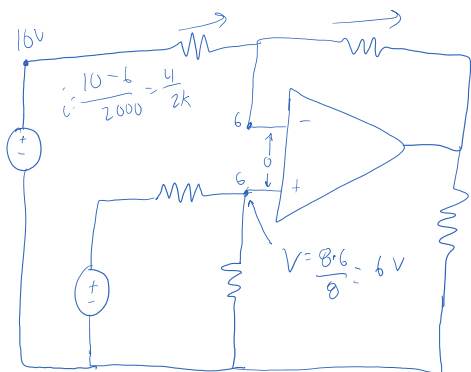


4) $V = \frac{3(10)}{15} = \frac{30}{15} = 2V$



$$V_{out} = -8000 \left(\frac{1}{2000} \right) + 2V$$

$$V_{out} = -4 + 2 = -2V$$

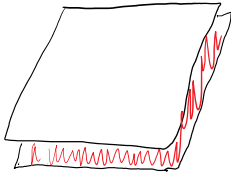


$$6 - 4k \left(\frac{4}{2k} \right)$$

$$6 - 8 = -2V$$

Inductors and Capacitors

Friday, October 7, 2016 9:02 AM

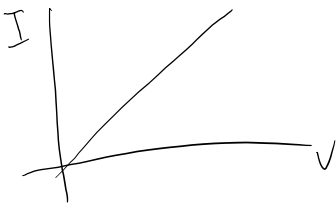
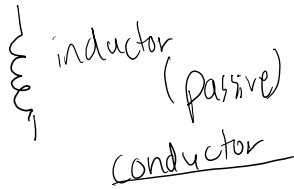


Constant ϵ_r
 $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$
 $C = \frac{\epsilon_r \epsilon_0 A}{d}$

$$V = L \frac{di}{dt} \qquad V = \frac{1}{C} \int i dt$$

$$i = \frac{1}{L} \int v dt \qquad i = C \frac{dv}{dt}$$

$$U = \frac{1}{2} L I^2 \qquad U = \frac{1}{2} C V^2$$



$$q = C V$$

$$\frac{\text{Coul}}{\text{Volt}} = \text{Farad} = \frac{\text{Amp} \cdot \text{Sec}}{\text{Volt}}$$

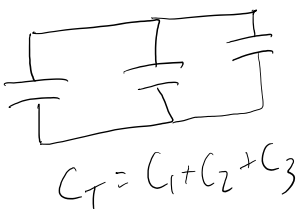
$$i = \frac{dq}{dt} = C \frac{dv}{dt}$$

$$i = C \frac{dv}{dt}$$

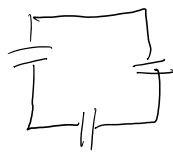
$$V = \frac{1}{C} \int i dt$$

$$U = \int P dt = \int v \cdot I dt = V C \frac{dv}{dt} \int v \frac{dv}{dt} dt = \frac{C v^2}{2} = U$$

Caps in parallel

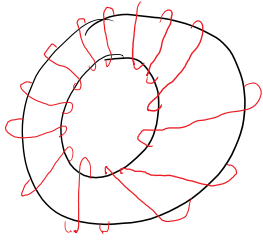


Caps in Series



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Inductance $L = \text{Henry} = \frac{\text{Volt} \cdot \text{Sec}}{\text{Amp}}$



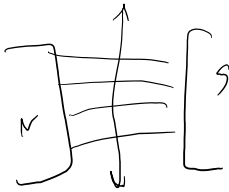
$$\lambda = LI = N \Phi$$

$$V = \frac{d\lambda}{dt} = \frac{d}{dt} LI$$

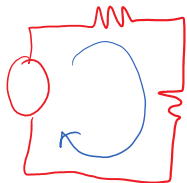
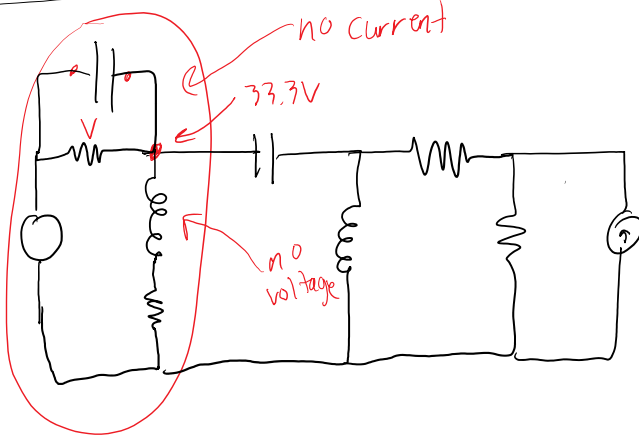
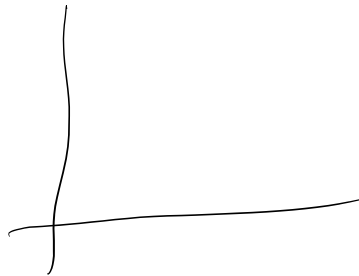


$$V = \frac{d}{dt} LI$$

$$= L \frac{dI}{dt}$$



inductor

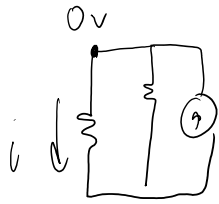


$$V = 100 \cdot \frac{100}{150}$$

$$U = \frac{1}{2} (10 \text{ G.S. } 6)$$

$$I = \frac{160V}{150\Omega} = \frac{2}{3} A$$

$$U = \frac{1}{2} LI^2$$



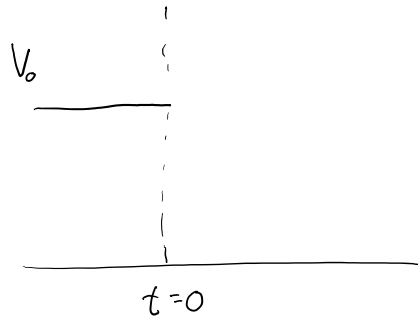
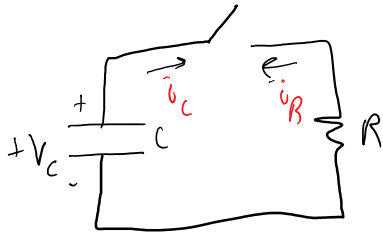
$$\hat{i} = 5A \left(\frac{40}{40+20} \right) = 3.33A$$

$$u = \frac{1}{2} L I^2$$

Transient Response of 1st Order Circuits

Monday, October 10, 2016 9:01 AM

Natural Response



$$i_R = \frac{V_c}{R}$$

$$i_c = C \frac{dV_c}{dt}$$

$$\frac{V_c}{R} + C \frac{dV_c}{dt} = 0$$

$$V_c + RC \frac{dV_c}{dt} = 0$$

$$k e^{st} + sRC k e^{st} = 0$$

$$\left. \begin{aligned} V_c &= k e^{st} \\ V_c' &= s k e^{st} \end{aligned} \right\}$$

$$(1 + sRC) k e^{st} = 0$$

$$1 + sRC = 0$$

$$s = -\frac{1}{RC}$$

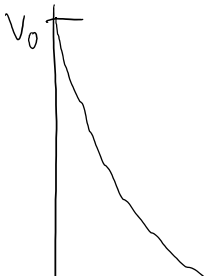
$$V_{0-} = V_{0+} = V_c = k e^{-t/RC}$$

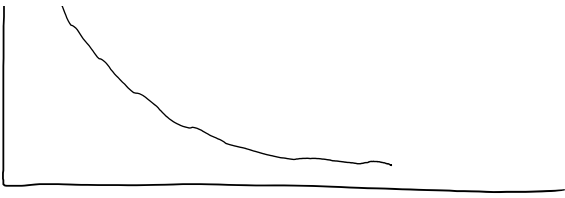
$$V_c(0) = k$$

$$V_c = V_0 e^{-t/\tau}$$

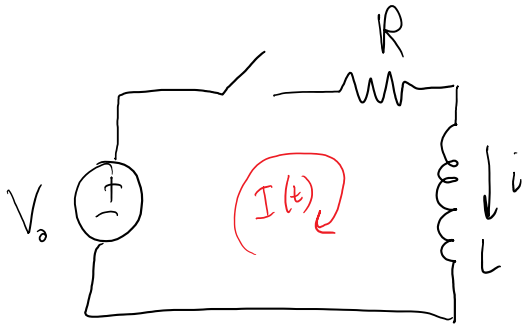
$$\tau = RC$$

$$V_c = V_0 e^{-t/\tau}$$





RL - Natural Response



$$-V + i_t R + L \frac{di_t}{dt} = 0$$

$$i_t R + L \frac{di_t}{dt} = V$$

$$i_t = k_1 + k_2 e^{st}$$

$$R(k_1 + k_2 e^{st}) + k_2 L s e^{st} = V$$

$$R(k_1) + (R + sL)k_2 e^{st} = V$$

$$R + sL = 0$$

$$sL = -R \quad s = -\frac{R}{L}$$

$$Rk_1 = V$$

$$k_1 = \frac{V}{R}$$

$$i_t = \frac{V_s}{R} + k_2 e^{-\frac{R}{L}t} \quad \frac{L}{R} = \tau$$

$$i_t = \frac{V_s}{R} + k_2 e^{-t/\tau}$$

$$i_{0+} = i_{0-} = 0 = \frac{V_s}{R} + k_2 e^{\frac{R}{L}(0)}$$

$$0 = \frac{V_0}{R} + k_2 \quad k_2 = -\frac{V_0}{R} \quad V_s = V_0$$

$$i(t) = \frac{V_0}{R} - \frac{V_0}{R} e^{-t/\tau}$$

$$i(t) = \frac{V_0}{R} (1 - e^{-t/\tau})$$

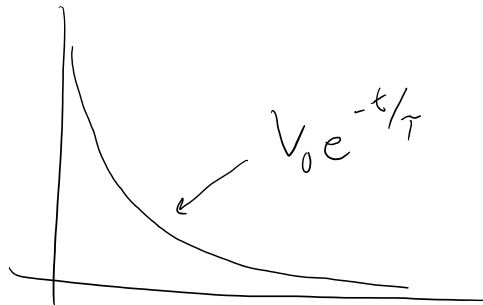
$i(t)$



$$i(t) = \frac{V_0}{R} (1 - e^{-t/\tau})$$



$$V_c = V_0 - i_t R$$



Process for Switched LR & RC Circuits

Step 1) Find the time constant for RC: $T = R_{eq} \cdot C$
 for LR: $\tau = L/R_{eq}$

Step 2) Evaluate the initial value

for L: i_0 is often ϕ at $t=0$

for C: V_0 is the voltage on the capacitor at $t=0$

Step 3) Evaluate the final value at t very large

Capacitors are open circuits at t large

Inductors are short circuits at t large

Step 4) $x(t) = x_f + (x_0 - x_f) e^{-t/\tau}$

for cap: $v(t) = 0 + (v_0 - 0) e^{-t/RC} \rightarrow v_0 e^{-t/RC}$

for ind: $I(t) = \frac{V}{R} + (0 - \frac{V}{R}) e^{-\frac{Rt}{L}} \rightarrow \frac{V}{R} - \frac{V}{R} e^{-Rt/L}$

Transient Analysis: Step by Step Procedure

Step 1: Find the time constant

Cap: $T = R_{eq} \cdot C$

Ind: $T = L / R_{eq}$

Step 2: Find Initial Value X_0

Cap: $V_{s0-} = V_{s0+}$

Ind: $I_{s0-} = I_{s0+}$

Step 3: Find the Final Value X_F (> 5 time constants later)

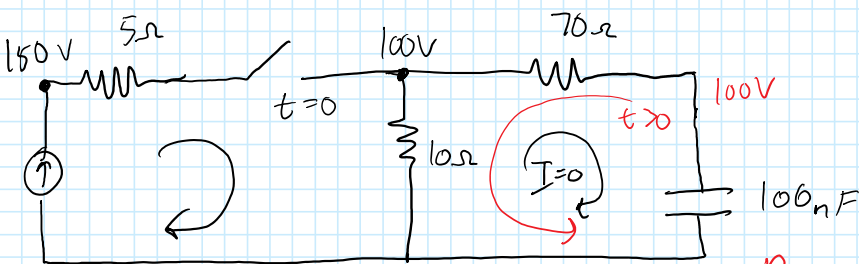
Step 4: $X_t = X_F + (X_0 - X_F) e^{-t/\tau}$

V_t

I_t

$X_t = X_F + (X_0 - X_F) e^{-t/\tau}$

HW #2



$100V = V_{C0-} = V_{C0+}$

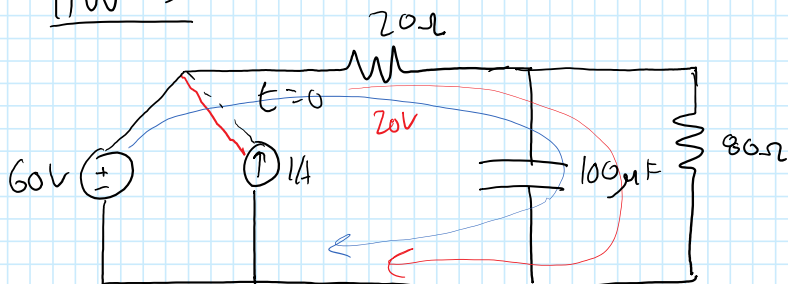
$R_{eq} = 70 + 10 \quad \tau = 100nF \times 80\Omega$

$V_{CF} = 0V$

$V_C(t) = 0 + (100 - 0) e^{-t/\tau}$

$V_C(t) = 100 e^{-t/\tau}$

HW #3



$V_{C0} = \frac{60(80)}{80+20} = 48V$

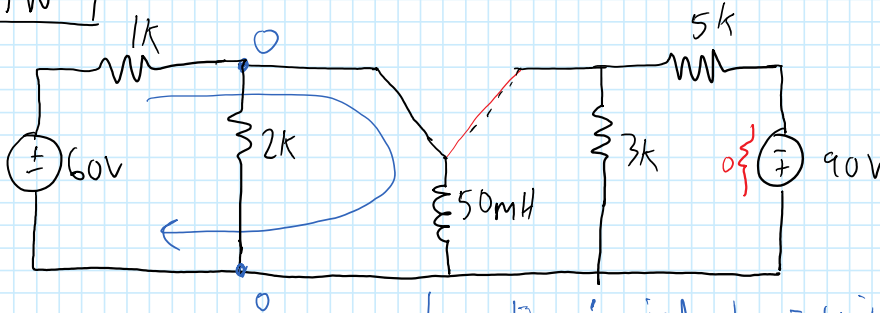
$V_{C0+} = 48V$

$V_{CF} = 80V$

$$x(t) = 80 + (-32)e^{-t/\tau}$$

$$\tau = 100 \mu\text{F} \cdot (80 \Omega)$$

HW #4



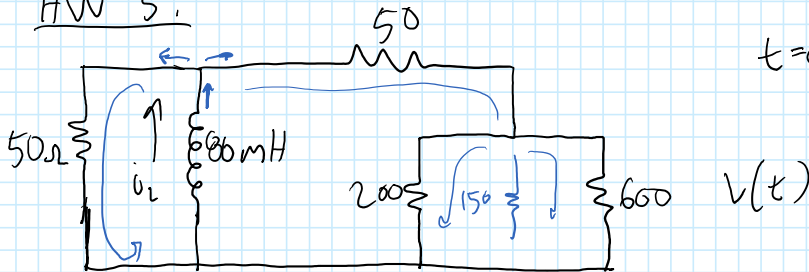
$$I_L(t_0) = I_L(t_0^+) = \frac{\text{long time: inductor = wire}}{60 \text{ V}} = \frac{60 \text{ V}}{1 \text{ k}} = 0.06 \text{ A} = I_0$$

$$R_{\text{eq}} = \frac{3 \times 5}{3+5} = 1.875 \text{ k}$$

$$\tau = \frac{0.05 \text{ H}}{1.875 \text{ k}}$$

$$I_f; \text{ final, inductor = wire, so } \bar{v} = \frac{-90 \text{ V}}{5 \text{ k}} = -0.018 \text{ A}$$

HW #5:



$t=0$ is 5A

$$I_{0+}(t) = 5 \text{ A}$$

$$R_{\text{eq}} = 150 + 50 \parallel 50$$

$$R_{\text{eq}} = \frac{200(50)}{250} \quad R_{\text{eq}} = 40 \Omega$$

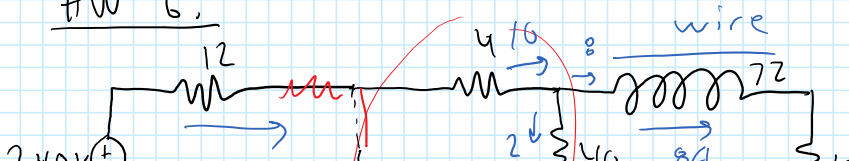
$$\tau = \frac{L}{R_{\text{eq}}} = \frac{0.08 \text{ H}}{40 \Omega}$$

$$I_f = 0 \text{ A} \quad I(t) = 0 + (5 - 0)e^{-t/\tau}$$

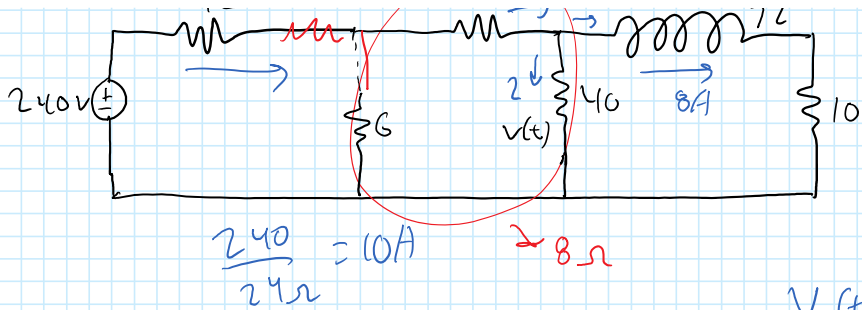
$$V_{tF} = 0 \text{ V}$$

$$V_{tU} =$$

HW #6:



$$R_{\text{eq}} = 10(40) = 8 \Omega$$



$$R_{eq} = \frac{10(40)}{50} = 8\Omega$$

$$I_o = \frac{10(40)}{50} = 8A$$

$$V_o(t) = 40(2) = 80V$$



$$X_F = 0$$

$$X(t) = 0 + (80 - 0)e^{-t/\tau}$$

Sequential Switching

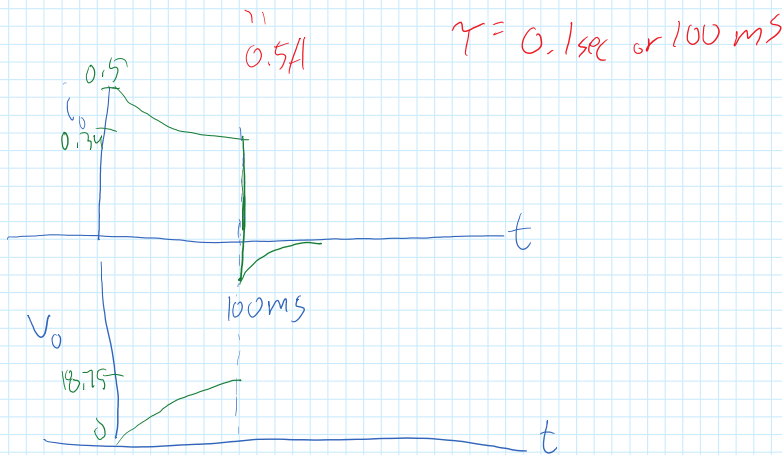
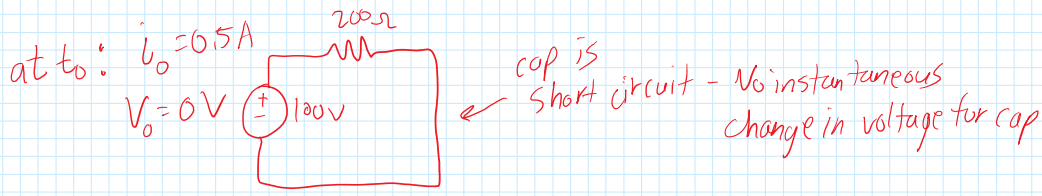
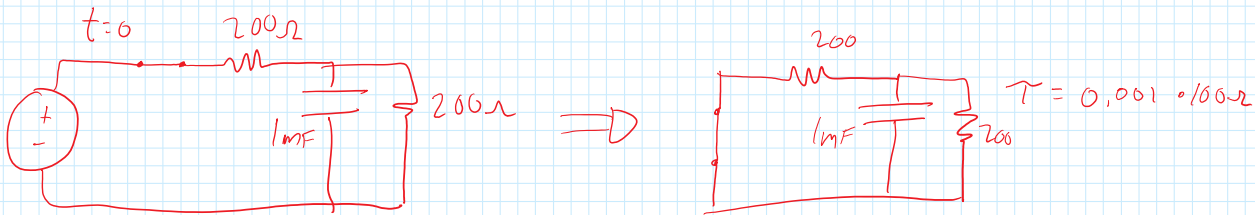
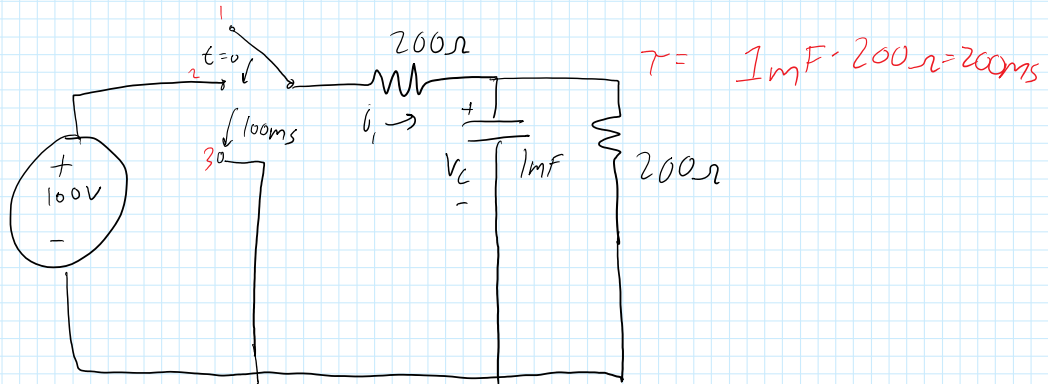
Friday, October 14, 2016 9:02 AM

General solution

$$X(t) = X_{Final} + [X_{initial} - X_{Final}] e^{-t/\tau}$$

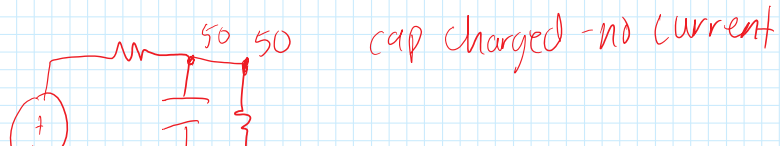
Find $i_1(t)$

Find $V_o(t)$



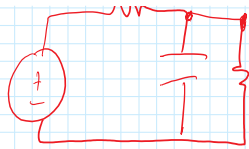
$V_{C \text{ Final}} = 50V$

$i_{\text{final}} = 0.25A$



• Circuit

$$i_{final} = 0.25A$$



$$i(t) = 0.25 + [0.5 - 0.25]e^{-t/\tau}$$

$$i(t) = 0.25 + 0.25e^{-t/\tau}$$

$$i(0.1) = 0.25 + 0.25e^{-0.1/0.1}$$

$$i(0.1) = 0.3425 A$$

$$v_c(t) = 50 + [0 - 50]e^{-t/\tau}$$

$$v_c(0.1) = 50 - 50e^{-0.1/0.1}$$

$$v_c(0.1) = 18.5 V$$

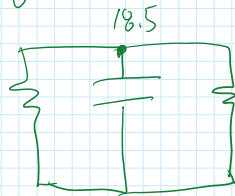
$$i_{new0+} = -0.0925A \quad i_{f(new)} = 0$$

$$v_{cnew0+} = 18.5 V$$

$$v_{f(new)} = 0$$

$$\frac{18.5}{200}$$

$$= 0.0925A$$



$$\tau = 0.1 \text{ sec}$$

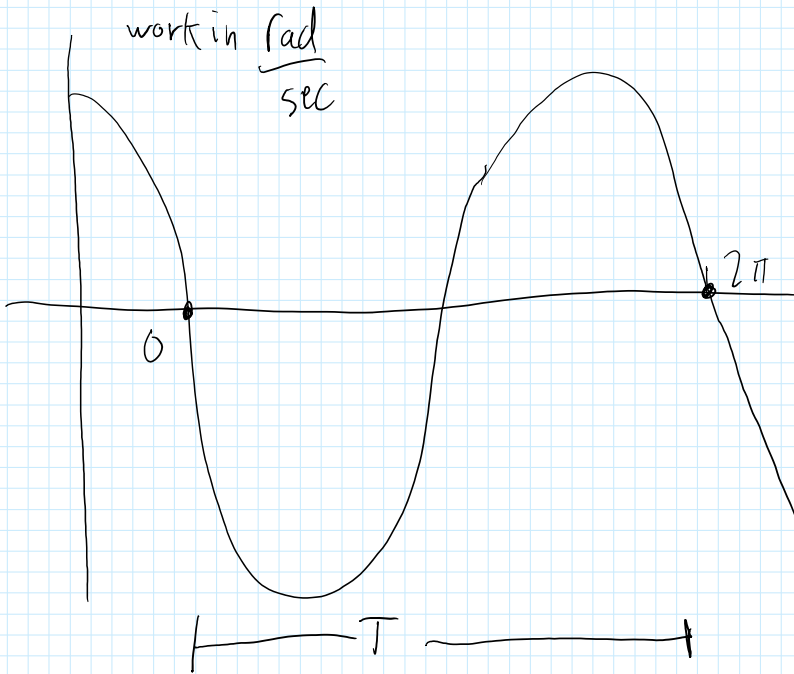
$$i_{new}(t) = 0 + [-0.0925 - 0]e^{-t/0.1}$$

$$i_{new}(t) = -0.0925e^{-t/0.1}$$

$$v_{new}(t) = 0 + [18.5 - 0]e^{-t/0.1}$$

$$v_{new}(t) = 18.5e^{-t/0.1}$$

$$f = \text{frequency} = \frac{1}{T} = \frac{\text{cycles}}{\text{sec}} = \text{Hz}$$



$$v(t) = \text{Amplitude} - VA$$

$T = \text{period of sine wave} = \text{sec}$

$\omega = \text{radian Frequency; /sec}$

$$\omega = 2\pi f$$

$$v(t) = \cos(\omega t + \phi)$$

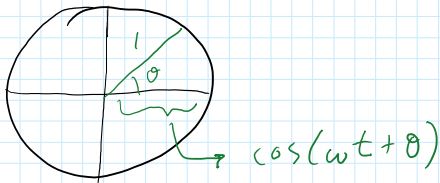
Note: $\sin \theta = \cos \theta + 90^\circ$

Euler's Identity $e^{j\phi} = \cos \phi + j \sin \phi$

$$j = \sqrt{-1} \quad j^2 = -1$$

$$v(t) = \text{Re} \{ V_m e^{j(\omega t + \theta)} \}$$

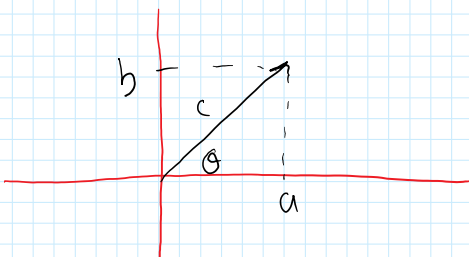
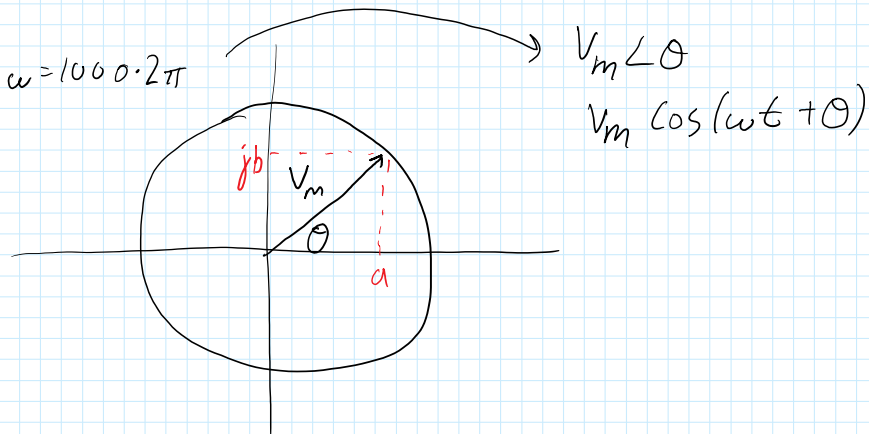
$$v(t) = \text{Re} \{ V_m \cos(\omega t + \theta) + j \sin \omega t + \theta \}$$



Phasor

$$f = 1000 \text{ Hz} \quad \omega = 1000 \cdot 2\pi$$

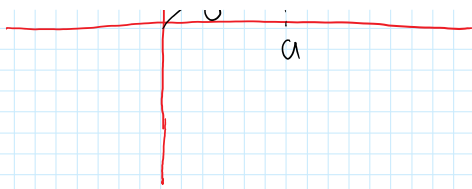
$$\bar{V} = V_m \angle \theta = V \angle$$



Rectangular: $a + jb$

Polar: $\angle \theta$

Rect \rightarrow Polar



rect. \rightarrow \angle \cup

Rect \rightarrow Polar

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Polar \rightarrow Rect

$$a = C \cos \theta$$

$$b = C \sin \theta$$

Basic Arithmetic:

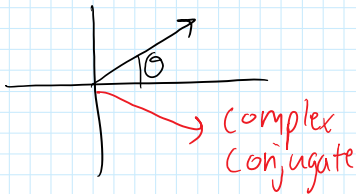
Add + Sub \rightarrow use Rectangular

Mult + Div + Exponential \rightarrow Use Polar

Complex Conjugate: $n = a + jb$

$n^* = a - jb \rightarrow$ complex conjugate

$$n^k = (a + jb)^k = (C e^{j\theta})^k = C^k e^{j\theta k} = (\overline{V} \angle)^k = \overline{V}_m^k \angle k\theta$$



$$n_1 = 4 + j3 = 5 \angle 36^\circ$$

$$n_2 = 13 \angle 67.4^\circ = 5 + j12$$

$$n_1 + n_2 = 4 + j3 + 5 + j12 = 9 + j15 = 17.5 \angle 59^\circ$$

$$\frac{n_1}{n_2} = \frac{4 + j3}{5 + j12} = \frac{5 \angle 36^\circ}{13 \angle 67.4^\circ} = \frac{5}{13} \angle -30.51^\circ$$

$$(5 \angle 36.9^\circ)^4 = 5^4 \angle 36.9^\circ \cdot 4 = 625 \angle 147.48^\circ$$

$$(n_1^*)(n_2) = (5 \angle -36^\circ)(13 \angle 67.4^\circ) = 65 \angle 31.4^\circ$$

Circuit Elements in the Frequency Domain

Wednesday, October 19, 2016 9:02 AM

$$V(t) = V_m \cos(\omega t + \theta) \quad \text{Re} \{ V_m e^{j(\omega t + \theta)} \}$$

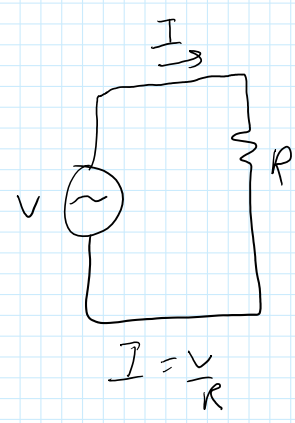
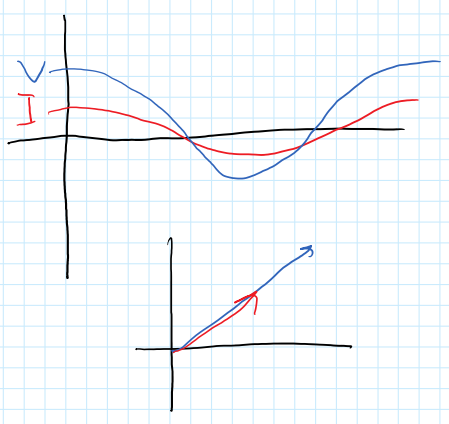
$$i(t) = I_m \cos(\omega t + \theta) \quad \text{Re} \{ I_m e^{j(\omega t + \theta)} \}$$

$$\frac{V_m e^{j\omega t + \theta_i}}{e^{j\omega t}} = \frac{RI e^{j\omega t + \theta_i}}{e^{j\omega t}}$$

$$V_m e^{j\theta_i} = RI e^{j\theta_i}$$

$$V_L = RI_L$$

$$\theta_v = \theta_i$$



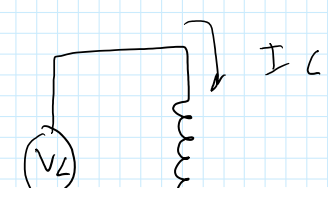
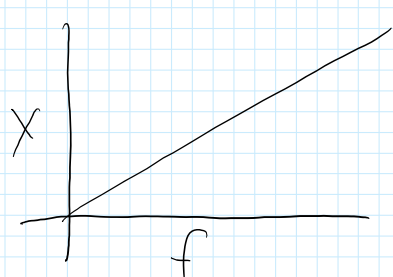
$$I_L = \frac{V_L}{R}$$

Inductor: $V_L = L \frac{di}{dt}$ $L d I_m e^{j(\omega t + \theta)}$

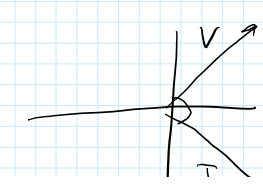
$$\frac{V e^{j(\omega t + \theta_v)}}{e^{j\omega t}} = L j\omega I e^{j(\omega t + \theta_i)} dt$$

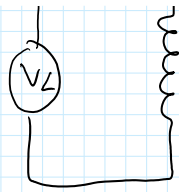
$$V e^{j\theta_v} = j\omega L I e^{j\theta_i}$$

$$V_L = \underbrace{j\omega L I}_{X_L} \rightarrow \text{inductive reactance, frequency dependent, in ohms}$$



$$I_L = \frac{V_L}{j\omega L} = \frac{-j}{\omega L} V_L$$



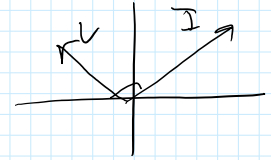


$$+ \sim \frac{-jV}{\omega L} \sim -j$$

$$I = \frac{-jV}{\omega L}$$



$$V = j\omega L I$$



Capacitor: $i(t) = C \frac{dV(t)}{dt}$

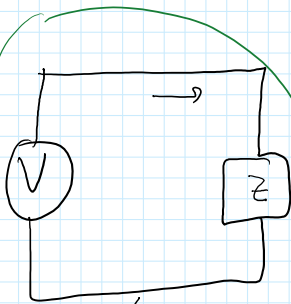
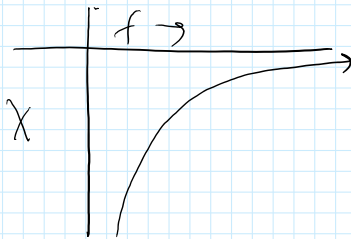
$$I_m e^{j(\omega t + \theta_v)} = C \frac{dV_m e^{j(\omega t + \theta_i)}}{dt}$$

$$\frac{I_m e^{j(\omega t + \theta_i)}}{e^{j\omega t}} = \frac{j\omega C V_m e^{j(\omega t + \theta_v)}}{e^{j\omega t}}$$

$$I_m e^{j\theta_i} = C j\omega V_m e^{j\theta_v}$$

$$\frac{1}{j\omega C} = \frac{V}{I} = jX$$

$$X = \frac{1}{j\omega C} \quad X_C = \frac{-j}{\omega C}$$



$$I = \frac{V}{Z}$$

$$\theta_i = \theta_v$$

$$I = \frac{V}{jX_C}$$

$$\theta_i = \theta_v - 90^\circ$$

Resistor

$$V = IR$$

$$V_{\theta_v} = I_{\theta_i} R$$

$$\theta_v = \theta_i \quad \text{for resistors}$$

Capacitor

$$V = I(-jX_C)$$

$$V_{\theta_v} = I_{\theta_i} (X_C - 90^\circ)$$

$$\theta_v = \theta_i - 90^\circ$$

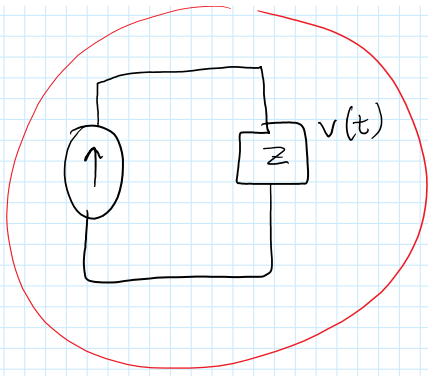
Inductor

$$V = I jX_L$$

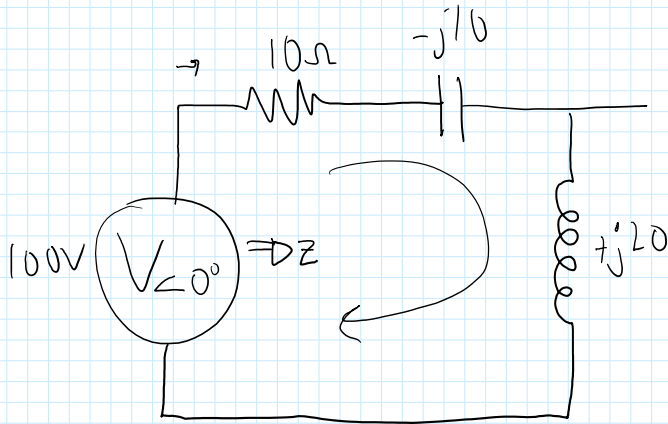
$$V_{\theta_v} = I_{\theta_i} (X_L + 90^\circ)$$

$$\theta_v = \theta_i + 90^\circ$$

$$I = \frac{V}{jX_C} \cdot \frac{j}{j} = \frac{jV}{X_C} \quad \theta_i = \theta_v + 90^\circ$$



$$I = \frac{V}{-jX_C} \cdot \frac{j}{j} = \frac{jV}{X_C} \quad V_i = 0_v + 10$$



$$Z = R \pm jX$$

$$Z = 10 - j10 - j20 = 10 + j10$$

$$\hat{i} = \frac{100V}{10 + j10}$$

$$V_{out} = \frac{j20(100V)}{10 + j10}$$

- Reactance of Elements is frequency dependent

- Impedance: Resistor

$$R$$

Inductor

$$j\omega L$$

Capacitor

$$\frac{-j}{\omega C}$$

Element	Impedance	Reactance	Conductance Susceptance	Admittance
Resistor	$R + j0$	$j0$	$\frac{1}{R} = G$	$\frac{1}{R} + j0$
Inductor	$0 + j\omega L$	$j\omega L$	$\frac{1}{j\omega L} = \frac{-j}{\omega L}$	$0 - \frac{j}{\omega L}$
Capacitor	$0 - \frac{j}{\omega C}$	$\frac{-j}{\omega C}$	$\frac{1}{-j/\omega C} = j\omega C$	$0 + j\omega C$

Sinusoidal Steady State Analysis

- 1) Identify the frequency
- 2) Convert from the time domain to the frequency domain
- 3) Solve the linear system:

$$\underline{V} = \underline{I} Z$$

$$\underline{I} = \frac{\underline{V}}{Z} = \underline{I} = \underline{V} \cdot \underline{Y} \rightarrow \text{(admittance)}$$

- 4) When requested, convert back to the time domain

$$V \angle \theta \Rightarrow V_m \cos(\omega t + \theta)$$

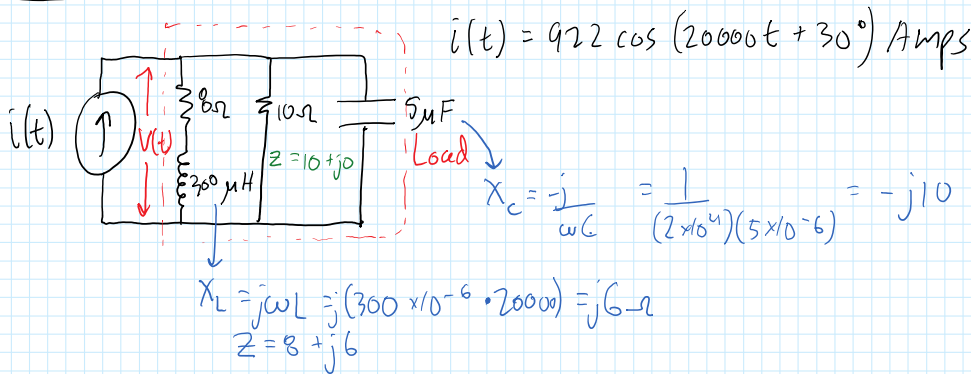
$$v(t) = V_m \cos(\omega t + \theta) \quad \text{frequency} = \omega = 2\pi f$$

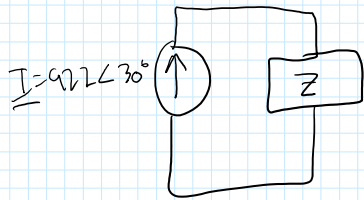
$$i(t) = I_m \cos(\omega t + \theta) \quad \text{Angle} = \theta$$

\downarrow magnitude \downarrow phase \angle
 $f = 1000 \text{ Hz}$
 $\omega = 2\pi f = 6280 \frac{\text{rad}}{\text{sec}}$

Phasor $V = V_m \angle \theta$

$I = I_m \angle \theta$





$$Z = \frac{1}{\frac{1}{8+j6} + \frac{1}{j6} + \frac{-1}{j10}}$$

$$Y = (0.08 - j0.06) + 0.1 + 0.1j = 0.18 + 0.04j$$

$$Z = \frac{1}{Y} = \frac{1}{0.18 + 0.04j} = 5.294 - j1.176$$

$$Z = \frac{1}{0.184 \angle 12.52^\circ} \quad \boxed{Z = 5.423 \angle -12.52^\circ}$$

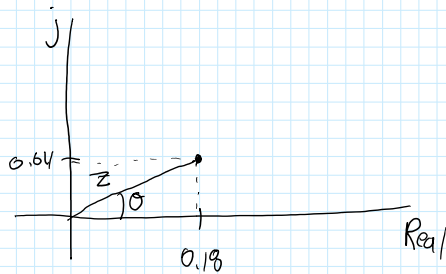
$$\underline{V} = (922 \angle 30^\circ) \cdot (5.423 \angle -12.52^\circ)$$

$$\boxed{\underline{V} = 5000.24 \angle 17.47^\circ}$$

time domain
equivalent \rightarrow

$$\boxed{V(t) = 5000.24 \cos(2000t + 17.47^\circ)}$$

$$\frac{1}{0.18 + j0.04}$$

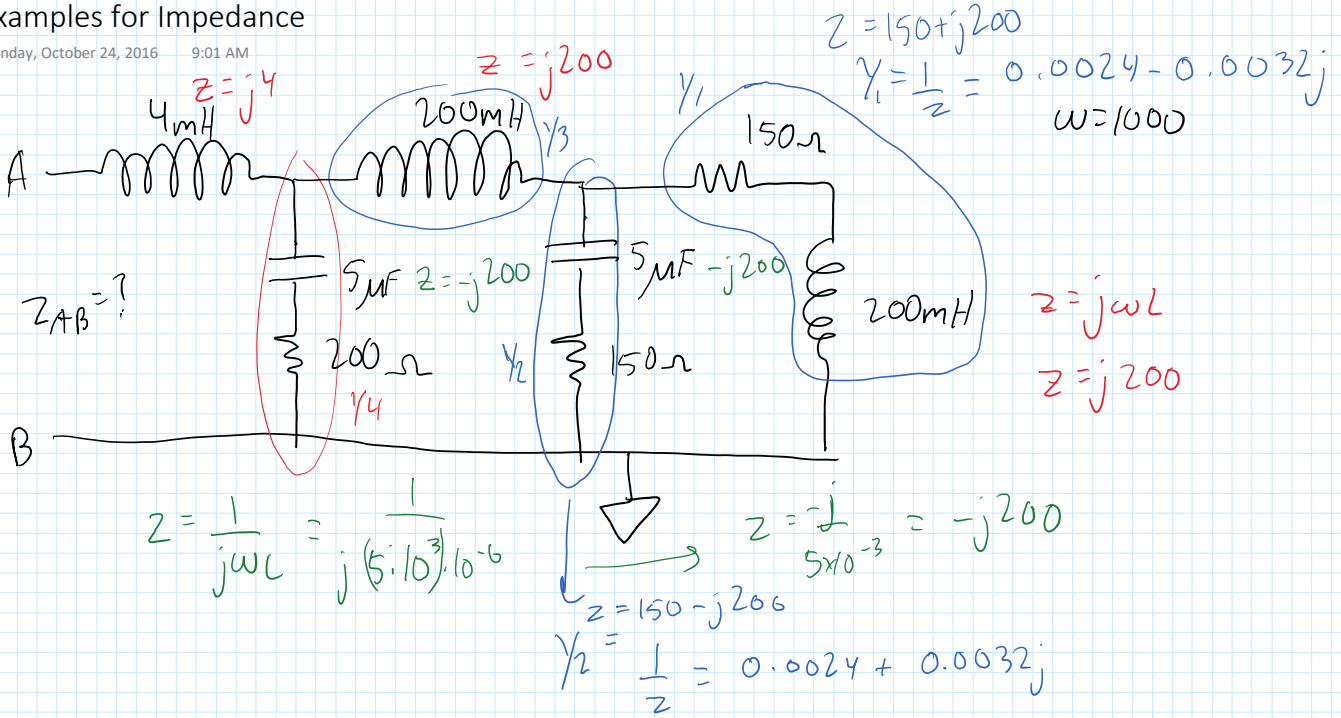


$$|Z| = \sqrt{0.18^2 + 0.04^2} = 0.184$$

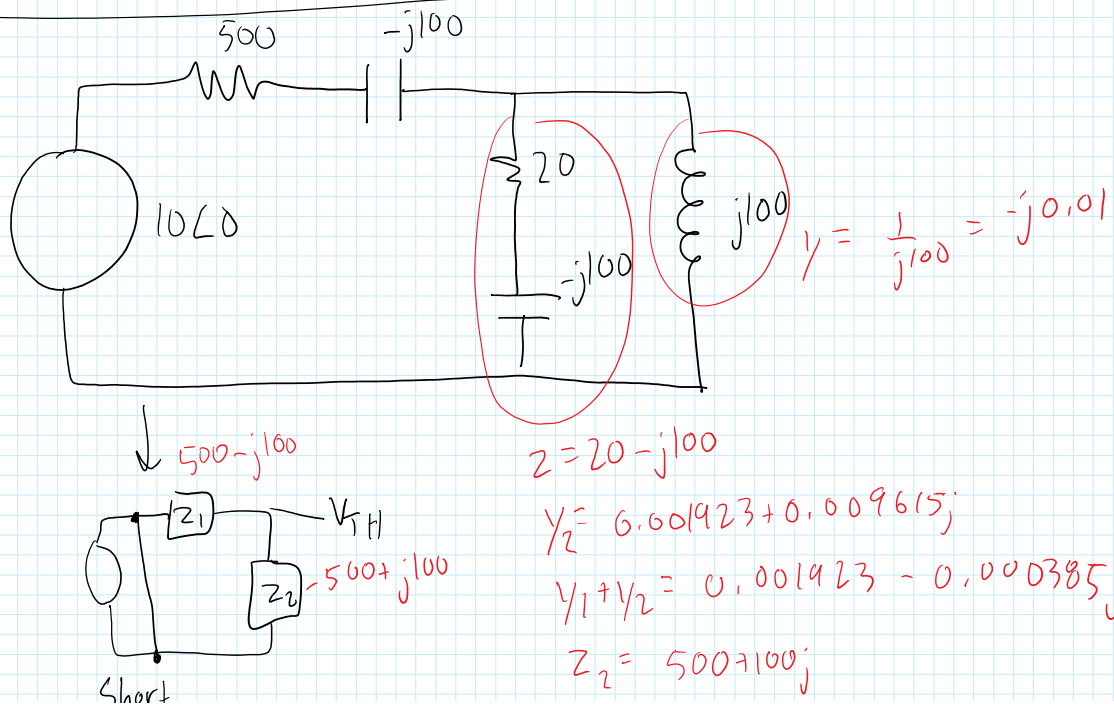
$$\theta = \tan^{-1}\left(\frac{0.04}{0.18}\right) = 12.528^\circ$$

Examples for Impedance

Monday, October 24, 2016 9:01 AM



$Y_T = Y_1 + Y_2 = 0.0048$
 $Z_T = \frac{1}{Y_T} = 208.3 \Omega$
 $Y_3 = Y_4 = (208.3 + j200)^{-1} = 0.002498 - 0.002398j$
 $Y_4 = (200 - j200)^{-1} = 0.0025 + 0.0025j$
 $Y_T = Y_3 + Y_4 = 0.004998 + 0.000102j$
 $Z_T = 200 - 4.08163j$
 $Z_{AB} = 200 - 4.08163j + j4$
 $Z_{AB} = 200 - 0.08163j = 200 \angle -0.02j$
 $Z_{AB} = 200 \angle -1.169$



Short
out source
for Z_{th}

$$Z_2 = 500 + j100$$

$$V_{TH} = \frac{10(Z_2)}{Z_1 + Z_2} = \frac{10(500 + j100)}{500 - j100 + 500 + j100} = \frac{10(500 + j100)}{1000}$$

$$\boxed{V_{TH} = 5 + j1} = 5.099 \angle 11.31^\circ = \overline{V_{TH}}$$

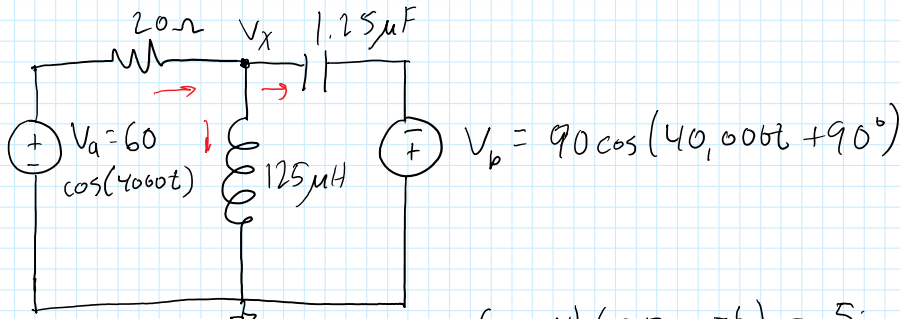
$$Z_{th} = Z_1 \parallel Z_2$$

$$Z_{th} = 500 - j100 \parallel 500 + j100$$

$$\boxed{Z_{th} = 260 \Omega}$$

More Transient Circuits

Wednesday, October 26, 2016 9:04 AM



$$\omega = 4000$$

$$X_L = j\omega L = (4 \times 10^4)(125 \times 10^{-6}) = 5j \Omega$$

$$X_C = \frac{-j}{\omega C} = \frac{-j}{(4 \times 10^4)(1.25 \times 10^{-6})} = -j20$$

$$V_a = 60 \angle 0^\circ$$

$$V_b = 90 \angle 90^\circ$$

$$\frac{V_a - V_x}{20 \Omega} = \frac{V_x - V_b}{j20 \Omega} + \frac{V_x}{j5 \Omega}$$

$$\frac{(60 \angle 0) - V_x}{20} = \frac{V_x - (90 \angle 90^\circ)}{-j20} + \frac{V_x}{j5}$$

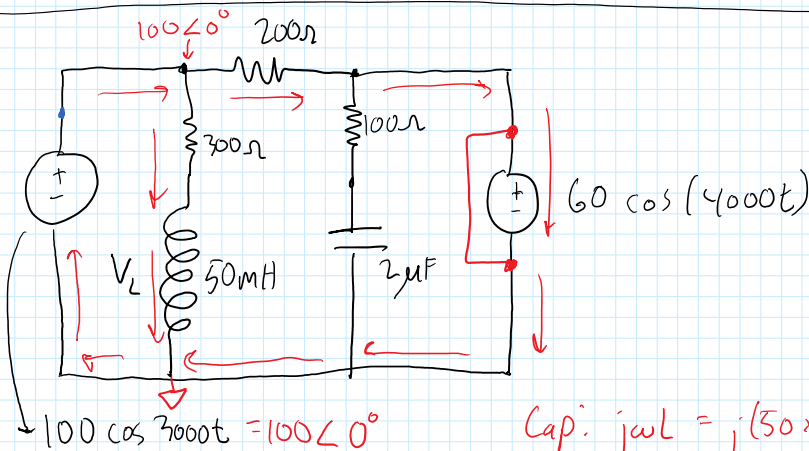
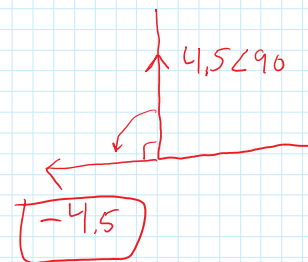
$$3 - 0.05 V_x = 0.05j V_x + j4.5 \angle 90^\circ - j0.2 V_x$$

$$7.5 - 0.05 V_x = 0.05j V_x - j0.2 V_x$$

$$7.5 = 0.05 V_x - 0.15j V_x$$

$$V_x = \frac{7.5}{0.05 - j0.15}$$

$$V_x = 47 \angle 71.56^\circ$$



Superposition

$\omega = 3000$

$$100 \cos 3000t = 100 \angle 0^\circ$$

$$\text{Cap: } j\omega L = j(50 \times 10^{-3})(3000) = j150$$

$$V_L = \frac{(100 \angle 0^\circ)(j150)}{300 + j150}$$

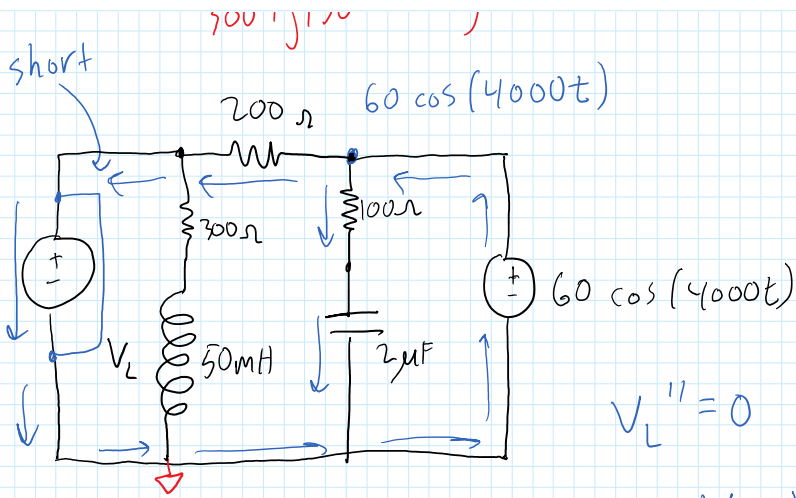
} voltage divider

$$V_L' = 44 \angle 63.4^\circ$$

$$V_L'(t) = 44 \cos(3000t + 63.4^\circ)$$

short

$$60 \cos(4000t)$$



$$V_L'(t) = 44 \cos(3000t + 63.4^\circ)$$

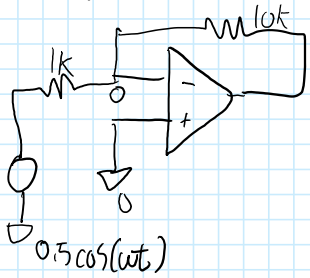
$$V_L'' = 0$$

$$V_L = V_L'' + V_L'$$

$$V_L(t) = 44 \cos(3000t + 63.4^\circ)$$

Exam 2 Review

Friday, October 28, 2016 9:00 AM

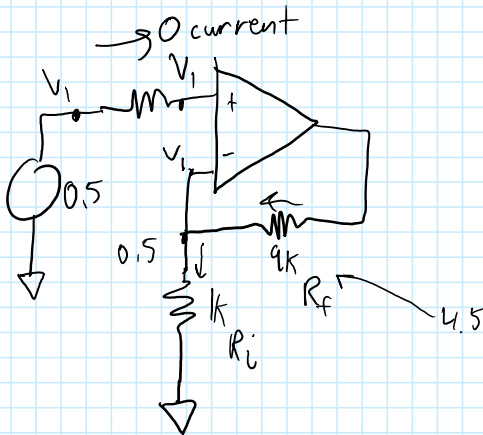


inverting (negative feedback)

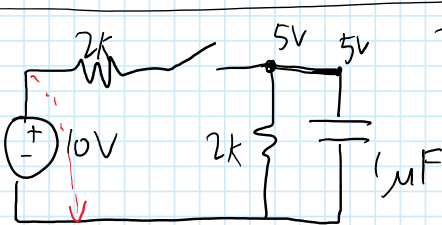
$$\text{Gain} = \left(-\frac{R_f}{R_i} \right) \quad \text{Gain} = -10$$

1) OP AMPS

$$\text{Output} = 5 \cos(\omega t + 180^\circ) \text{ or } -5 \cos(\omega t)$$



$$\text{Gain} = \left(\frac{R_f}{R_i} + 1 \right)$$



$t=0$ switch closes

2) Transient Response

$$V_{\text{initial}} = 0 \text{ V}$$

$$\tau = R_{\text{eq}} C$$

$$V_{\text{final}} = 5 \text{ V}$$

$$\tau = (1\text{k})(10^{-6}) = 10^{-3}$$

$$V(t) = V_f + (V_i - V_f) e^{-t/\tau}$$

$$V(t) = 5 + (0 - 5) e^{-t/10^{-3}}$$

$$V(t) = 5 - 5 e^{-t/0.001}$$

$2\text{k} \parallel 2\text{k} = 1\text{k}$

$$\bar{V} = V_m \angle \theta \rightarrow \text{frequency domain}$$

$$v(t) = V_m (\cos \omega t + \theta) \rightarrow \text{time domain}$$

$$\omega = 2\pi f = \frac{\text{rad}}{\text{sec}}$$

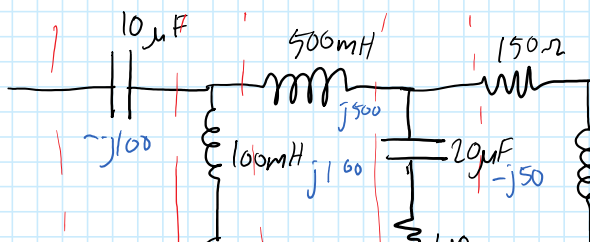
Impedance: $Z = R + x$

$$X_C = \frac{-j}{\omega C}$$

$$X_L = j\omega L$$

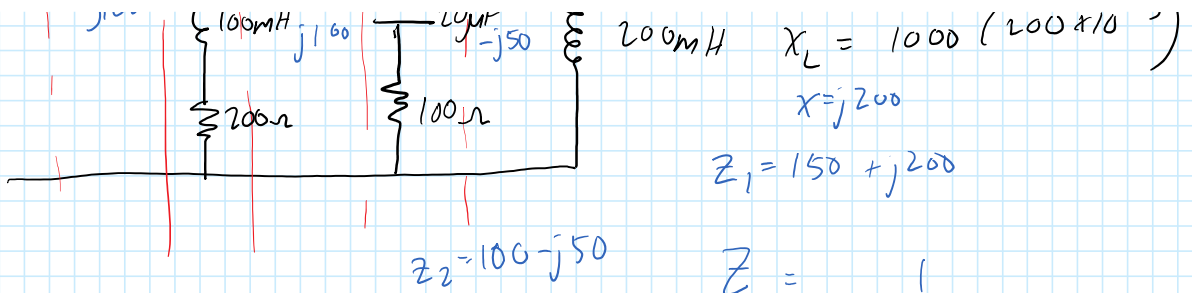
Admittance: $Y = \frac{1}{Z}$

Ex:



$$X_L = 1000 \quad (200 \times 10^{-3})$$

$x = j200$

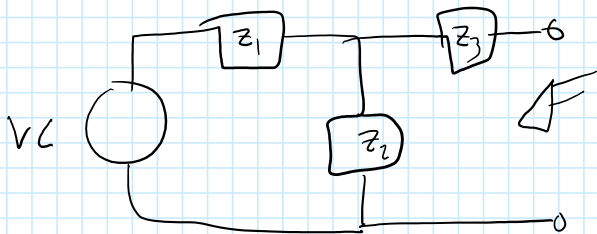


$$Z_3 = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$$

$$Z_4 = Z_3 + j500$$

$$Z_5 = \left((Z_4)^{-1} + (200 + j100)^{-1} \right)^{-1}$$

$$Z_{AB} = Z_5 - j100$$



$$V_{TH} = \frac{V_C (Z_2)}{Z_1 + Z_2}$$

$$Z_{TH} = Z_3 + (Z_2 || Z_1)$$

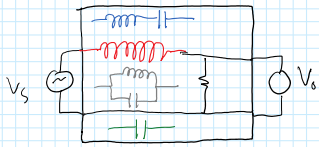
Power: 50/60 Hz + harmonics

Communications/Control

Frequency Spectrum

Transfer Functions

Steady State Analysis



$$V_o = \frac{V_s(R)}{R + jX_L}$$

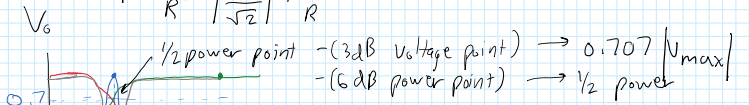
$$V_o = \frac{V_s(R)}{R - jX_C}$$

$$V_o = \frac{V_s(R)}{R + jX_L - jX_C}$$

$$V_o = \frac{V_s(R)}{R + \left(\frac{1}{X_L} + \frac{1}{X_C}\right) \rightarrow X_o}$$

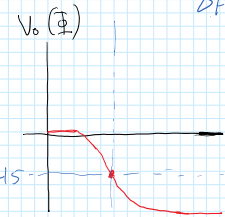
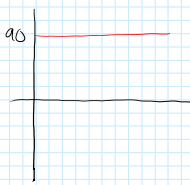
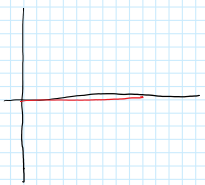
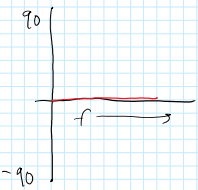
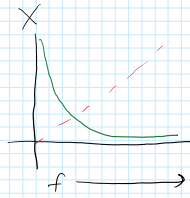
at res, $\frac{1}{Y_{res}} \rightarrow 0$ so $X_{res} = \infty$

$$P = \frac{V^2}{R} = \left| \frac{V_s}{\sqrt{2}} \right|^2 \cdot \frac{1}{R}$$



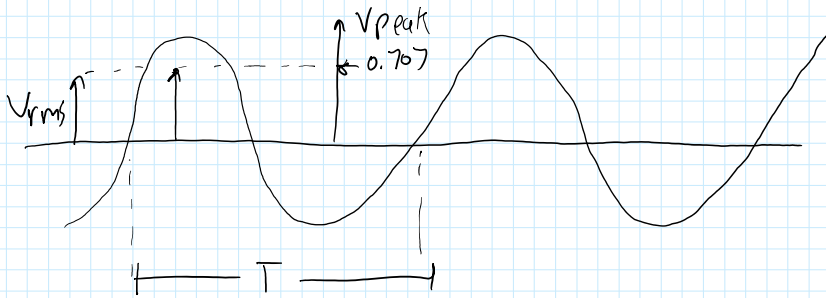
- Lowpass filter
- High pass filter
- Band pass filter
- Band reject filter

$$Q = \frac{F}{\Delta F} = \frac{X}{R}$$

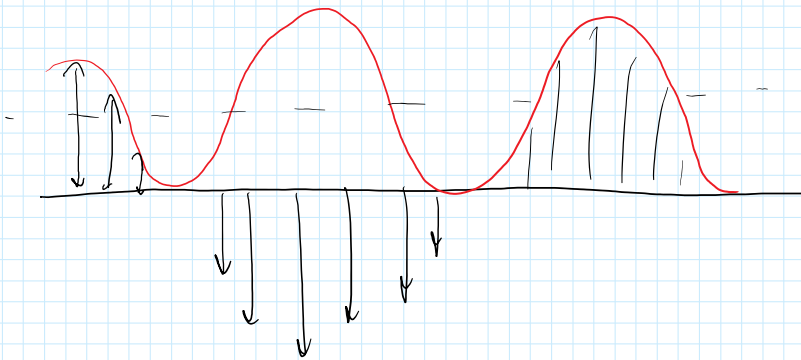


Reactive Power

Friday, November 4, 2016 9:04 AM



$$P = \frac{V^2}{R}$$

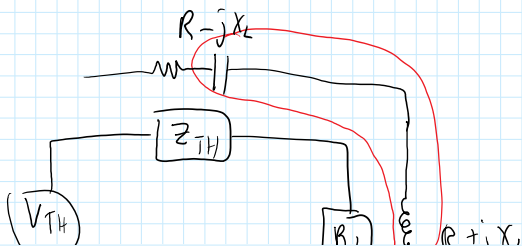
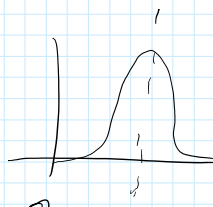
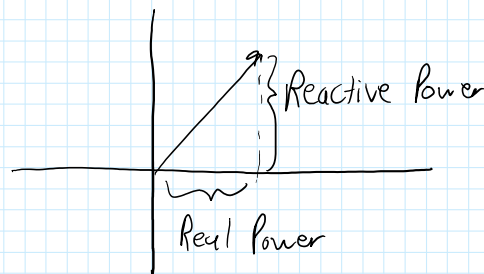
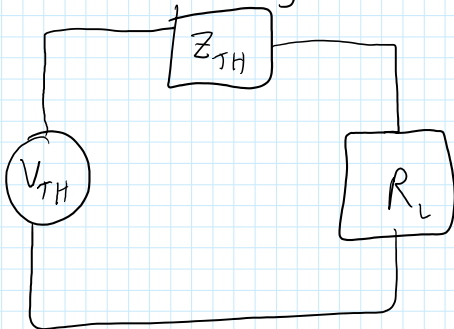


$$V_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} v^2 \cos(\omega t + \theta) dt}$$

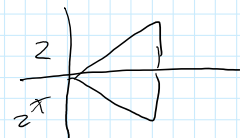
Appendix G:

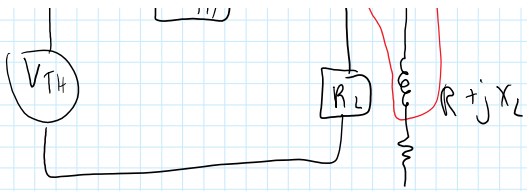
$$\int \cos^2 \alpha x dx = \frac{x}{2} + \frac{\sin^2 \alpha x}{4\alpha}$$

$$V_{rms} = \left[\frac{V_m^2}{T} \left(\frac{T}{2} \right) \right]^{1/2} = \frac{V_m}{\sqrt{2}} = \frac{V_m}{1.414} \approx 0.707 V_m$$

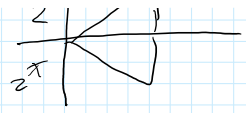


$$(R - jX_L)^* = (R + jX_L)$$





$(1 + j^2) (1 + j^2)$



Sinusoidal Steady State Power

Monday, November 7, 2016 9:02 AM

$$P(t) = v_i$$

$$V(t) = V_{max} \cos(\omega t + \theta_v)$$

$$i(t) = I_{max} \cos(\omega t + \theta_i)$$

$$\vec{V} \angle (\theta_v - \theta_i) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$\vec{I} \angle 0 = I_m \cos(\omega t)$$

$$P(t) = V_m I_m [\cos(\omega t + \theta_v - \theta_i) \cdot \cos(\omega t)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$$

$$P(t) = V_m I_m \left[\frac{1}{2} \cos(\omega t + \theta_v - \theta_i - \omega t) + \frac{1}{2} \cos(\omega t + \theta_v - \theta_i + \omega t) \right]$$

$$= V_m I_m \left[\frac{1}{2} \cos(\theta_v - \theta_i) + \frac{1}{2} \cos(2\omega t + \theta_v - \theta_i) \right]$$

$$\cos \alpha + \cos \beta = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

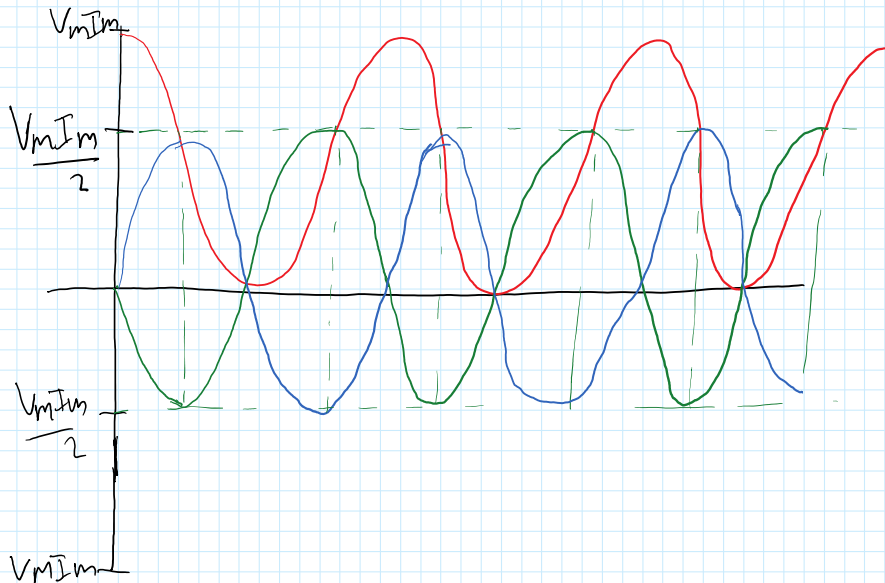
$\alpha = 2\omega t$ $\beta = \theta_v - \theta_i$

$$= \cos(\theta_v - \theta_i) \cos(2\omega t) - \sin(\theta_v - \theta_i) \sin(2\omega t)$$

$$P(t) = \frac{V_m I_m}{2} \left[\cos(\theta_v - \theta_i) \right] + \frac{V_m I_m}{2} \left[\cos(\theta_v - \theta_i) \cos(2\omega t) - \sin(\theta_v - \theta_i) \sin(2\omega t) \right]$$

For a resistor (R), $\theta_v - \theta_i = 0$

$2\omega = 2 \cdot$ line frequency [rad/sec]



For an inductor, $\theta_v - \theta_i = 90^\circ$

For a capacitor, $\theta_v - \theta_i = -90^\circ$

$$S = P + jQ$$

$$S = \frac{V_m I_m}{2} [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)] = V_m I_m e^{j(\theta_v - \theta_i)} = \frac{1}{2} V_m I_m \angle (\theta_v - \theta_i)$$

$$S = \frac{V_m I_m}{2} \angle \theta_v - \theta_i = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$I_{rms} e^{-j\theta_i} = I_{rms} (\cos \theta_i + j \sin(-\theta_i)) = I_{rms} (\cos \theta_i - j \sin \theta_i)$$

$$S = V_{rms} I_{rms}^*$$

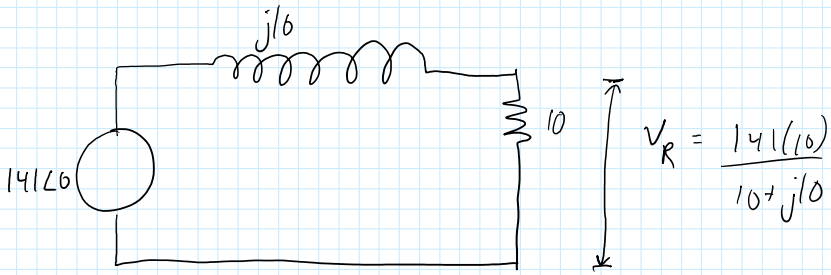
Note Also: $V_{rms} = I_{rms} (Z)$ $A \cdot A^* = |A|^2$

$$S = (I_{rms})(I_{rms})^* Z = |I_{rms}|^2 Z = \frac{V_{rms}^2}{Z}$$

$$S = P + jQ; S = |I_{rms}|^2 Z = \frac{|V_{rms}|^2}{Z}$$

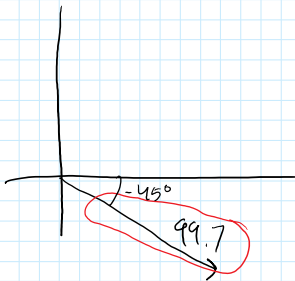
Power Factor

Wednesday, November 9, 2016 9:05 AM



$$V_R = \frac{141(10)}{10 + j6}$$

$$V_{RMS} = 0.707 V_m$$



$$S = P + jQ$$

$$S = V_{rms} + I_{rms}^*$$

$$V_{rms} = I_{rms} Z$$

$$I_{rms} Z I_{rms}^*$$

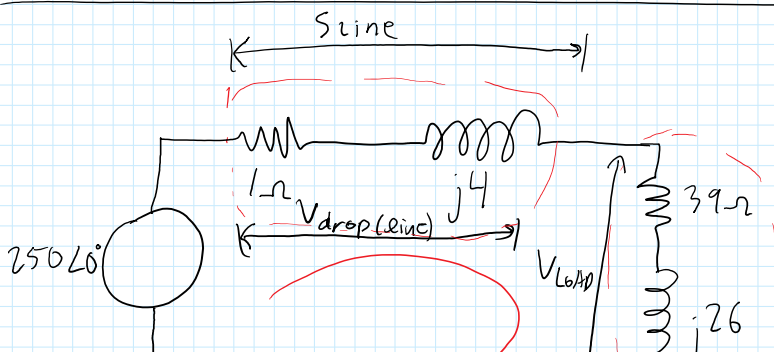
↓

$$|I_{rms}|^2$$

$$S = P + jQ$$

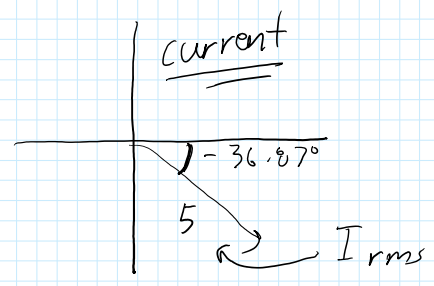
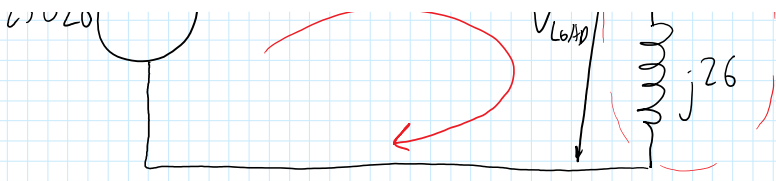
$$S = |I_{rms}|^2 R + |I_{rms}|^2 X$$

$$S = \frac{|V_{rms}|^2}{R} + \frac{|V_{rms}|^2}{X}$$



$$I = \frac{250 \angle 0^\circ}{40 + j30} = 4 - j3 \quad \underline{\underline{OR}}$$

current



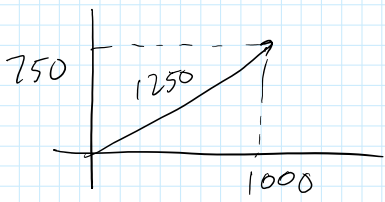
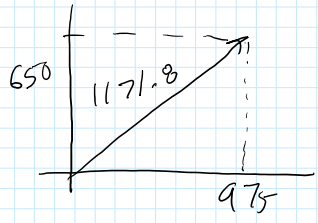
$$V_{Load} = (39 + j26)(4 - j3) = 234 - j13 \quad \text{watts} \quad \text{VAR} = \text{Volt Amperes Reactive}$$

$$S_{Load} = V_L I^* = (234 - j13)(4 + j3) = 975 + j650$$

$$S_{Line} = I_{Line} Z_{Line} I_{Line}^* = (4 - j3)(1 + j4)(4 + j3) = 25 \text{ watts} + j160 \text{ VAR}$$

$$S_{Total} \rightarrow 1000 + j750$$

$$S = 1250 \angle 36.87 = 1000 + j750$$



OR

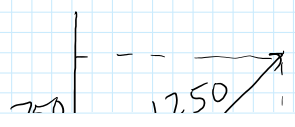
$$S_{Line} = I_{rms}^2 R + I_{rms}^2 X = (5)^2 \cdot 1 + (5)^2 (j4) = 25 + j100 \checkmark$$

$$S_{Load} = I_{rms}^2 R + I_{rms}^2 X = (5)^2 \cdot 39 + (5)^2 (-j26) = 975 - j650 \checkmark$$

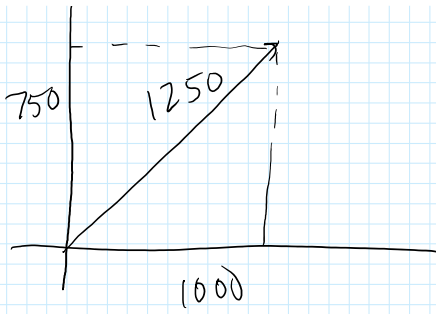
$$\underline{1000 + j750 \checkmark}$$

Leading Implies Current Leads Voltage (Inductive)

Lagging Implies Current Lags Voltage (Capacitive)



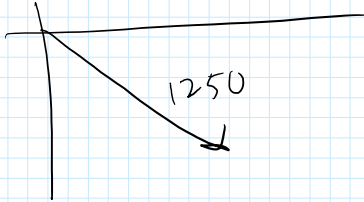
$$P.F. (\text{Power Factor}) = \frac{1000 \text{ watts}}{1250 \text{ watts}} = 0.8$$



P.F. (Power Factor)
Leading

$$\frac{1000 \text{ watts}}{1250 \text{ watts}} = 0.8$$

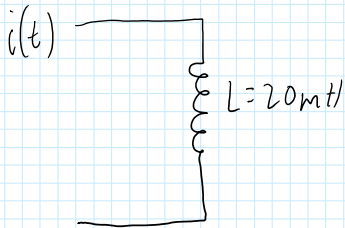
$$36.87 = \cos^{-1}\left(\frac{1000}{1250}\right)$$



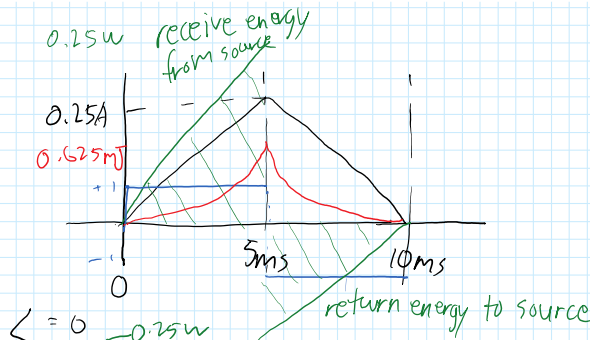
Lagging

Mutual Inductance

Monday, November 14, 2016 9:06 AM



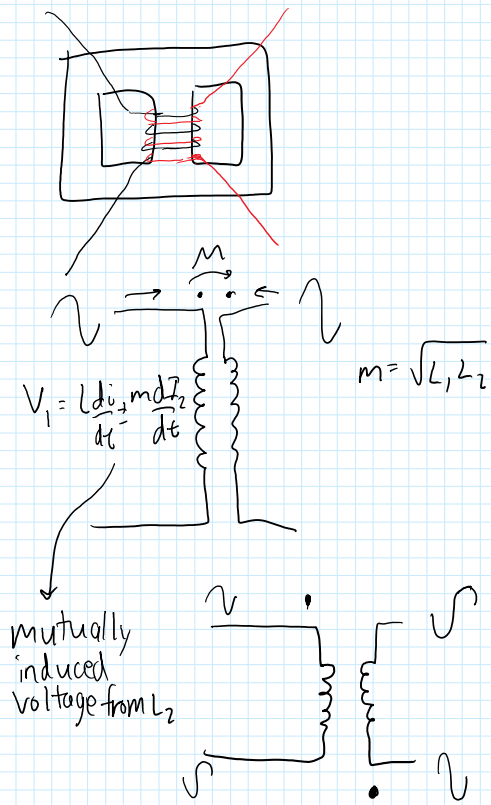
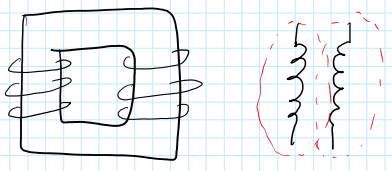
$$V(t) = L \frac{di(t)}{dt}$$



Energy = Power * Time = $\frac{1}{2} L I^2$

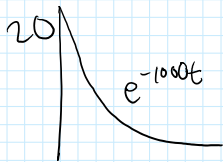
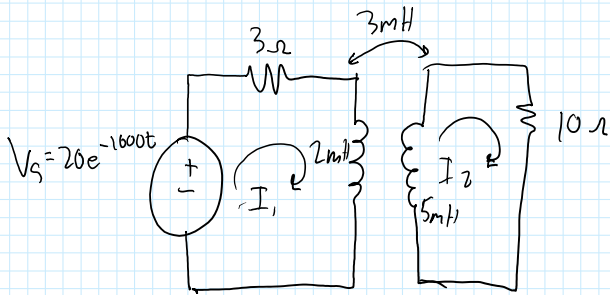
$L = 0$
 $0 - 5 \frac{di(t)}{dt} = 50 \frac{A}{s}$
 $\frac{250 \mu\text{A}}{5 \text{ ms}} = \frac{50 \text{ A}}{s}$
 $L \frac{di(t)}{dt} = 20 \times 10^{-3} \times 50 = +1 \text{ Volt}$
 $> 10 \text{ ms} = 0$
 $5 - 10 \frac{di(t)}{dt} = 0.25 - 50t$
 $L \frac{di(t)}{dt} = 20 \times 10^{-3} \times (-50) = -1 \text{ Volt}$

Power = $V(t) I(t)$



$$V_1 = L \frac{dI_1(t)}{dt} + m \frac{dI_2(t)}{dt}$$

$$V_2 = L \frac{dI_2(t)}{dt} + m \frac{dI_1(t)}{dt}$$



Mesh 1

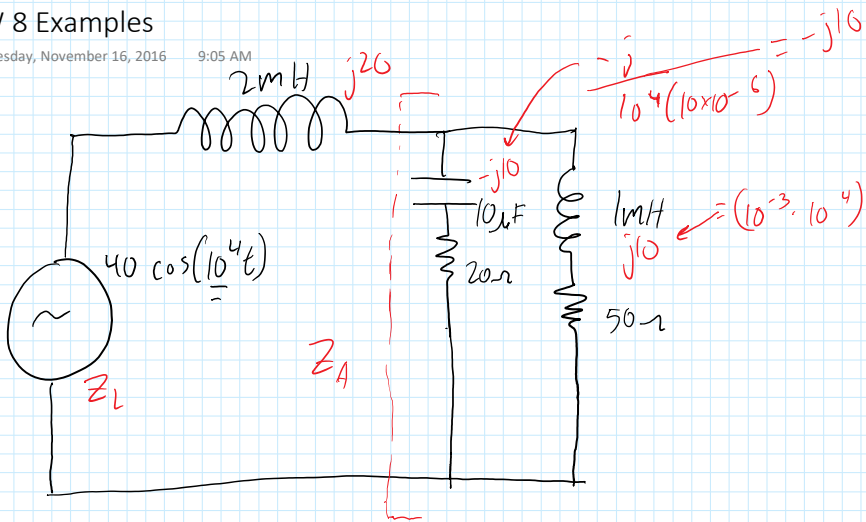
$$-V_s + 3i_1 + 2 \times 10^{-3} \frac{dI_1(t)}{dt} - 3 \times 10^{-3} \frac{dI_2}{dt} = 0$$

Mesh 2

$$10i_2 + 5 \times 10^{-3} \frac{dI_2}{dt} - 3 \times 10^{-3} \frac{dI_1}{dt} = 0$$

HW 8 Examples

Wednesday, November 16, 2016 9:05 AM



$$Z_L = 2mH + Z_A$$

$$Z_A = \left((50 + j10)^{-1} + (20 - j10)^{-1} \right)^{-1}$$

$$Z_L = 15.7 + j15.7$$

$$= 15.7 - j4.28$$

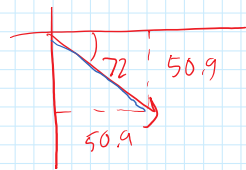
$$S = \frac{V^2}{Z_L} = \frac{40^2}{15.7 + j15.7}$$

a) Real and Reactive Power

$$S = 50.9 - j50.9$$

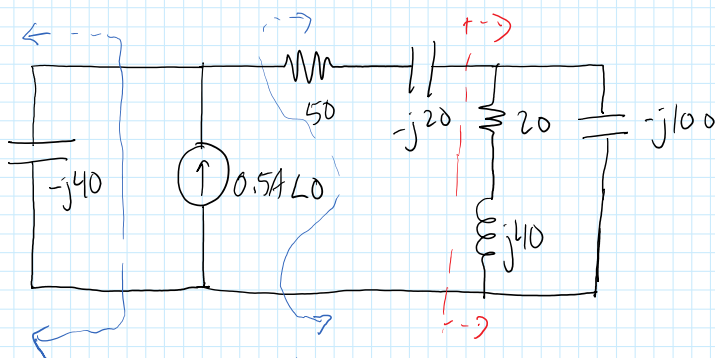
b) Apparent power

$$S = 72 \angle -45^\circ$$



c) Power Factor

$$\cos(45) = 0.707 = 70.7\%$$



$$\left((20 + j40)^{-1} + (-j100)^{-1} \right)^{-1} = 50 - j50$$

$$50 - j20 + 50 + j50 = 100 + j30$$

$$\left((100 + j30)^{-1} + (-j40)^{-1} \right)^{-1} = 15.84 - j38.4 = Z_T$$

$$S = (0.5)^2 (Z_T) \quad S = (0.5)^2 (15.84 - j38.4) = 3.96 - j9.6$$

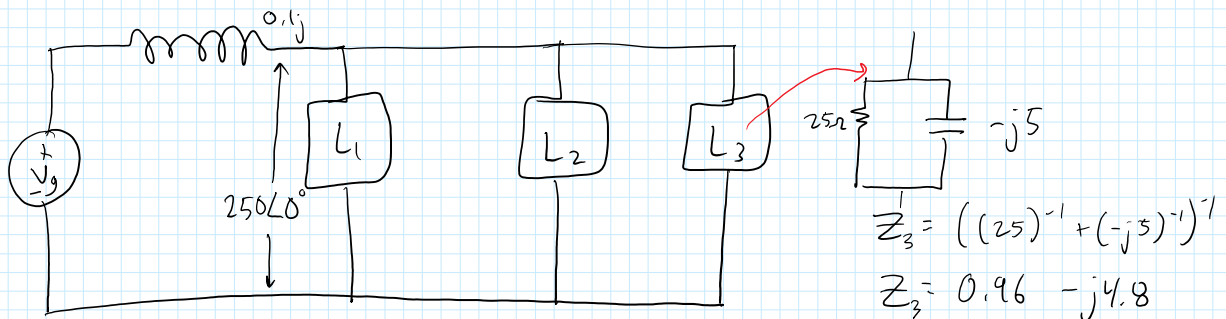
$$= 10.388 \angle -67.59^\circ$$

$$VA = 10.388 = \sqrt{3.96^2 + 9.6^2}$$

WATTS = 5.96

VAR = 9.6

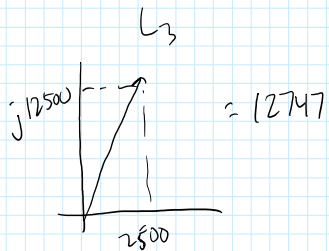
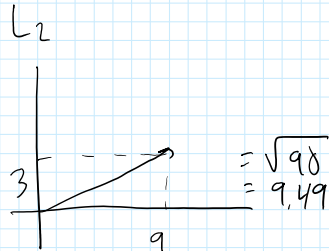
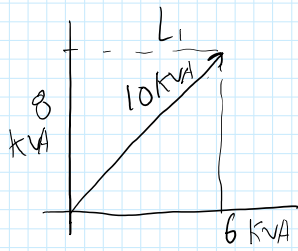
PF = $\cos(67.59) = 0.38 = 38\%$



$$\frac{1}{Z_3} = ((25)^{-1} + (-j5)^{-1})^{-1}$$

$$Z_3 = 0.96 - j4.8$$

$$S_3 = \frac{250^2}{Z_3} = 2500 + j12500$$



$$V_g = 250 + j0.1 (I_T) \quad I = \frac{S}{V}$$

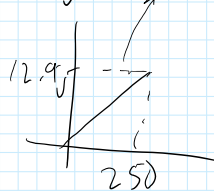
$$I = \frac{9490}{250} + \frac{10,000}{250} + \frac{12747}{250}$$

$$I = 128.95$$

$$V_g = 250 + j0.1 (128.95)$$

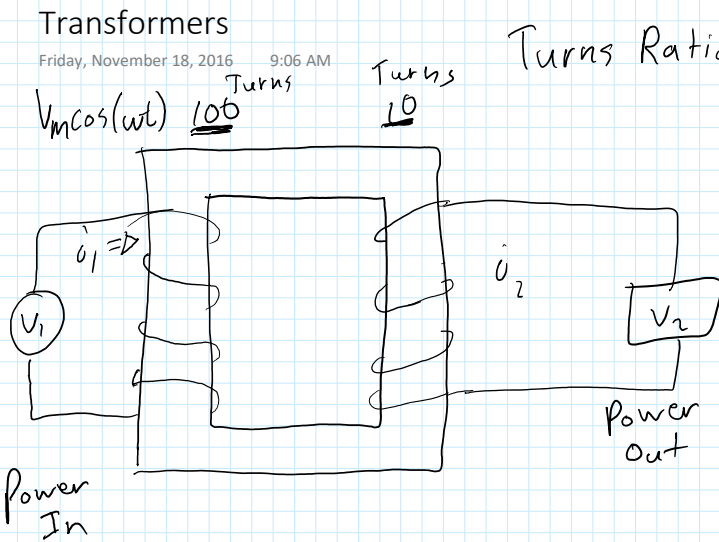
$$V_g = 250 + 12.9j$$

$$V_g = 250.3$$



Transformers

Friday, November 18, 2016 9:06 AM



$$\text{Turns Ratio} = \frac{N_1}{N_2} = \frac{100}{10} = 10:1$$

N_1 turns
Primary
(Source side)

N_2 turns
Secondary
(Load side)

$$V_1 = N_1 \frac{d\Phi}{dt}$$

$$V_2 = N_2 \frac{d\Phi}{dt}$$

$$\frac{V_1}{N_1} = \frac{d\Phi}{dt}$$

$$\frac{V_2}{N_2} = \frac{d\Phi}{dt}$$

$$V = N \frac{d\Phi}{dt}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_2 = V_1 \left(\frac{N_2}{N_1} \right)^{\leftarrow 10}$$

$$V_2 = 0.1 V_1$$

$$V_2 = V_1 \left(\frac{N_2}{N_1} \right)$$

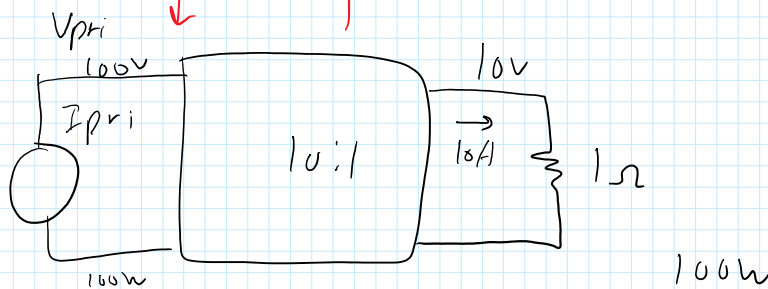
Turns Ratio
100 : 10
 $P_{in} = P_{out}$

$$V_1 I_1 = V_2 I_2$$

100w

100v · 1A 10v · 10A

$V \downarrow, I \uparrow$



$$V = 10V$$

$$I = 10A$$

$$P = 100W$$

$$V = 100V$$

$$I = 1A$$

$$P = 100W$$

→
Voltage Ratio = Turns Ratio
Current Ratio = $\frac{1}{\text{Turns Ratio}}$

$$S = S$$

$$Z_{pri} = \frac{V_{pri}}{I_{pri}}$$

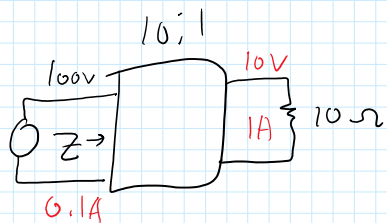
$$Z_2 = 1\Omega$$

Z_{sec}

$$Z_{pri} = \frac{V_{pri}}{I_{pri}} \quad Z_2 = 1 \Omega$$

$$\frac{V_{pri}}{I_{pri}} = \frac{V_s \times 10}{I_s \times 0.1} = N \frac{V_{sec}}{\frac{1}{N} I_{sec}} = N^2 Z_{sec}$$

$$Z_{pri} = N^2 Z_{sec} \quad Z_{pri} = 100 \Omega$$



Given
Find

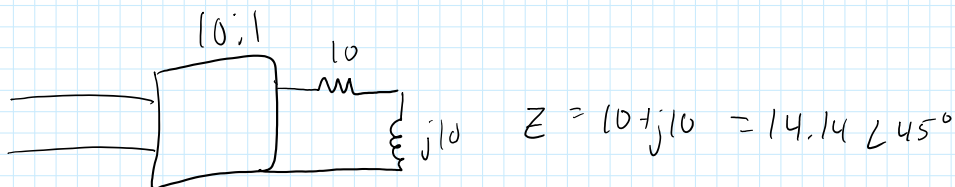
$$V_{pri} = TR \times V_{sec}$$

$$I_{pri} = \frac{I_{sec}}{TR}$$

$$Z_{pri} = |TR|^2 \times Z_{sec}$$

$$Z = (10)^2 \cdot 10 \Omega = 1000 \Omega$$

$$Z = \frac{100V}{0.1A} = 1000 \Omega$$

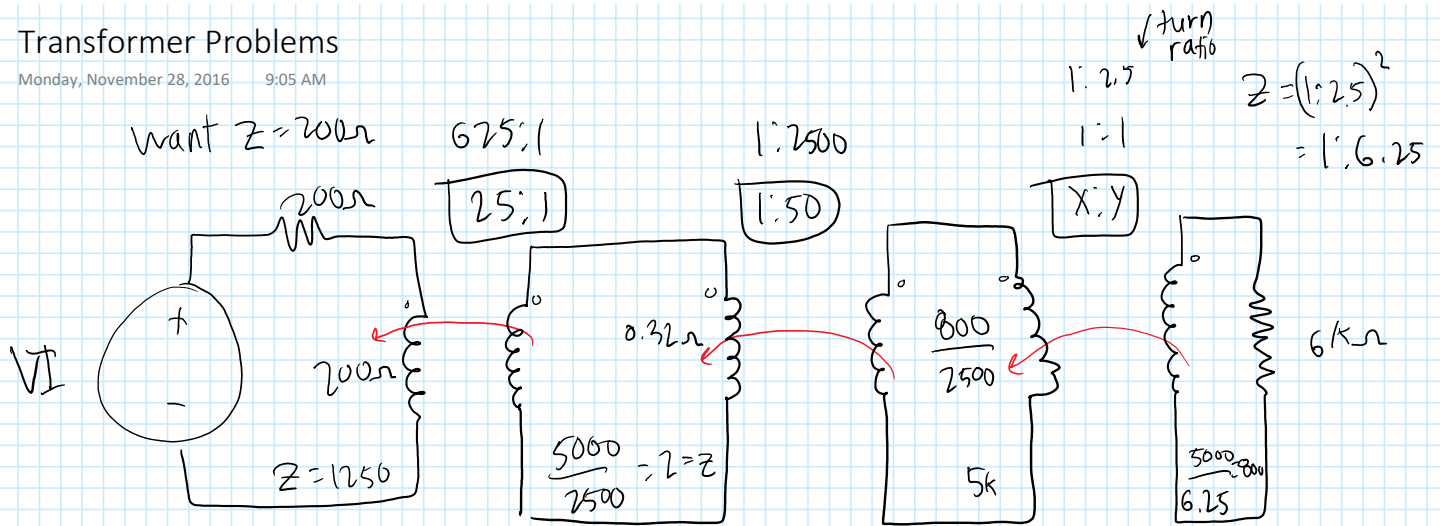


$$Z = 10 + j10 = 14.14 \angle 45^\circ$$

$$100 (10 + j10) = 100 \times 14.14 \angle 45^\circ = 1000 + j1000 \text{ or } 1414 \angle 45^\circ$$

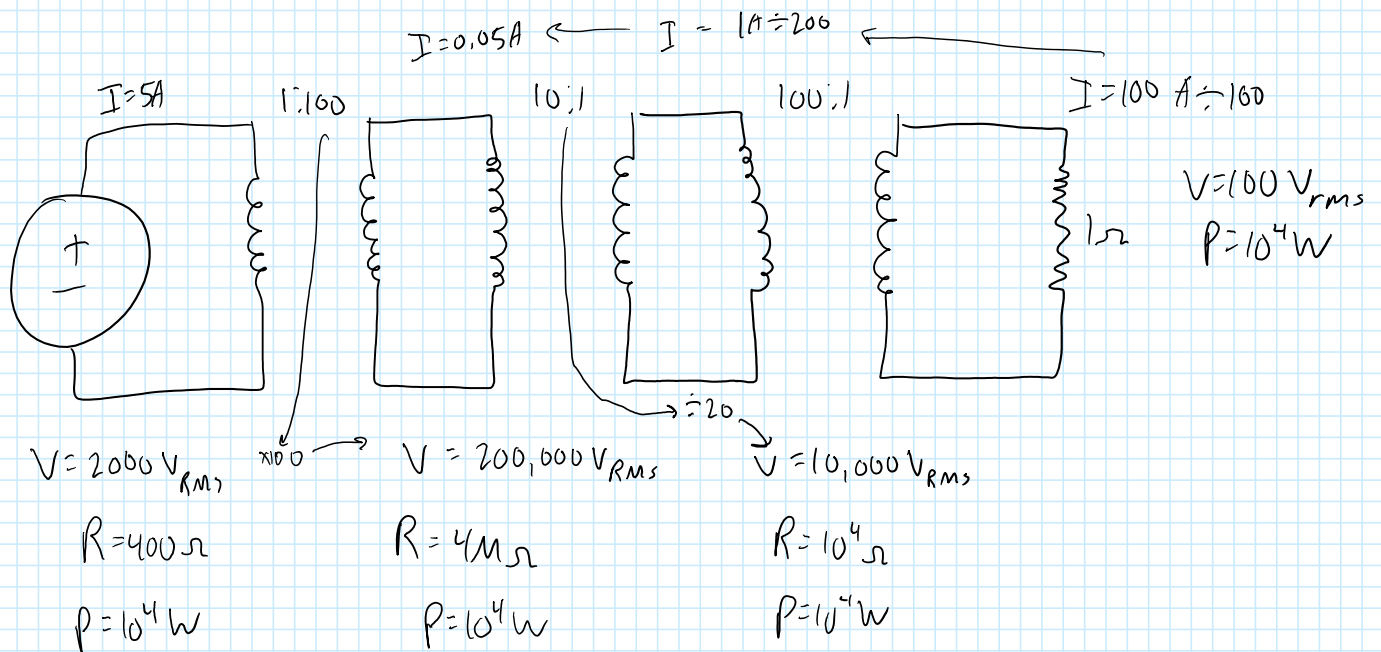
Transformer Problems

Monday, November 28, 2016 9:05 AM

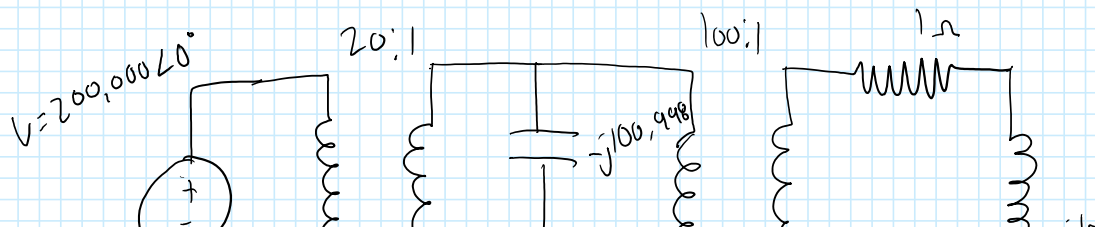


$$\frac{\sqrt{1250}}{200} = \sqrt{6.25} = 2.5$$

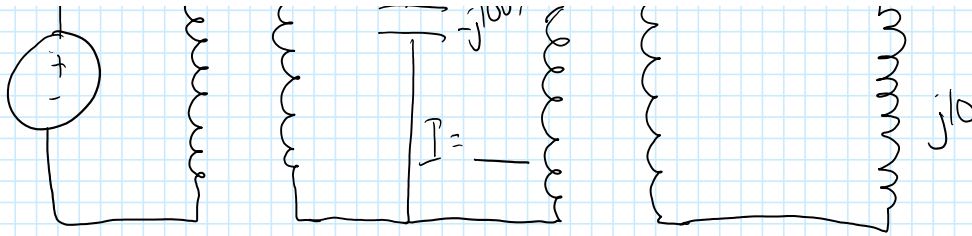
$$Z_{pri} = Z_{sec} \times (N_{ps})^2 \quad \text{or} \quad Z_{sec} = \frac{Z_{pri}}{(N_{ps})^2}$$



$$(20)^2 \times 10^4 = 4 \times 10^6 = 4M\Omega$$



v



$Z =$ _____
 $V = 200,000$
 $I =$ _____

$Z = 1.01 \times 10^6$
 $V = 10,000$
 $I = 0.0095$

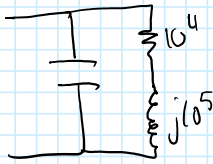
$Z = 10^4 + j10^5$
 $V = 10,000$
 $I = 0.5649 - j0.09$

$Z = 1 + j10$
 $V = 100$
 $I = 0.99 - j9.9$

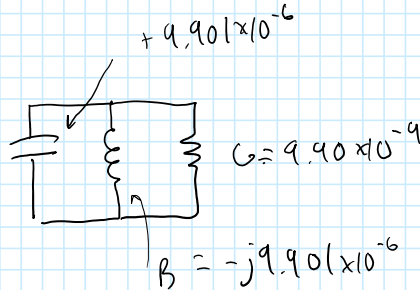
$I = \frac{V}{Z}$

$S = P$ _____ $+ jQ$ _____

$S = P$ 9.84 $+ jQ$ 98.4



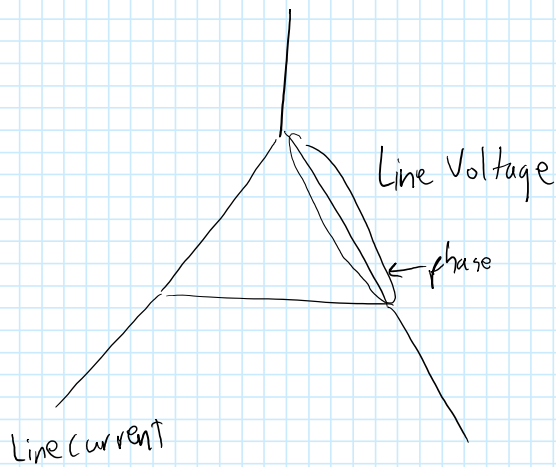
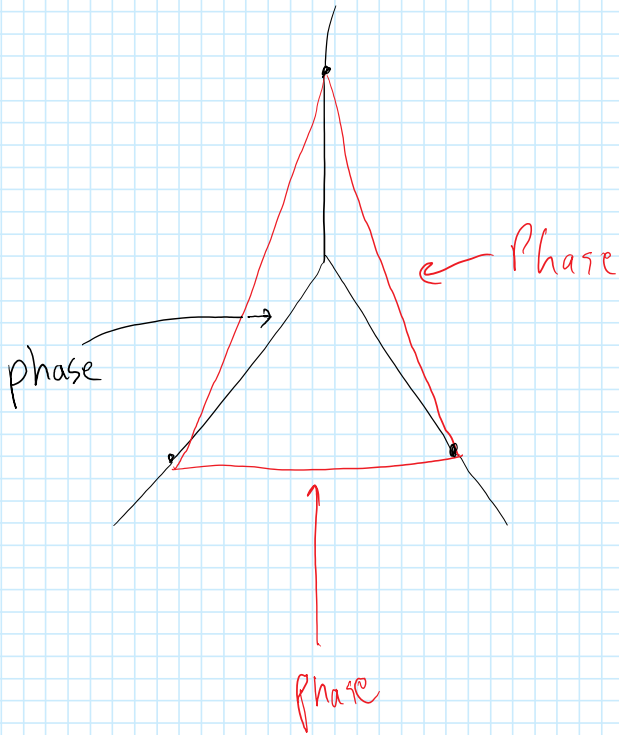
$(10^4 + j10^5)^{-1}$



$X_c = -j(9.901 \times 10^{-6})^{-1}$
 $= -j100,998$

3-Phase Delta and Wye

Wednesday, November 30, 2016 9:05 AM



For Delta Config

Delta phase voltage = Line-to-line phase voltage
and the phase current = is the vector "sum"
or "difference" of the line current

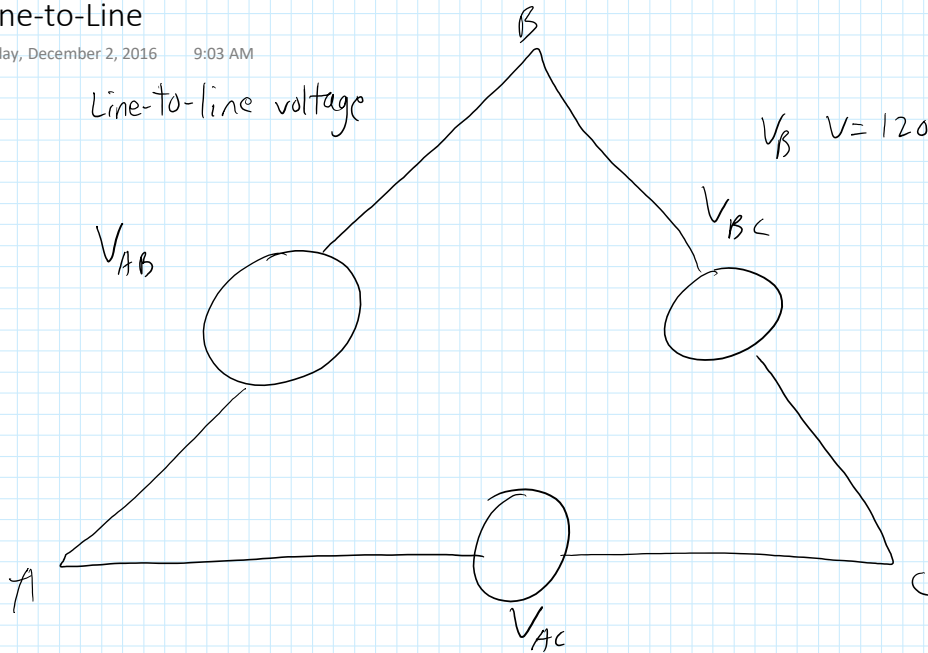
For Wye Config

the phase current = line current of the phase
voltage is vectorally related to the line-to-line voltage

Line-to-Line

Friday, December 2, 2016 9:03 AM

Line-to-line voltage



Line voltage = phase voltage

Line current = $\sqrt{3} \times$ phase current

phase current = $\frac{\text{Line current}}{\sqrt{3}}$

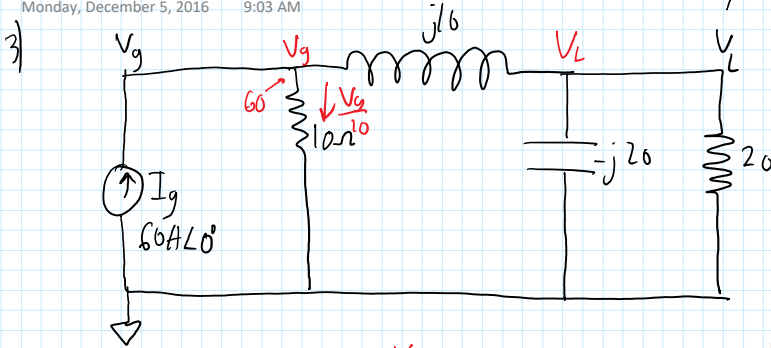
$$\begin{aligned} I_{\text{line A}} &= I_{AB} - I_{AC} = I_{\text{rms}} \angle -120^\circ = I_{\text{rms}} (e^{-j0} - e^{-j120^\circ}) \\ &= I_{\text{rms}} (1 - \cos(-120^\circ) - j \sin(-120^\circ)) \\ &= I_{\text{rms}} (1 - (-1/2 - j \frac{\sqrt{3}}{2})) = I_{\text{rms}} (\frac{3}{2} + j \frac{\sqrt{3}}{2}) \\ &= (\sqrt{3}) (I_{\text{rms}}) \left(\frac{\sqrt{3} + j}{2} \right) = \frac{\sqrt{3}}{2} + j \frac{1}{2} \end{aligned}$$

$$I_{\text{line A}} = \sqrt{3} \times I_{\text{rms}} \angle 30^\circ$$

$$I_{\text{line B}} = \sqrt{3} \times I_{\text{rms}} \angle 150^\circ$$

$$I_{\text{line C}} = \sqrt{3} \times I_{\text{rms}} \angle -90^\circ$$

NODE VOLTAGE



$V_g =$
 $V_L =$

Node V_g : $60 - \frac{V_g}{10} - \frac{V_g - V_L}{j10} = 0$ $j100 [] = 6000j - j10V_g - 10V_g + 10V_L = 0$
 $6000j - V_g(10 + j10) + 10V_L = 0$

① $V_g(10 + j10) - 10V_L = 6000j$

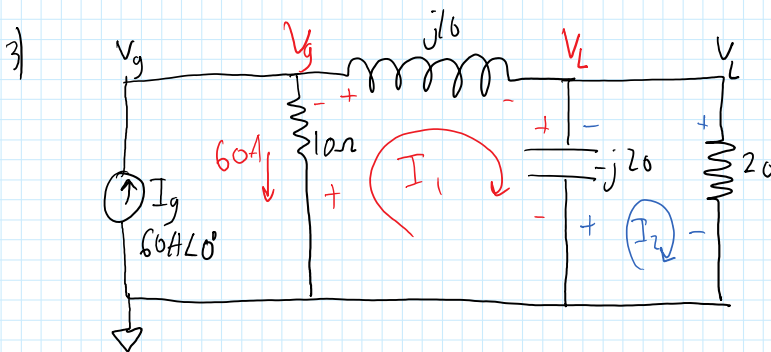
Node V_L : $\frac{(V_g - V_L)}{j10} - \frac{V_L}{-j20} - \frac{V_L}{20} = 0$ $200 [] \Rightarrow -20j(V_g - V_L) - 10jV_L - 10V_L = 0$

$-20j(V_g) + 20j(V_L) - 10j(V_L) - 10(V_L) = 0$

② $-20j(V_g) + V_L(10j - 10) = 0$

Solve ① and ② $\Rightarrow V_g = 300 + j0 = 300 \angle 0^\circ$
 $V_L = 300 - j300 = 424.3 \angle -45^\circ$

MESH CURRENT



$V_g =$
 $V_L =$

Loop 1: $10(I_1 - 60) + j10(I_1) - j20(I_1 - I_2) = 0$
 $-600 + 10I_1 + j10(I_1) - j20(I_1) + j20(I_2) = 0$
 $-600 + I_1(-j10 + 10) + I_2(j20) = 0$

$$\textcircled{1} \quad I_1(10 - j10) + I_2(j20) = 600$$

Loop 2: $-j20(I_2 - I_1) + 20(I_2) = 0$

$$\textcircled{2} \quad I_1(j20) + I_2(20 - j20) = 0$$

Solve $\textcircled{1}$ and $\textcircled{2} \implies I_1 = 30 + j0$

$$I_2 = 15 - j15$$

$$V_g = 10(I_1 - 60) \quad V_g = 10(30 - 60) = -300$$

$$\begin{array}{|c|} \hline - \\ \hline + \\ \hline \end{array} V_g = -300 \quad \text{Reality} \implies \begin{array}{|c|} \hline + \\ \hline - \\ \hline \end{array} 300 = V_g$$

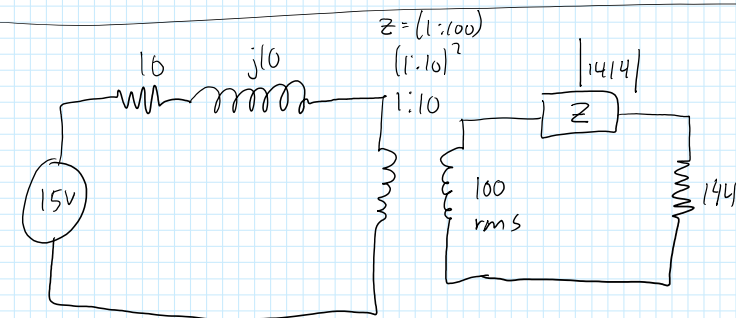
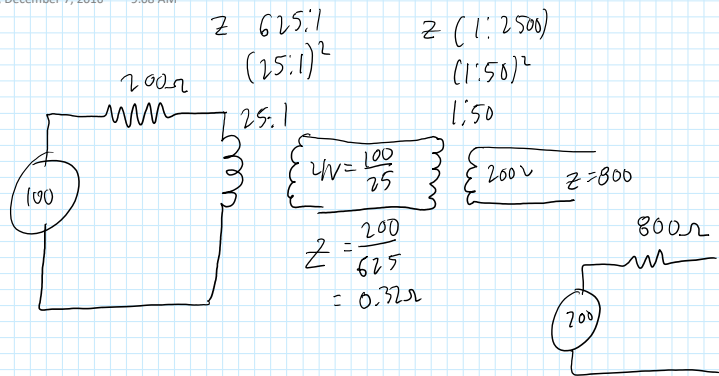
$$V_g = 300 \text{ V}$$

$$V_L = 20(15 - j15) = 300 - j300$$

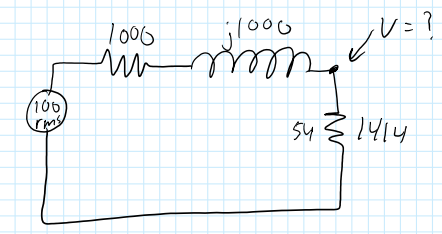
$$V_L = 300 - j300$$

More Final Practice

Wednesday, December 7, 2016 9:08 AM

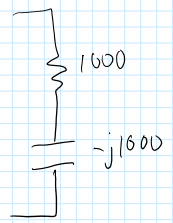


$Z = 1000 + j1000 \quad |Z| = 1414$



$V = \frac{100(1414)}{2414 + j1000} = 44.9956 - 20.7j = 54.1\angle -22^\circ$

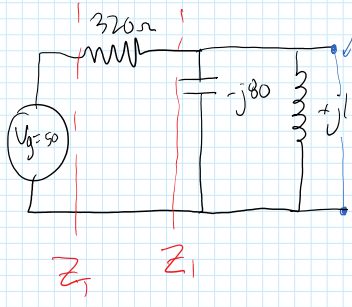
Complex Conjugate



$\frac{100}{1000 + j1000 + 1000 - j1600} = \frac{100}{2000\Omega} = 0.05A$

$= (0.05)^2 (1000\Omega) = 2.6W$

$I_N = \frac{50V}{320\Omega} = 0.15625A$ (cap and ind are shorted)

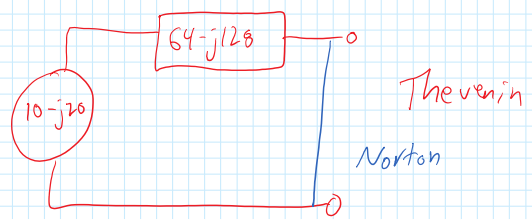


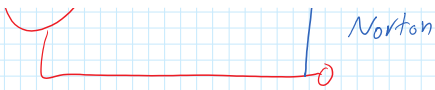
$V_{TH} \rightarrow 50(Z_1)$
 $Z_1 + 320$
 $Z_{TH} \leftarrow$
 (look in from the outside)

$Z_1 = ((-j80)^{-1} + (j160)^{-1})^{-1} = -j160$

$\frac{50(-j160)}{-j160 + 320} = 10 - j20$

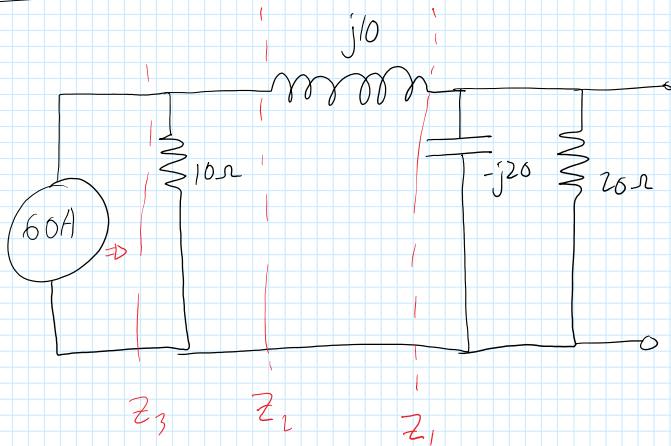
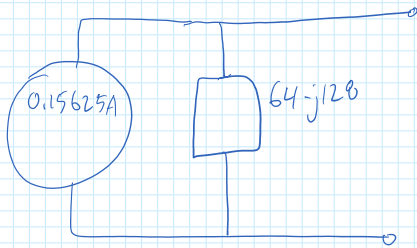
$Z_{TH} = ((320)^{-1} + (-j80)^{-1} + (j160)^{-1})^{-1} = 64 - j128$





$$I_N = \frac{10 - j20}{64 - j128} \rightarrow I = \frac{V}{R}$$

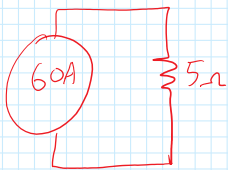
$$I_N = 0.15625 \text{ A}$$



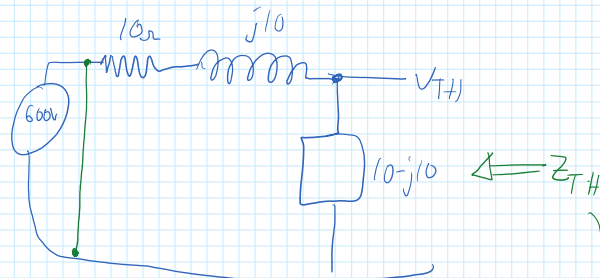
$$Z_1 = ((-j20)^{-1} + (20)^{-1})^{-1} = 10 - j10$$

$$Z_2 = Z_1 + j10 = 10$$

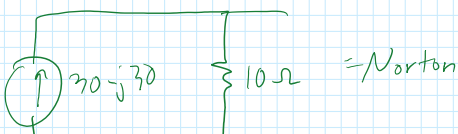
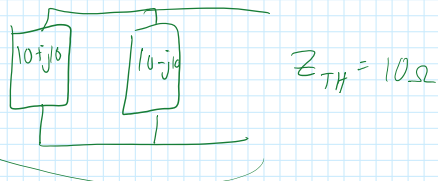
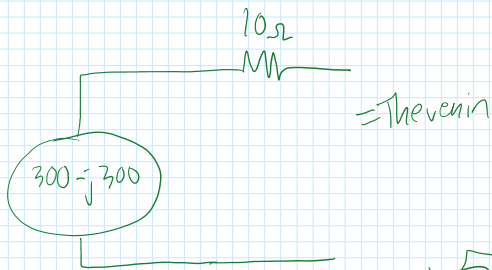
$$Z_3 = 10 \parallel Z_2 = 5\Omega$$

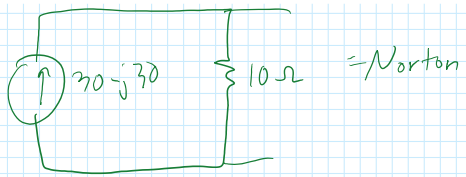


OR



$$= \frac{600(10 - j10)}{10 - j10 + 10 + j10} = \frac{600(10 - j10)}{20\Omega} = 300 - j300$$





John Schad

E211

Dr. Galassi

HW #9

Review Problems

1) 120°

2) Delta, wye

3) $480V = V_{Line}$ $V_{phase} = \frac{V_{Line}}{\sqrt{3}}$ $V_{phase} = \frac{480}{\sqrt{3}}$ $V_{phase} = 277.13V$

4) $I_{phase} = 25A$ $I_{line} = I_{phase}$ $I_{line} = 25A$

5) $V_{Line} = 560V$ $V_{phase} = V_{Line}$ $V_{phase} = 560V$

6) $I_{phase} = 30A$ $I_{line} = I_{phase}(\sqrt{3})$ $I_{line} = 51.96A$

7) $V_{phase} = 240V$ $I_{phase} = 18A$ $VA = 3 \times V_{phase} \times I_{phase}$ $VA = 3(240)(18)$
 $VA = 12960 \text{ W} \cdot \sqrt{3}$

8) $VA = \sqrt{3} \times I_{Line} \times V_{Line}$ $V_{phase} = \frac{V_{Line}}{\sqrt{3}}$ $V_{Line} = V_{phase}(\sqrt{3})$ $V_{Line} = 240(\sqrt{3})$
 $V_{Line} = 415.69V$
 $I_{Line} = \frac{VA}{\sqrt{3}(V_{Line})}$ $I_{Line} = \frac{12960}{(\sqrt{3})(415.69)}$ $I_{Line} = 18A$

9) $V_L = 2400V$ $I_L = 40A$ $I_L = \frac{V_L}{Z}$ $Z = \frac{V_L}{I_L}$ $Z = \frac{2400}{40}$ $Z = 60\Omega$

10) $PF = 1$ $P = \sqrt{3} (PF) (V_L) (I_L)$ $P = \sqrt{3}(1)(2400)(40)$ $P = 166,276.88W$

Practice Problems

1) $V_p(A) = 138.56V$ $V_p(L) = 240V$
 $I_p(A) = 34.64A$ $I_p(L) = 20A$
 $V_L(A) = 240V$ $V_L(L) = 240V$
 $I_L(A) = 34.64A$ $I_L(L) = 34.64A$
 $P = 14399.58W$ $Z_{(phase)} = 12\Omega$

$V_L(A) = V_L(L)$ alternator directly connected to load

$$V_p(L) = V_L(L) \quad I_p(L) = \frac{V_p(L)}{Z} \quad I_p(L) = \frac{240}{12} = 20$$

$$I_L(L) = I_p(L) \times \sqrt{3} \quad I_L(L) = 34.64$$

$$I_L(L) = I_L(A) \quad I_p(A) = I_L(A) \quad V_p(A) = \frac{V_L(A)}{\sqrt{3}} \quad V_p(A) = \frac{240}{\sqrt{3}} \quad V_p(A) = 138.56$$

$$P = (\sqrt{3})(V_{L(A)})(I_{L(A)}) \text{ PF} \quad \text{PF} = 1 \quad P = (\sqrt{3})(240)(34.64)(1) = 14399.58$$

2)

$V_p(A) = 4160 \text{ V}$	$V_p(L) = 2401.78 \text{ V}$
$I_p(A) = 23.11 \text{ A}$	$I_p(L) = 40.03 \text{ A}$
$V_L(A) = 4160 \text{ V}$	$V_L(L) = 4160 \text{ V}$
$I_L(A) = 40.03 \text{ A}$	$I_L(L) = 40.03 \text{ A}$
$P = 288,429.41 \text{ W}$	$Z_{\text{(phase)}} = 60 \Omega$

$$V_L(L) = V_L(A) \quad V_p(L) = \frac{V_L(L)}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2401.78$$

$$I_p(L) = \frac{V_p(L)}{Z} = \frac{2401.78}{60} = 40.03 \quad I_p(L) = I_L(L) = I_L(A)$$

$$I_p(A) = \frac{I_L(A)}{\sqrt{3}} = \frac{40.03}{\sqrt{3}} = 23.11 \quad V_p(A) = V_L(A)$$

$$P = (\sqrt{3})(V_L)(I_L)(\text{PF}) \quad \text{PF} = 1 \quad P = (\sqrt{3})(4160)(40.03)(1) = 288,429.41 \text{ W}$$

3)

$V_p(A) = 323.32 \text{ V}$	$V_p(L_1) = 323.32 \text{ V}$	$V_p(L_2) = 560 \text{ V}$
$I_p(A) = 185.9 \text{ A}$	$I_p(L_1) = 64.66 \text{ A}$	$I_p(L_2) = 70 \text{ A}$
$V_L(A) = 560 \text{ V}$	$V_L(L_1) = 560 \text{ V}$	$V_L(L_2) = 560 \text{ V}$
$I_L(A) = 185.9 \text{ A}$	$I_L(L_1) = 64.66 \text{ A}$	$I_L(L_2) = 121.24 \text{ A}$
$P = 180,313.42 \text{ W}$	$Z_{\text{(phase)}} = 5 \Omega$	$Z_{\text{(phase)}} = 8 \Omega$

$$V_L(A) = V_L(L_1) = V_L(L_2) = V_p(L_2)$$

$$I_p(L_2) = \frac{V_p(L_2)}{Z} = \frac{560 \text{ V}}{8 \Omega} = 70 \quad I_L(L_2) = I_p(L_2)(\sqrt{3}) = 70(\sqrt{3}) = 121.24$$

$$I_p(L_2) = \frac{V_p(L_2)}{Z} = \frac{560V}{8\Omega} = 70 \quad I_L(L_2) = I_p(L_2)(\sqrt{3}) = 10(\sqrt{3}) = 141.42$$

$$V_p(L_1) = \frac{V_L(L_1)}{\sqrt{3}} = \frac{560}{\sqrt{3}} = 323.32 \quad I_p(L_1) = \frac{V_p(L_1)}{Z} = \frac{323.32}{5} = 64.66$$

$$I_p(L_1) = I_L(L_1) \quad I_L(A) = I_L(L_1) + I_L(L_2) = 64.66 + 121.24 = 185.9$$

$$I_p(A) = I_L(A) \quad V_p(A) = \frac{V_L(A)}{\sqrt{3}} = \frac{560}{\sqrt{3}} = 323.32$$

$$P = (\sqrt{3})(V_L)(I_L)(PF) \quad PF = 1 \quad P = (\sqrt{3})(560)(185.9)(1) = 180,313.42 \text{ W}$$

Three-Phase Power Practice

Friday, December 9, 2016 9:13 AM

8) Alternator

$$V_p = 508V$$

$$I_p = 381A$$

$$V_L = 880V$$

$$I_L = 381A$$

Load 1

$$V_p = 508V$$

$$I_p = 127A$$

$$V_L = 880V$$

$$I_L = 127A$$

Load 2

$$V_p = 880V$$

$$I_p = 146.7A$$

$$V_L = 880V$$

$$I_L = 254A$$

$$V_{Line} = \sqrt{3} (V_{phase}) \rightarrow \text{For Wye}$$

$$I_{Line} = \sqrt{3} I_{phase} \rightarrow \text{For Delta}$$

$$I_p = \frac{V_p}{Z}$$

$$I_{Line} = 254 + 127$$

$$\text{Power} = 580,644W$$

$$\text{Power} = (V_p)(I_p)(3) = 193,548W$$

$$\text{Power} = (V_p)(I_p)(3) = 387,288 \text{ watts}$$