Synchronized Phasor Measurements and State Estimation

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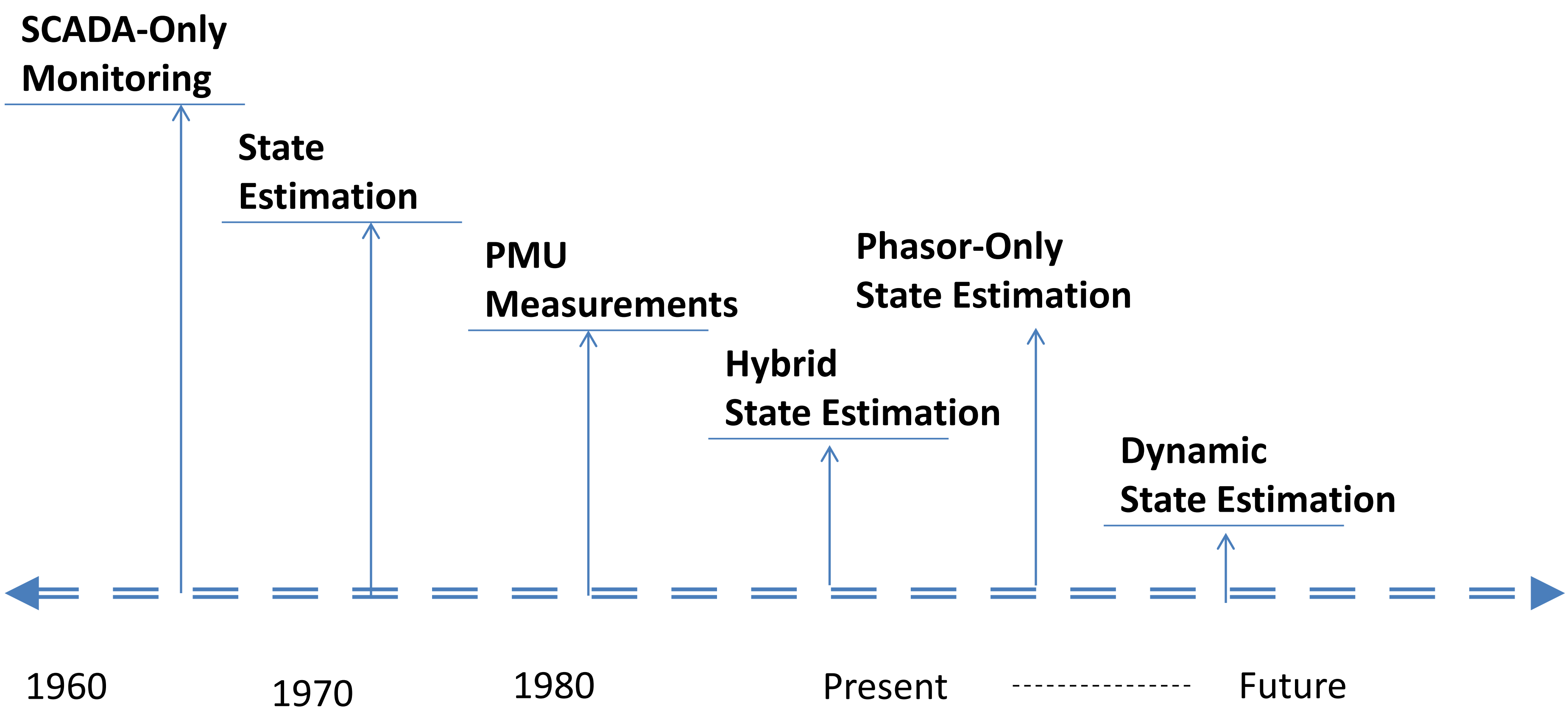
Outline

- Synchronized Phasor Measurements
- SCADA/PMU Hybrid State Estimation
- Phasor-Only Linear State Estimation
  - WLS formulation
  - LAV formulation
- Dynamic State Estimation and Observability
- Concluding Remarks

Historical Timeline



Historical Timeline

- SCADA only monitoring
  - Vulnerable to errors in measurements, network model parameters and topology
  - No direct measurement of phase angles
- Introduction of SE using SCADA
  - Phase angles can be estimated
  - Errors can be detected and removed

[*] Schweppe, F.C., Wildes, J., and Rom, D., "Power system static state estimation: Parts I, II, III", *Power Industry Computer Applications (PICA)*, Denver, CO, June 1969.

Static state estimation

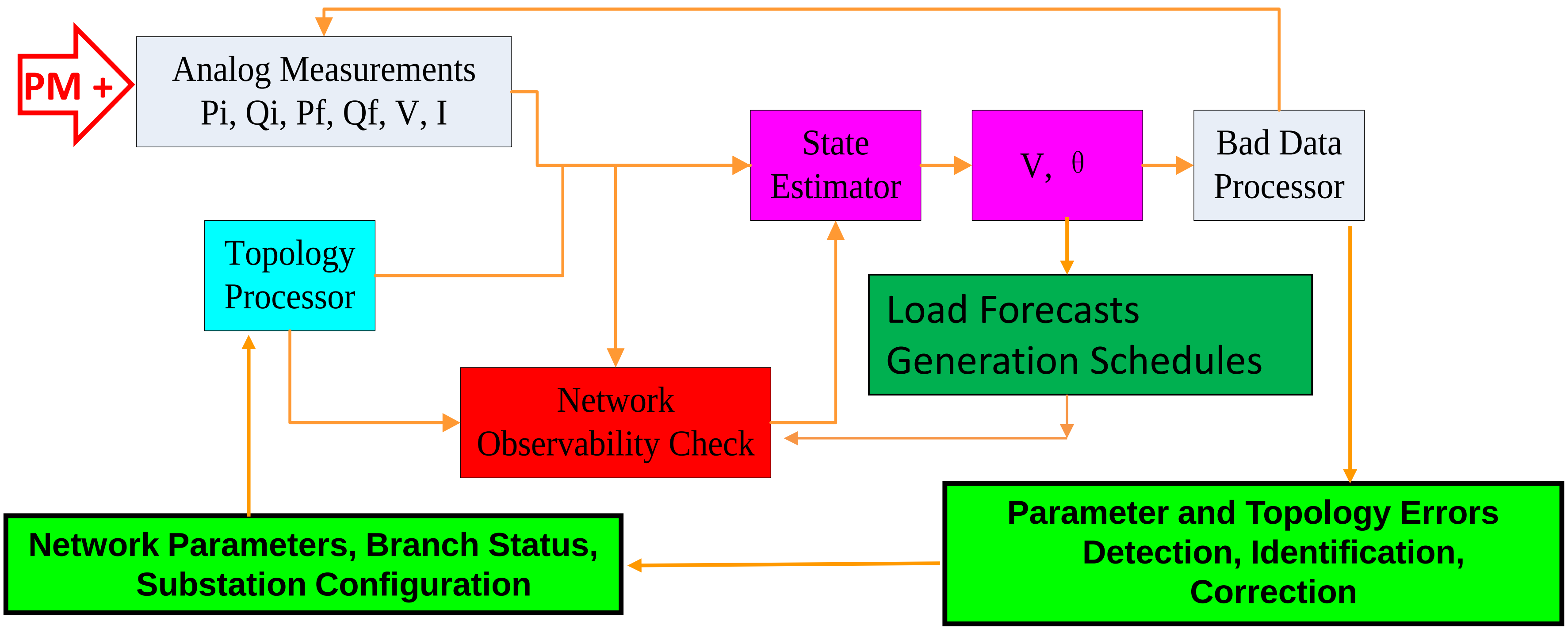
– State variables: voltage phasors at all system buses

– Measurements:

- Power injection measurements
- Power flow measurements
- Voltage/Current magnitude measurements
- **Synchronized phasor measurements**

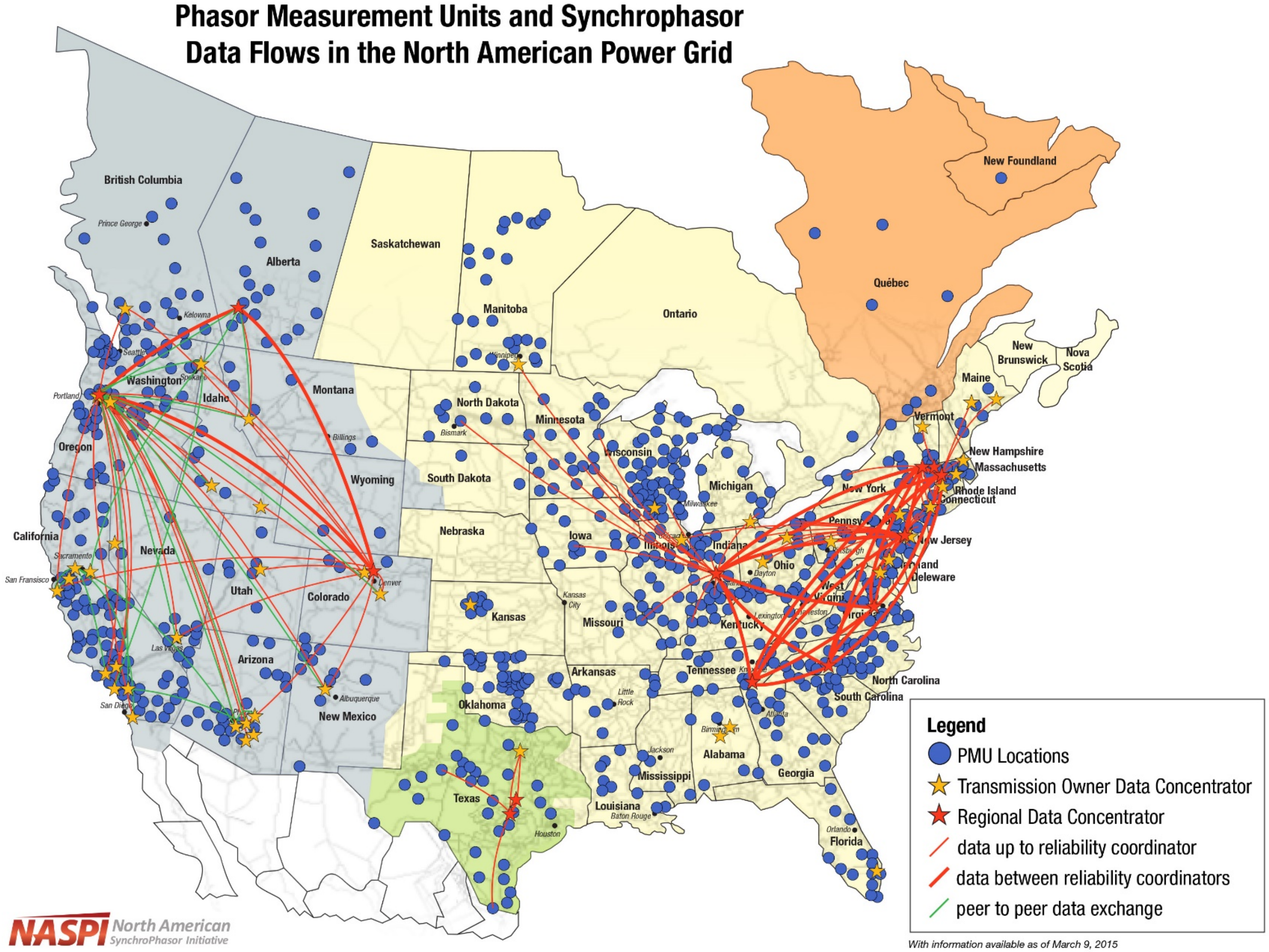
SCADA Measurements

State estimation: data/info flow diagram



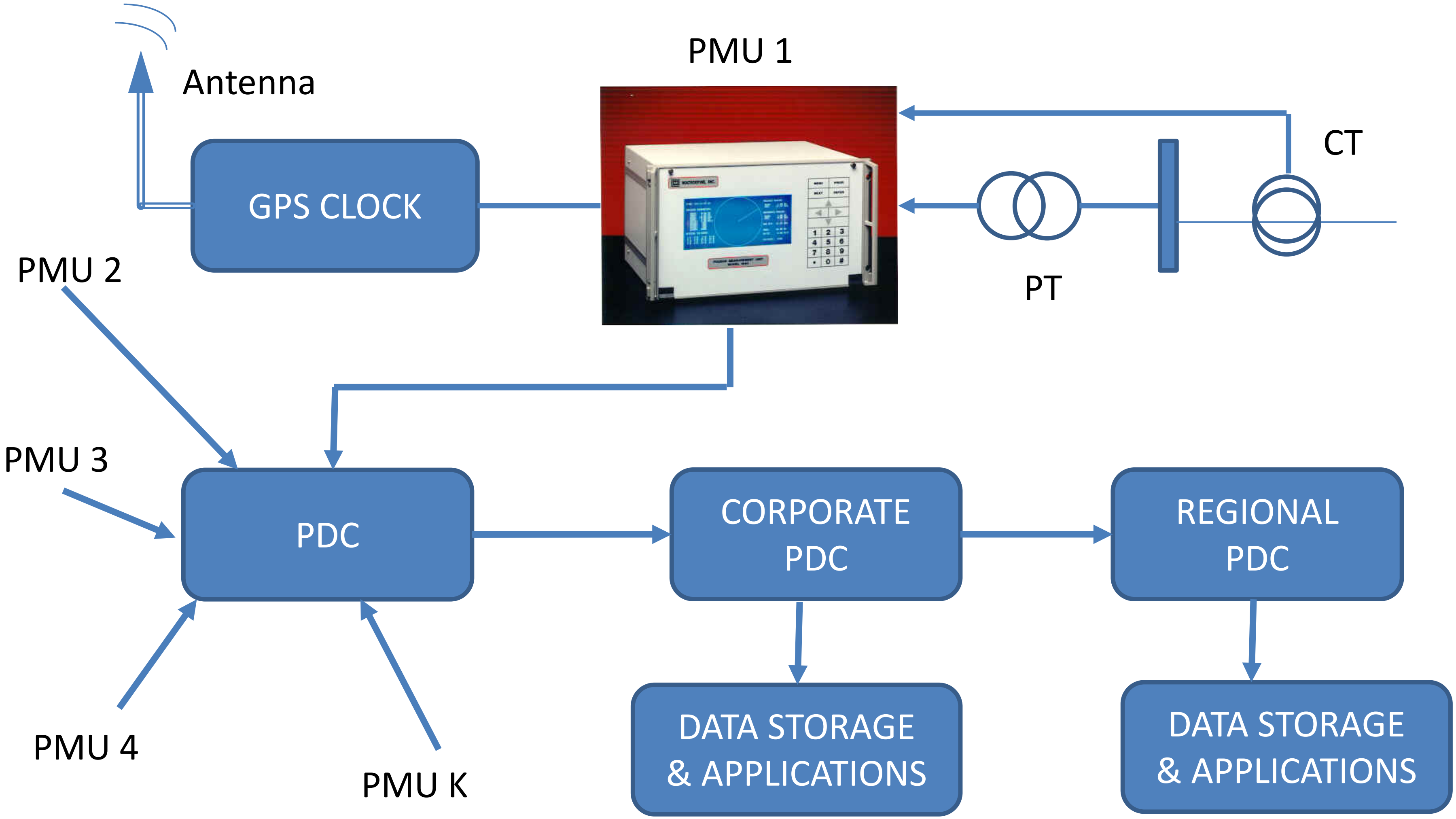
- Available every 1/30 seconds
- Both voltage and current phasors
- More accurate than SCADA but not error free

Under the Recovery Act SGIG and SGDP programs, their numbers rapidly increased : 166 → 1700



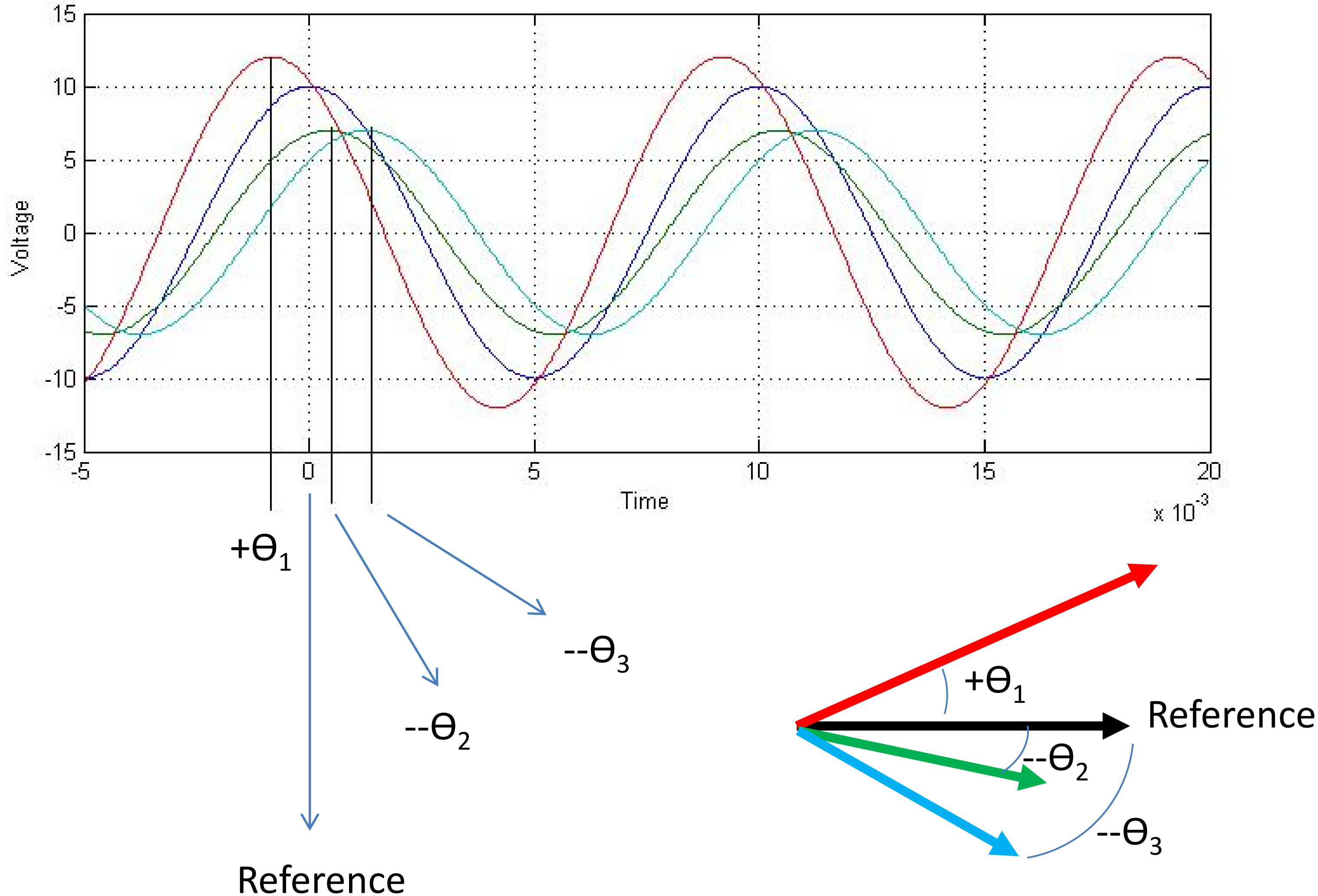
US DoE Office of Electricity Delivery and Energy Reliability
“Advancement of Synchrophasor Technology in projects funded by the American Recovery and Reinvestment Act of 2009”, March 2016.

Phasor Measurement Units (PMU) Phasor Data Concentrators (PDC)[*]

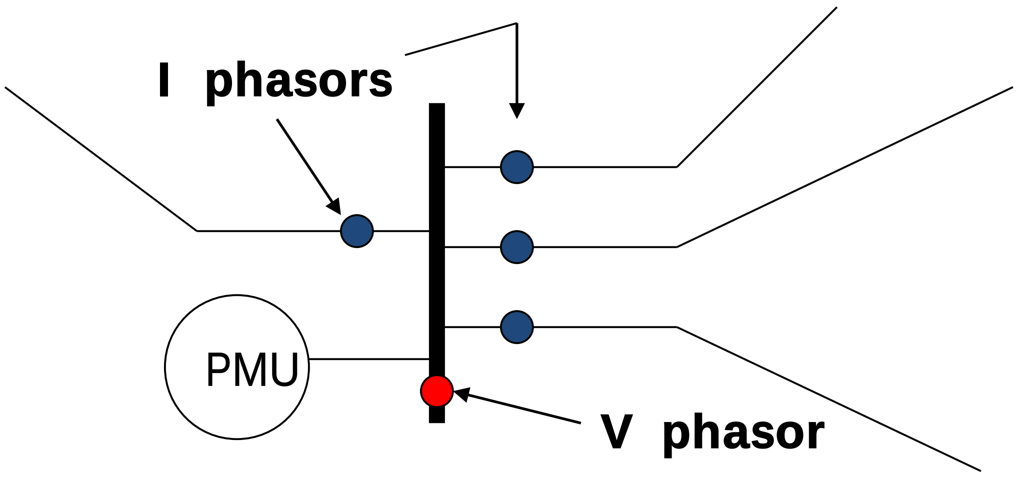


[*] IEEE PSRC Working Group C37 Report

Reference phasor



Measurements provided by PMUs



ALL 3-PHASES ARE TYPICALLY MEASURED

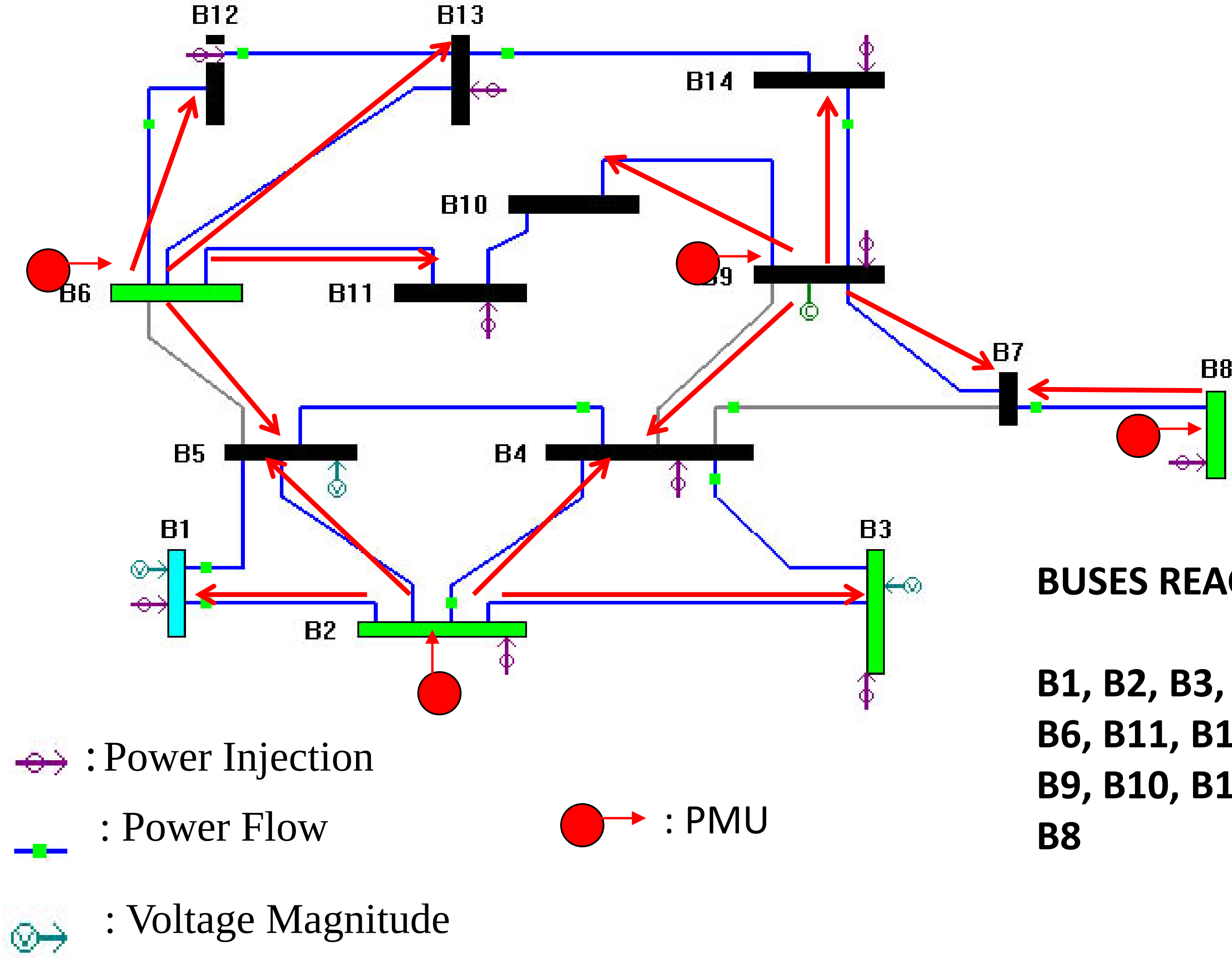
BUT

ONLY POSITIVE SEQUENCE COMPONENTS ARE REPORTED

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = [T] \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix} \Rightarrow V_+ = \frac{1}{3}[V_A + \propto V_B + \propto^2 V_C]$$

$$\propto = e^{\frac{j2\pi}{3}}$$

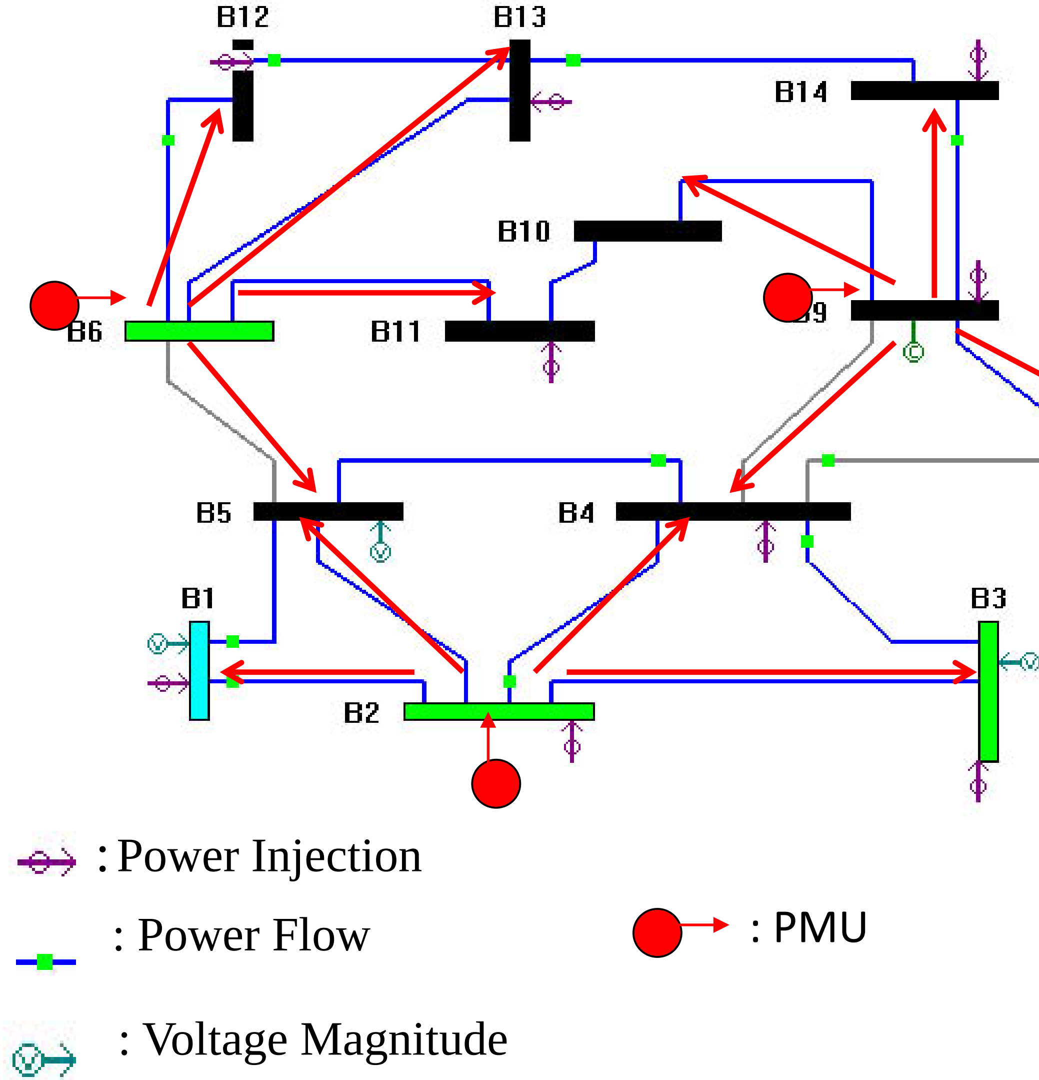
Tutorial Example: Placement of PMUs



BUSES REACHED:

B1, B2, B3, B4, B5, B6, B11, B12, B13, B9, B10, B14, B7, B8

Tutorial Example: Placement of PMUs

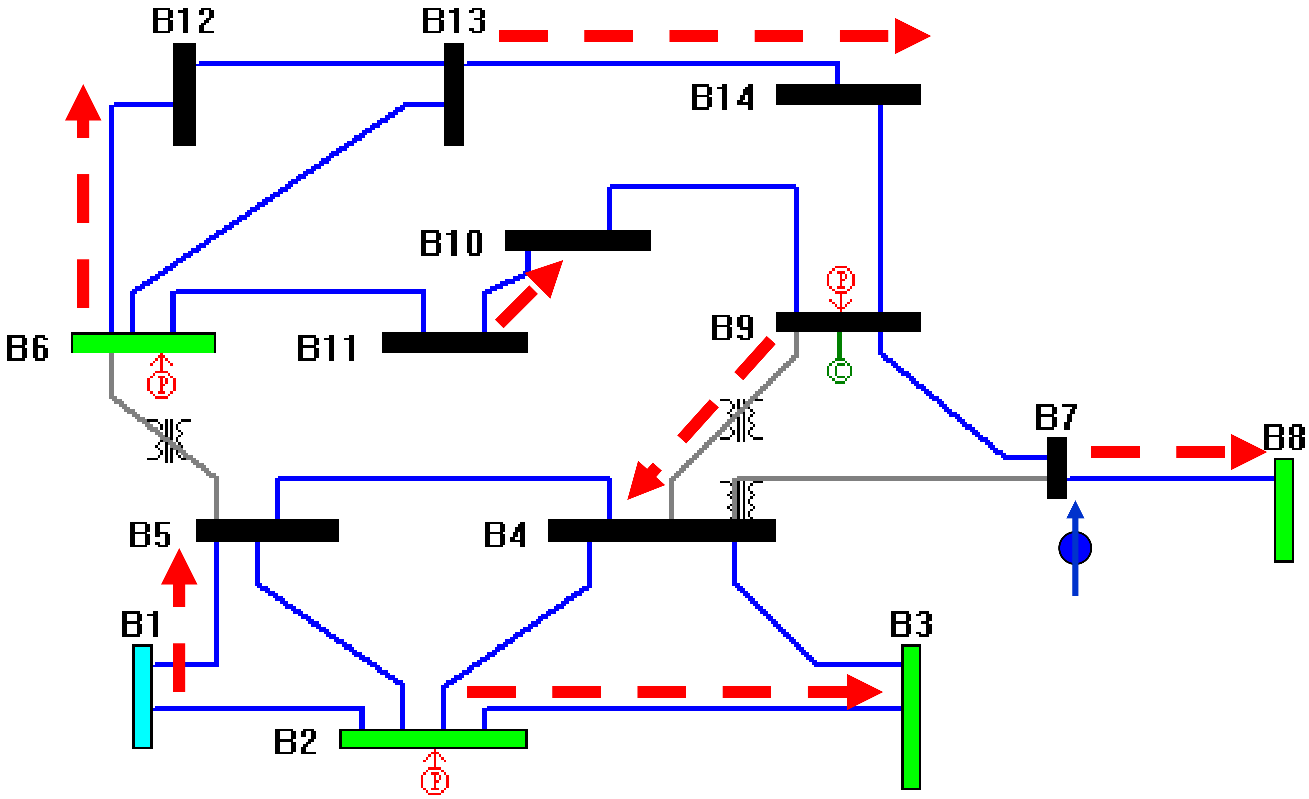


CONSIDERING ZERO INJECTION AT BUS B7 !

BUSES REACHED:

B1, B2, B3, B4, B5, B6, B11, B12, B13, B9, B10, B14, B7, B8

Branch PMU Placement for Full Observability



Only 7 branch PMUs make the entire system observable.

Use of Synchrophasor Measurements

Given unlimited number of available channels per PMU, it is sufficient to place PMUs at roughly 1/3rd of the system buses to make the entire system observable just by PMUs.

Systems	No. of zero injections	Number of PMUs	
		Ignoring zero Injections	Using zero injections
14-bus	1	4	3
57-bus	15	17	12
118-bus	10	32	29

Incorporation of PMUs in State Estimators: Hybrid Estimation

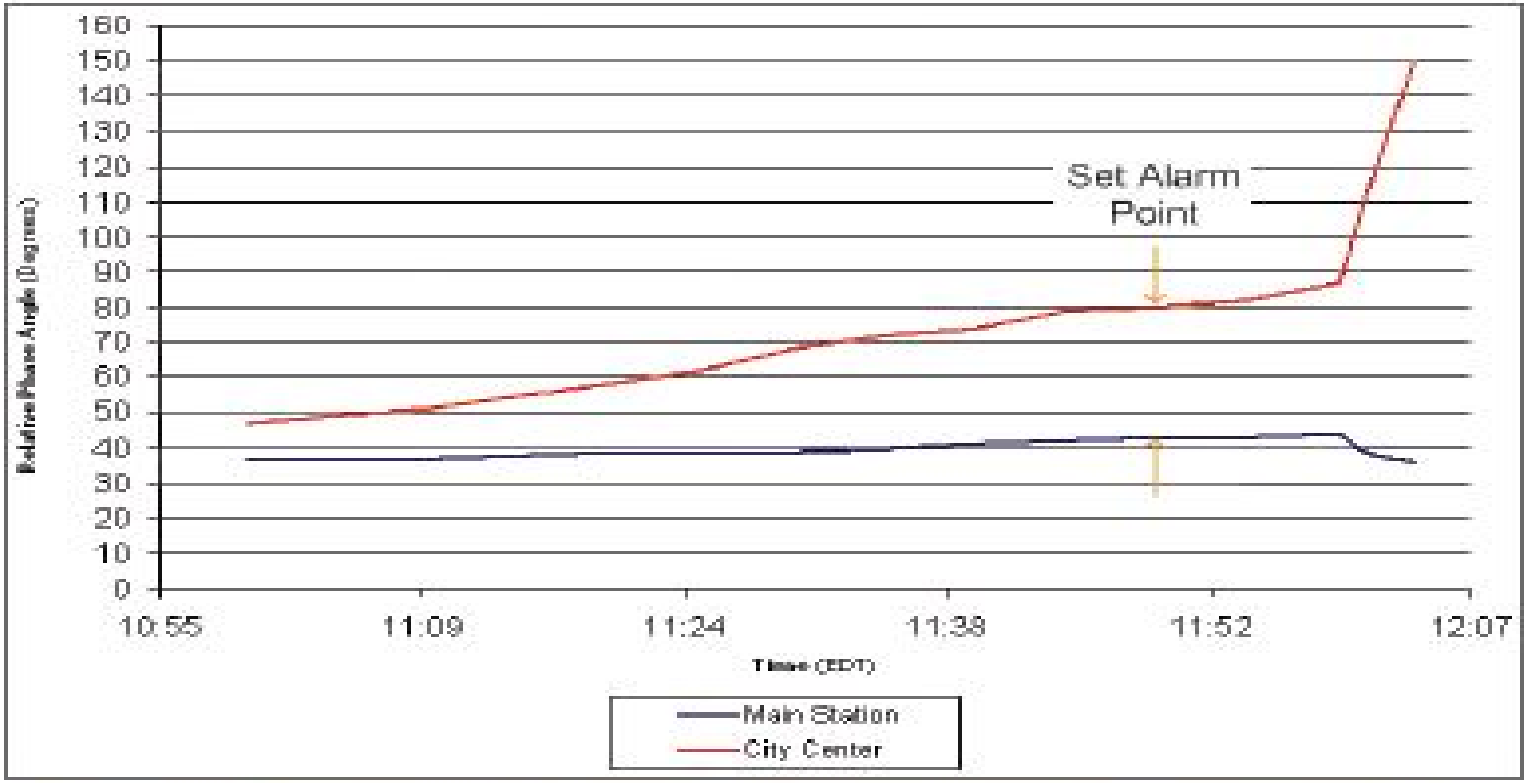
Hybrid State Estimation:

- Use of hybrid measurements
- Use of hybrid estimation methods

Challenges:

- Different scan rates of SCADA and PMUs
  Every 2-3 seconds versus every 33 ms
- Different accuracy classes
- Lack of full observability by PMU measurements
- Coordinating two different estimators running together

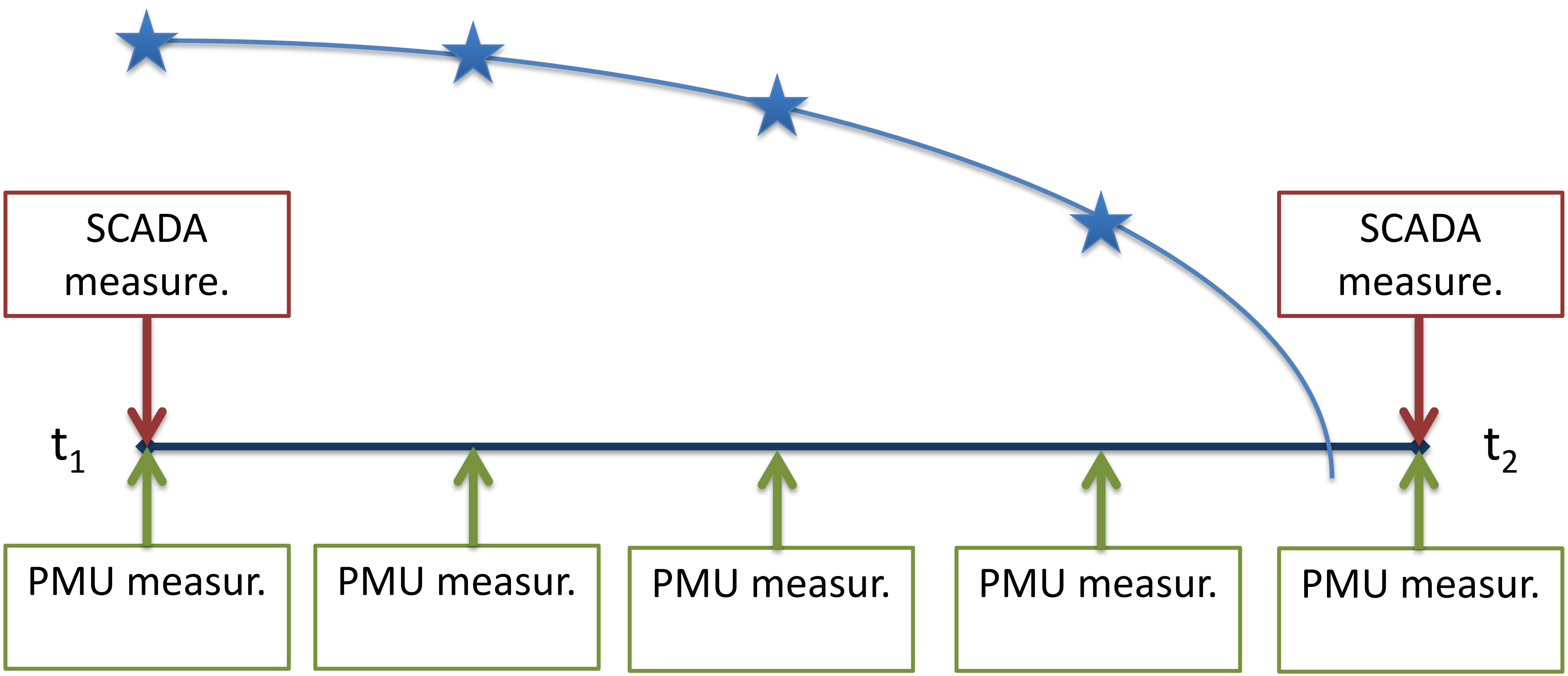
Re: SEL Synchrophasors Brochure



Phase angle measurement between critical locations on a power system provides operators with an early warning of potential system collapse.

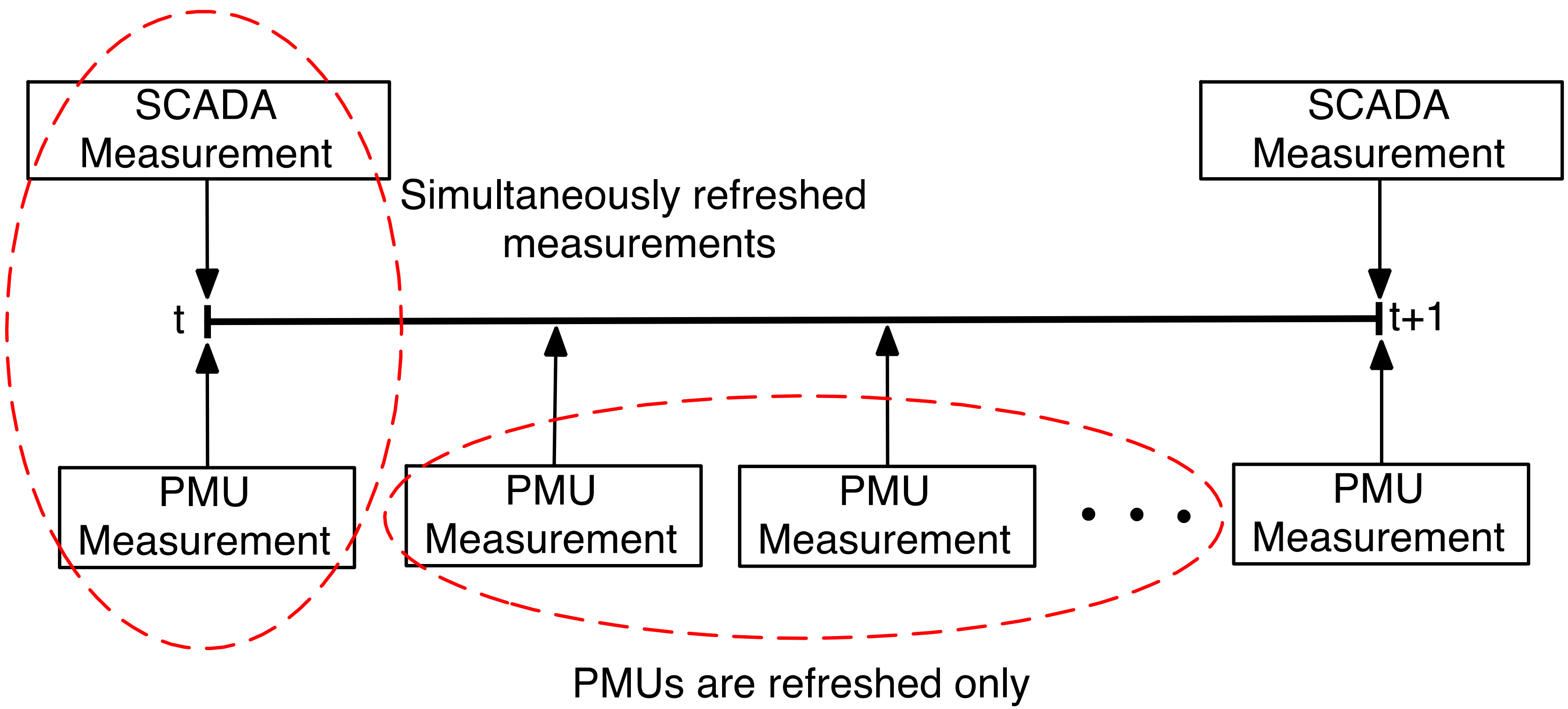
SCADA and PMU Measurements

Using conventional SCADA-based SE → System collapses without warning
Using mixed measurement based SE → Tracks state and takes control action

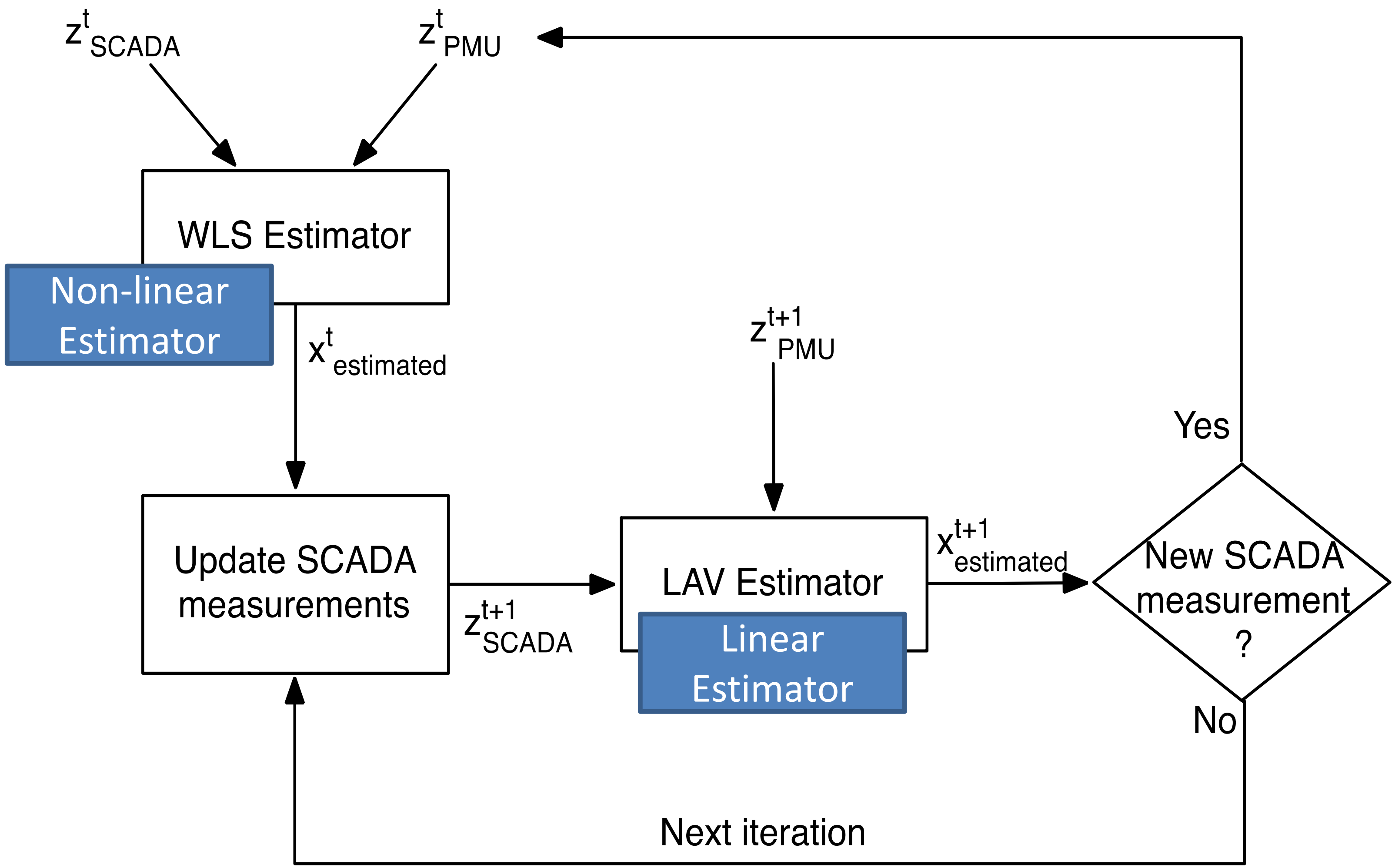


Hybrid SE Using Both SCADA and PMU Measurements

Challenge: Different scan rates of SCADA and PMU measurements.



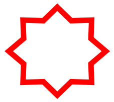
Possible Implementation of a Hybrid Scheme



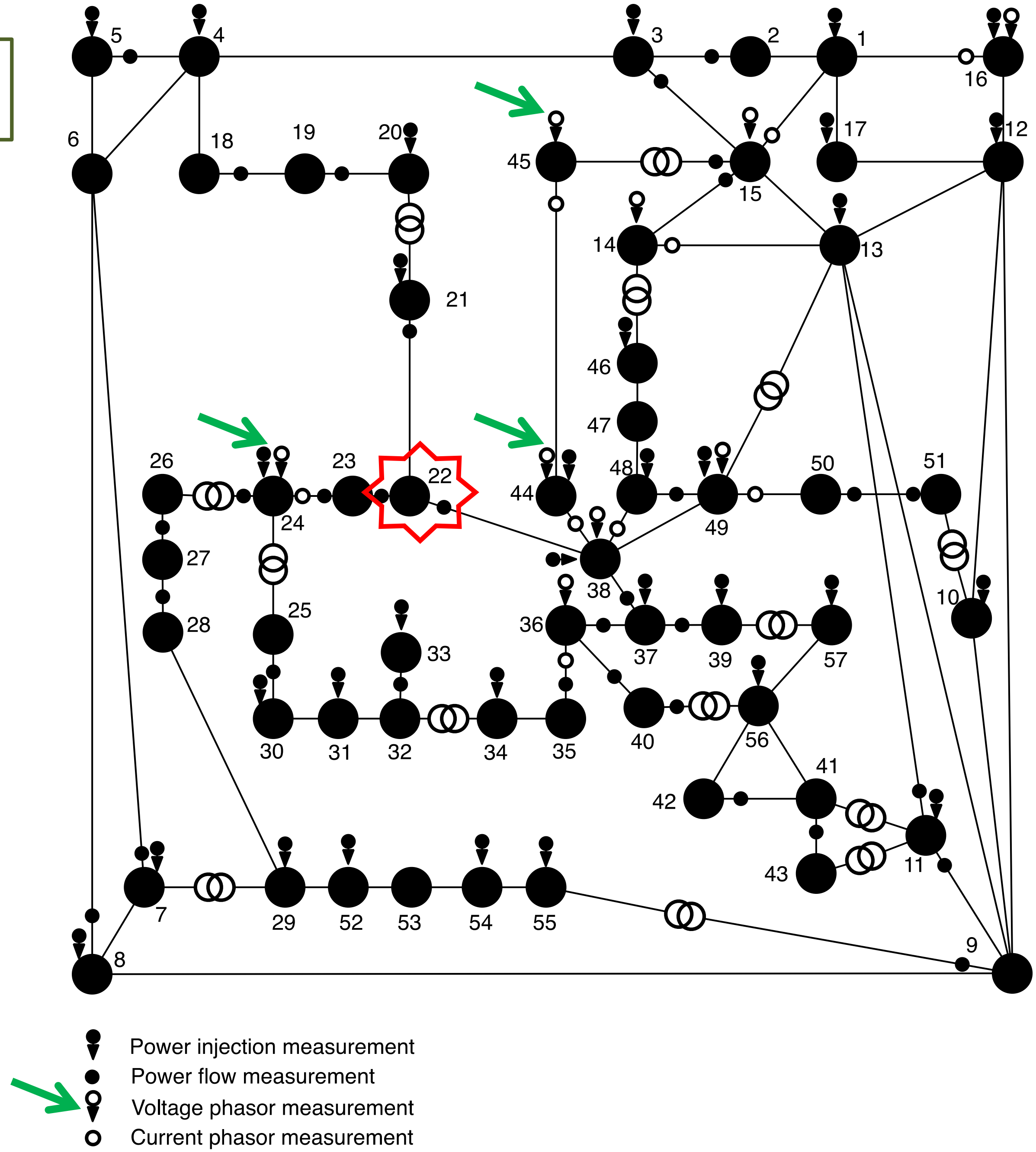
Test Case

- IEEE 57 bus system
  - 9 branch PMUs
  - 32 Power injection measurements
  - 32 Power flow measurements
- Voltage collapse at bus 22
  - No PMU at the bus.

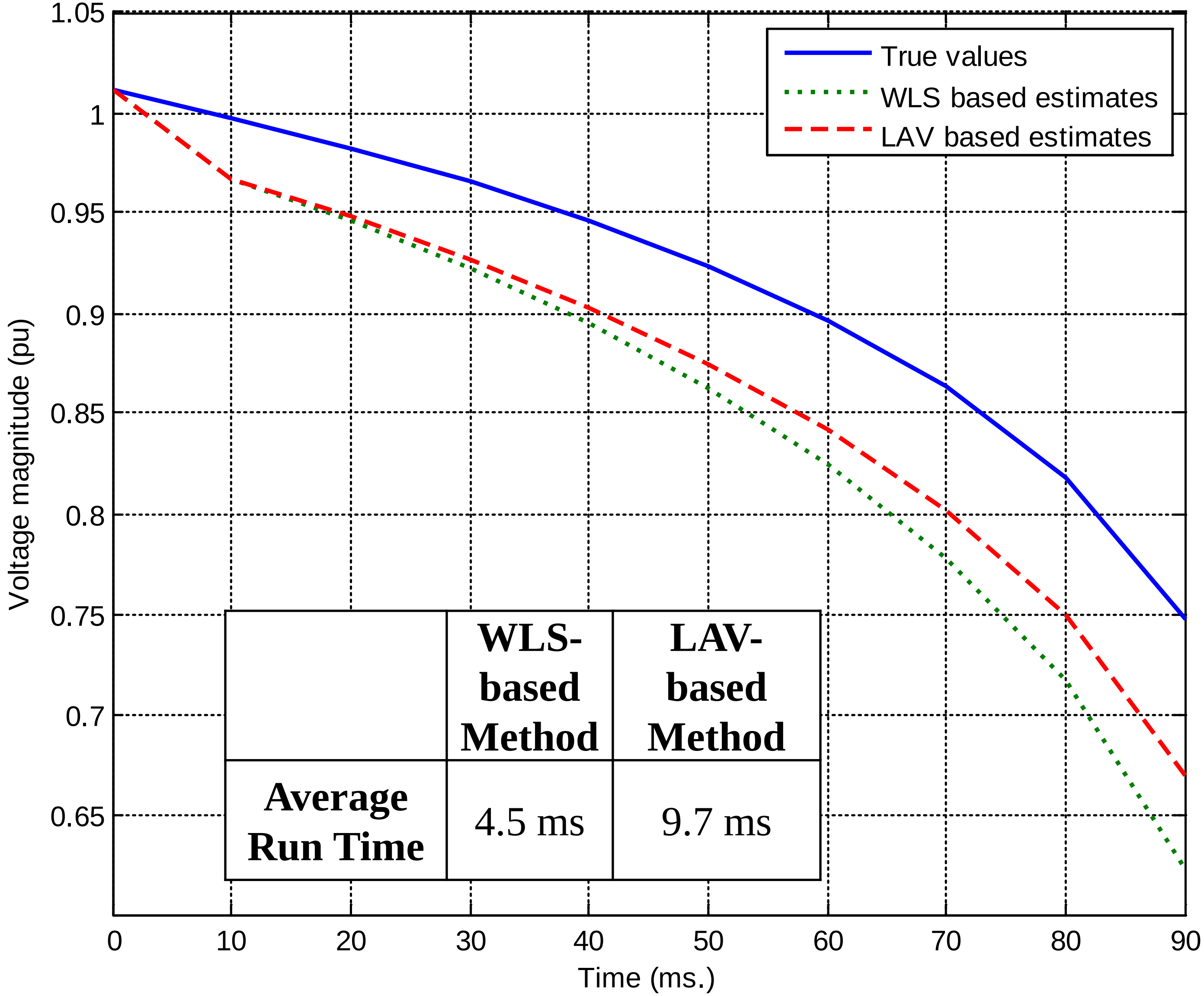
57-Bus System



Voltage Collapse
Bus Location



Voltage at Bus 22 Tracked by the Two Estimators



	WLS-based Method	LAV-based Method
Average Run Time	4.5 ms	9.7 ms

Historical Timeline

- Phasor Only State Estimation
  - Requires phasor based observability
  - Needs to be faster than scan rate of PMUs
  - Should handle bad data (detect and remove)

Incorporation of PMUs in State Estimators:
Using Only Synchronized Phasor Measurements

PMU-Only State Estimator:
- Use of only phasor measurements
- Use of robust estimation methods

Challenges:
- Requires a large number of PMUs for full observability
- Estimation should be faster than PMU scan rate
- Robustness should preferably be built-in

Measurement equations

$$SCADA\ Measurements$$

$$Z = h(X) + \upsilon \quad Non-linear\ Model$$

$$H_x : \nabla h(X)$$

$$Phasor\ Measurements$$

$$Z = H \cdot X + \upsilon \quad Linear\ Model$$

$$H \ : Function\ of\ network\ parameters\ only$$

A.G. Phadke, J.S. Thorp, and K.J. Karimi, "State Estimation with Phasor Measurements", IEEE Transactions on Power Systems, vol. 1, no.1, pp. 233-241, February 1986.

Phasor-only WLS state estimation

$$Z = H \cdot X + \upsilon \quad \textit{Linear Model}$$

$$\textit{WLS state estimation problem:}$$

$$\textit{Minimize} \quad \sum_{i}^{m} \frac{r_i^2}{\sigma_i^2}$$

$$\textit{Subject to} \quad r = Z - H \cdot \hat{X} \quad \textit{residual}$$

$$\hat{X} = G^{-1} H^T R^{-1} Z \quad \textit{Direct solution}$$

$$G = H^T R^{-1} H \; ; R = E\{\upsilon \cdot \upsilon^T\} = \operatorname{cov}(\upsilon)$$

$$\sigma_i^2 : R(i,i) \; \textit{error variance}$$

Phasor-only WLS state estimation

Consider a fully measured system:

$$Z^m = \begin{bmatrix} V^m \\ I^m \end{bmatrix} \begin{matrix} \Rightarrow \textit{Bus voltages} \\ \Rightarrow \textit{Branch currents} \end{matrix}$$

$$= \begin{bmatrix} U \\ Y_b \cdot A \end{bmatrix} \cdot [V] + \upsilon$$

U : identity matrix

Y_b : branch admittance matrix

A : branch - bus incidence matrix

Note: Shunt branches are neglected initially, they will be introduced later.

Phasor-only WLS state estimation

$$Let \begin{bmatrix} V^m \\ I^m \end{bmatrix} = \begin{bmatrix} e^m + jf^m \\ c^m + jd^m \end{bmatrix}$$

$$= \begin{bmatrix} U \\ (g + jb) \cdot A \end{bmatrix} \cdot \left[e + jf \right] + \upsilon$$

$$= \left[H_F \right] \cdot \left[e + jf \right] + \upsilon$$

Phasor-only WLS state estimation: Complex to real transformation

$$\begin{bmatrix} e^m \\ f^m \\ c^m \\ d^m \end{bmatrix} = \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix} + \upsilon$$

$$[Z] = [H] \cdot \begin{bmatrix} e \\ f \end{bmatrix} + \upsilon$$

Phasor-only WLS state estimation: Exact cancellations in off-diagonals of [G]

R is assumed to be identity matrix without loss of generality

$$G = H^T \cdot H = \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}^T \cdot \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}$$

$$= \begin{bmatrix} U + A^T\left(g^T g + b^T b\right)A & 0 \\ 0 & U + A^T\left(b^T b + g^T g\right)A \end{bmatrix}$$

[G] matrix:

- Is block – diagonal
- Has identical diagonal blocks
- Is constant, independent of the state

Phasor-only WLS state estimation: Correction for shunt terms

$$[Z] = (H + H_{sh}) \cdot X + \upsilon = H \cdot X + u$$

$$u = H_{sh} \cdot X + \upsilon$$

$$E\{u\} = H_{sh} \cdot E\{X\}$$

$$E\{X\} = \hat{X} = G^{-1} H^T R^{-1} Z$$

$$X^{corr} = G^{-1} H^T R^{-1} (Z - H_{sh} \cdot \hat{X})$$

$$= \hat{X} - G^{-1} \underbrace{H^T R^{-1} H_{sh}}_{\text{Very sparse}} \cdot \hat{X}$$

Fast Decoupled WLS Implementation Results

Test Systems Used

System Label	Number of Buses	Number of Branches	Number of Phasor Measurements
A	159	198	222
B	265	340	361
C	3625	4836	4982

Cases simulated:

Case-1: No bad measurement.

Case-2: Single bad measurement.

Case-3: Five bad measurements.

Fast Decoupled WLS Implementation Results

MEAN CPU TIMES OF 100 SIMULATIONS

System	Case	CPU Times (ms)	
		WLS	Decoupled WLS
A	1	5	2.4
	2	5.7	2.7
	3	9.3	3.9
B	1	7.5	3.5
	2	8.7	3.9
	3	14.8	5.8
C	1	137.4	75.9
	2	169.5	95.7
	3	284.7	165.6

L_1 (LAV) Estimator

$$Minimize \ \sum_{i=1}^{m} c^T \cdot |r|$$

$$Subject\ to \ \ Z = H \cdot \hat{X} + r$$

$$c^T = [1,1,\ldots,1]$$

Robust against gross errors

L_1 estimator

- Computationally efficient. Fast Linear Programming (LP) code exists to solve large scale systems.

- L_1 estimator automatically rejects bad data given sufficient local redundancy, hence bad data processing is built-in.

Conversion to Equivalent LP Problem

$$\min \mathrm{c}^T |r| \quad \text{s.t. } Hx + r = z$$

$$\min \mathrm{c}^T y \quad \text{s.t. } My = z, \quad y \geq 0$$

$$\mathrm{c}^T = [0_n \quad 0_n \quad c_m \quad c_m]$$

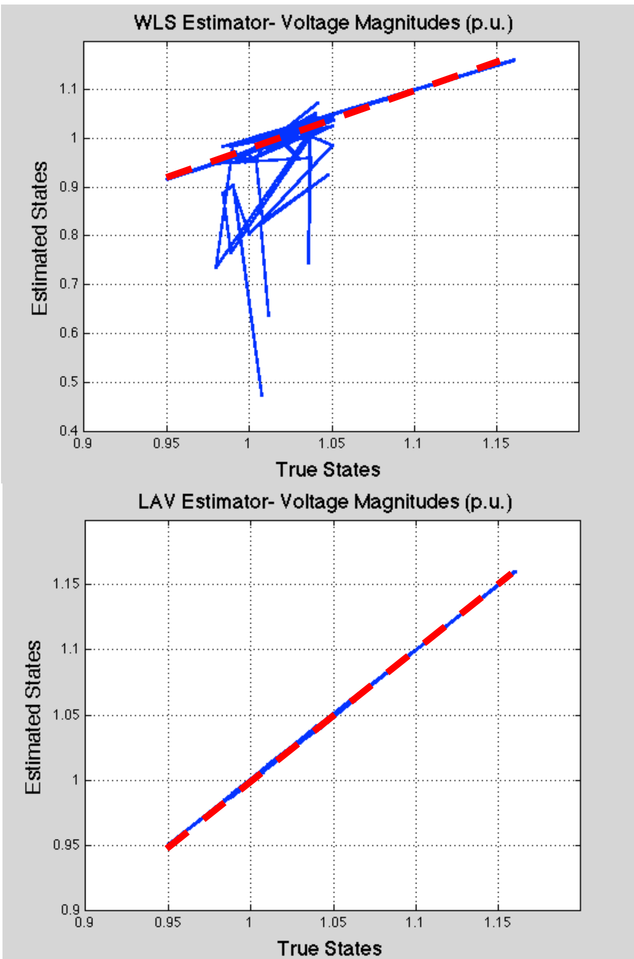
$$y = [X_a^T \quad X_b^T \quad U^T \quad V^T]^T \qquad x = X_a - X_b$$

$$M = [H \quad -H \quad I \quad -I] \qquad r = U \ \ -V$$

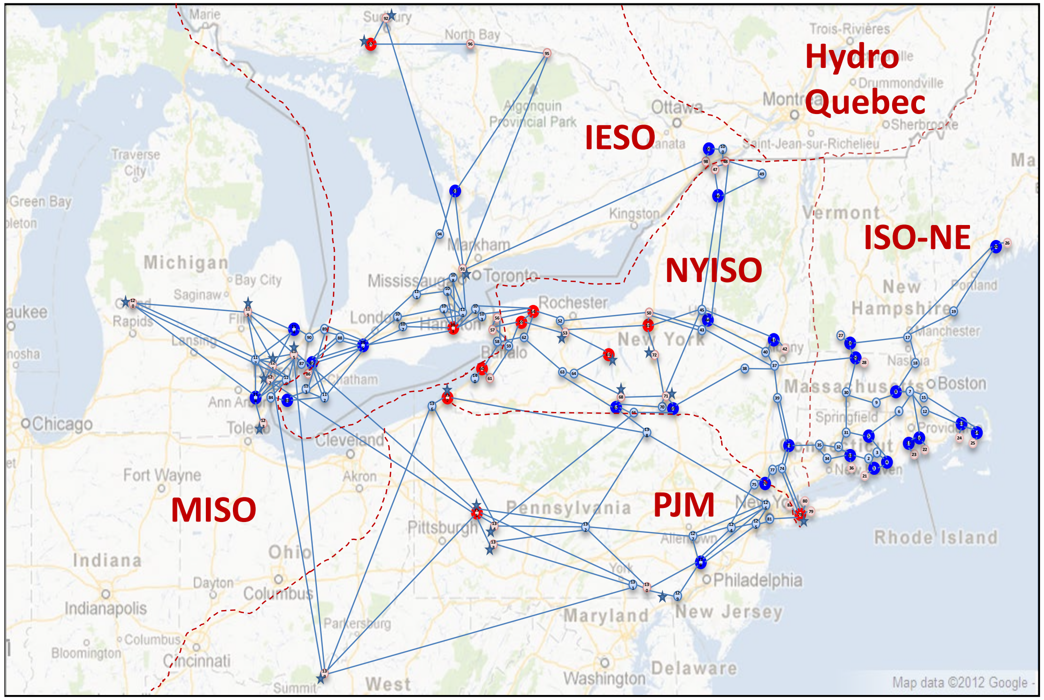
Phasor-Only Robust State Estimation

Objectives

- Perform static state estimation using a redundant set of PMU measurements
- Maintain robustness against bad data



Robust LAV Based Estimator Testbed



140-Bus NPCC System

3625 Bus + 4836 Branch Utility System

Case a: No bad measurement.

Case b: Single bad measurement.

Case c: Five bad measurements.

	Case a	Case b	Case c
LAV	3.33 s.	3.36 s.	3.57 s.
WLS	2.32 s.	9.38 s.	50.2 s.

Phasor-only state estimation: WLS versus L_1 (LAV)

WLS :

- Linear solution (exact cancellations in [G] leading to decoupled formulation)

- Requires bad-data analysis
  - Normalized residuals test (CPU increases with BD)

L_1 (LAV):

- Linear programming (computationally competitive with WLS)

- Built-in bad-data analysis (CPU is relatively insensitive to BD)

Historical Timeline

- Dynamic State Estimation
  - Load and generator dynamic models
  - Wide-area versus local estimation
  - Tool to facilitate dynamic security assessment

Basic Formulation

Dynamic state vector for the generators is augmented by the vector of all bus voltage magnitudes and phase angles.

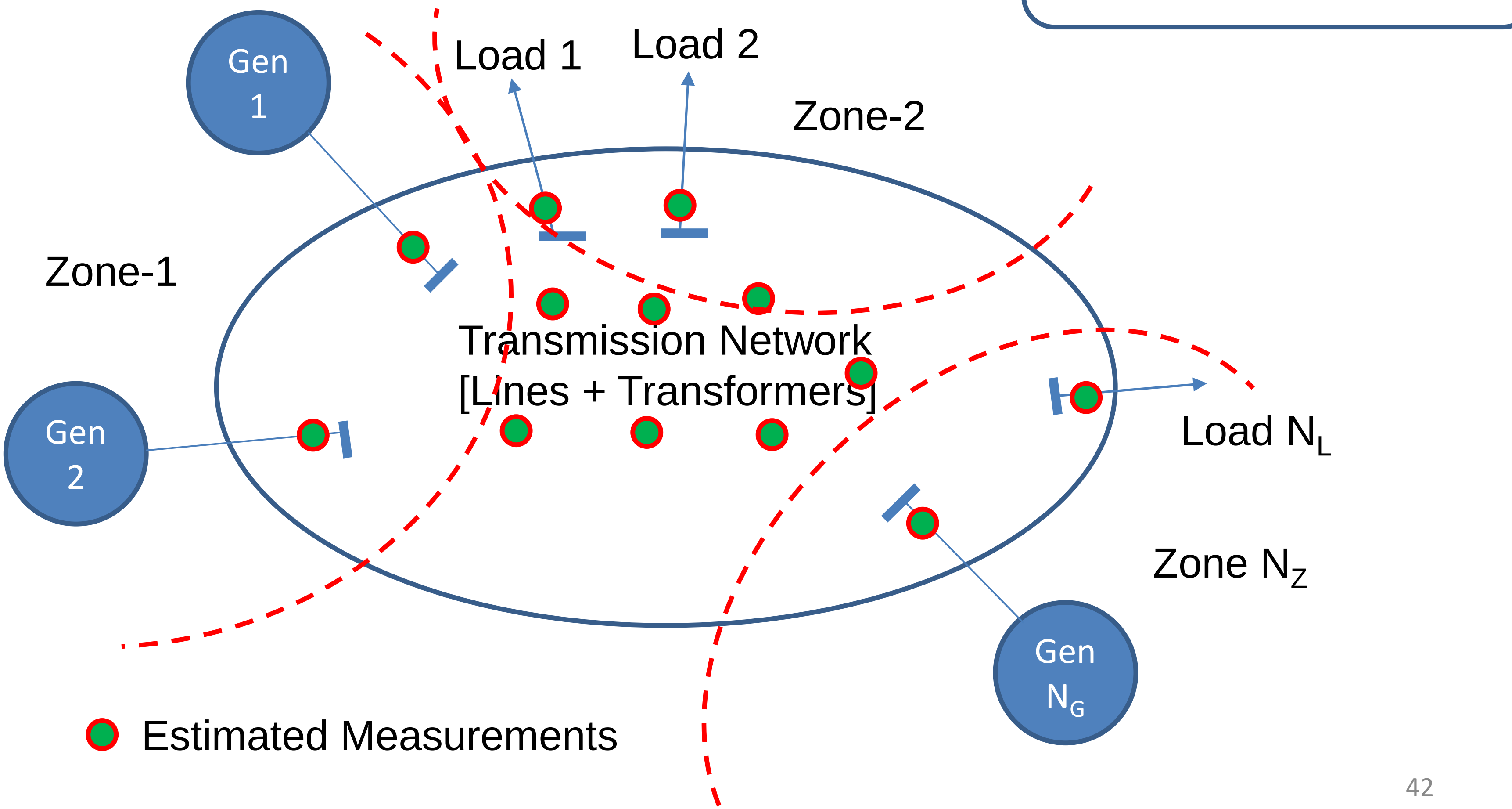
Considering a system with *N* buses, the augmented state vector will be:

$x_k^T = [\delta_k^T \ \omega_k^T \ V_k^T \ \theta_k^T]$ at time instant *k*

Modeling [DSE]

DSE: Single machine / zonal / wide-area
Detailed Gen and Load models
Frequency and power angle estimation

Tracking the network and dynamic gen/load state variables in real-time



Basic Formulation

- Use all available measurements to form *z*
- Discretize the dynamic state and measurement equations
- Form the set of discrete time equations:

$$x_{k+1} = f(x_k, k) + \upsilon_k$$

$$z_k = h(x_k, k) + e_k$$

Extended Kalman Filter

- ***Prediction:***

$$\boxed{\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+, k-1)}$$

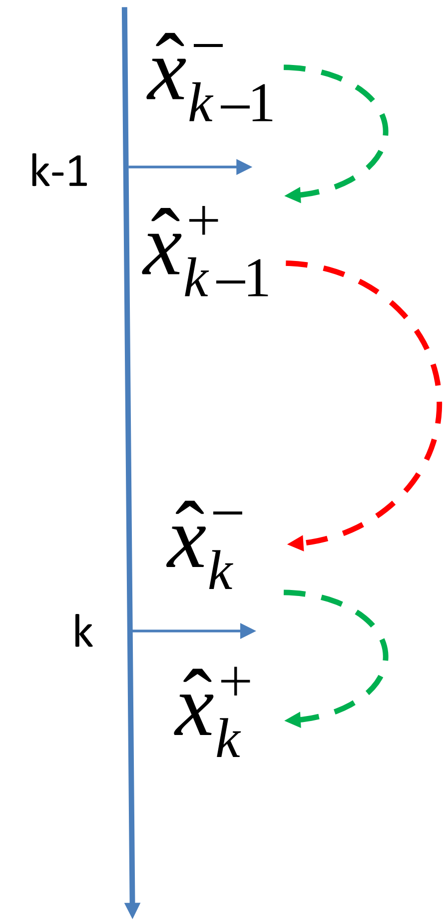
$$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T$$

- ***Correction:***

$$\boxed{\hat{x}_k^+ = \hat{x}_k^- + K_k \left(z_k - h(\hat{x}_k^-, k) \right)}$$

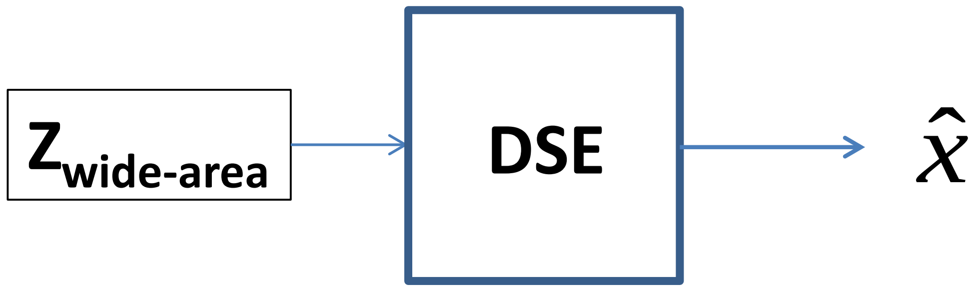
$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + M_k R_k M_k^T \right)^{-1}$$

$$P_k^+ = \left(I - K_k H_k \right) P_k^-$$

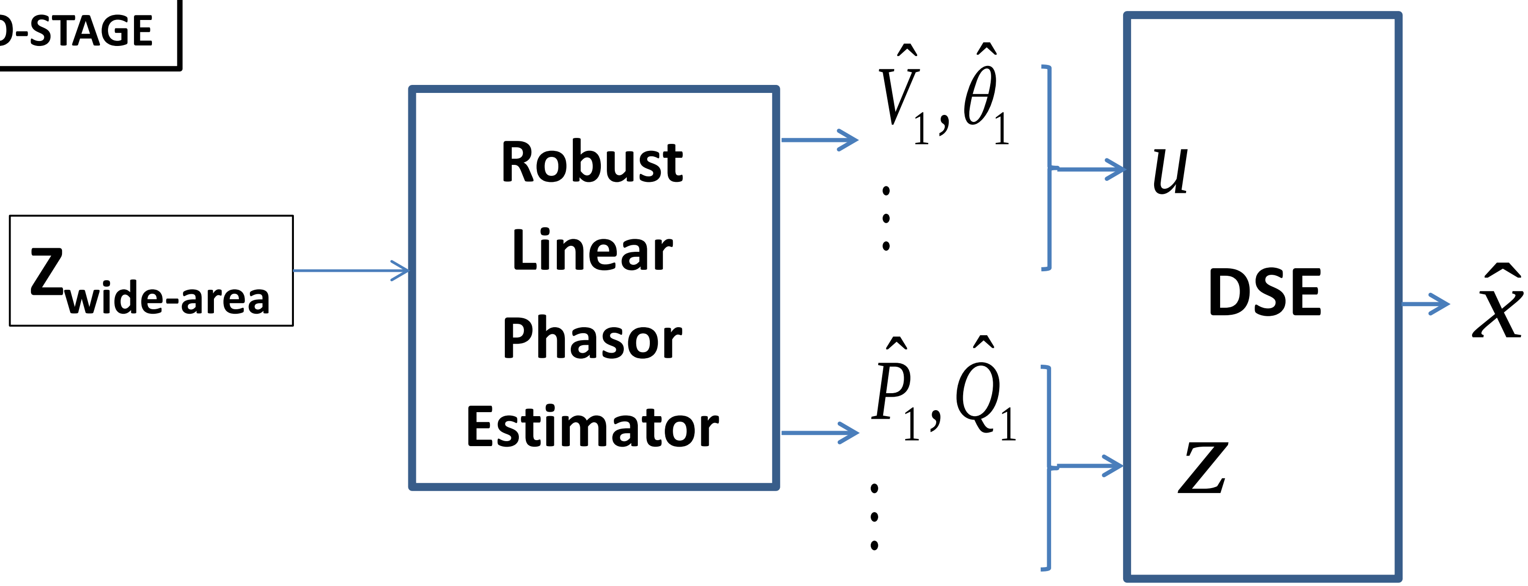


DSE: Direct and Two-Stage Implementations

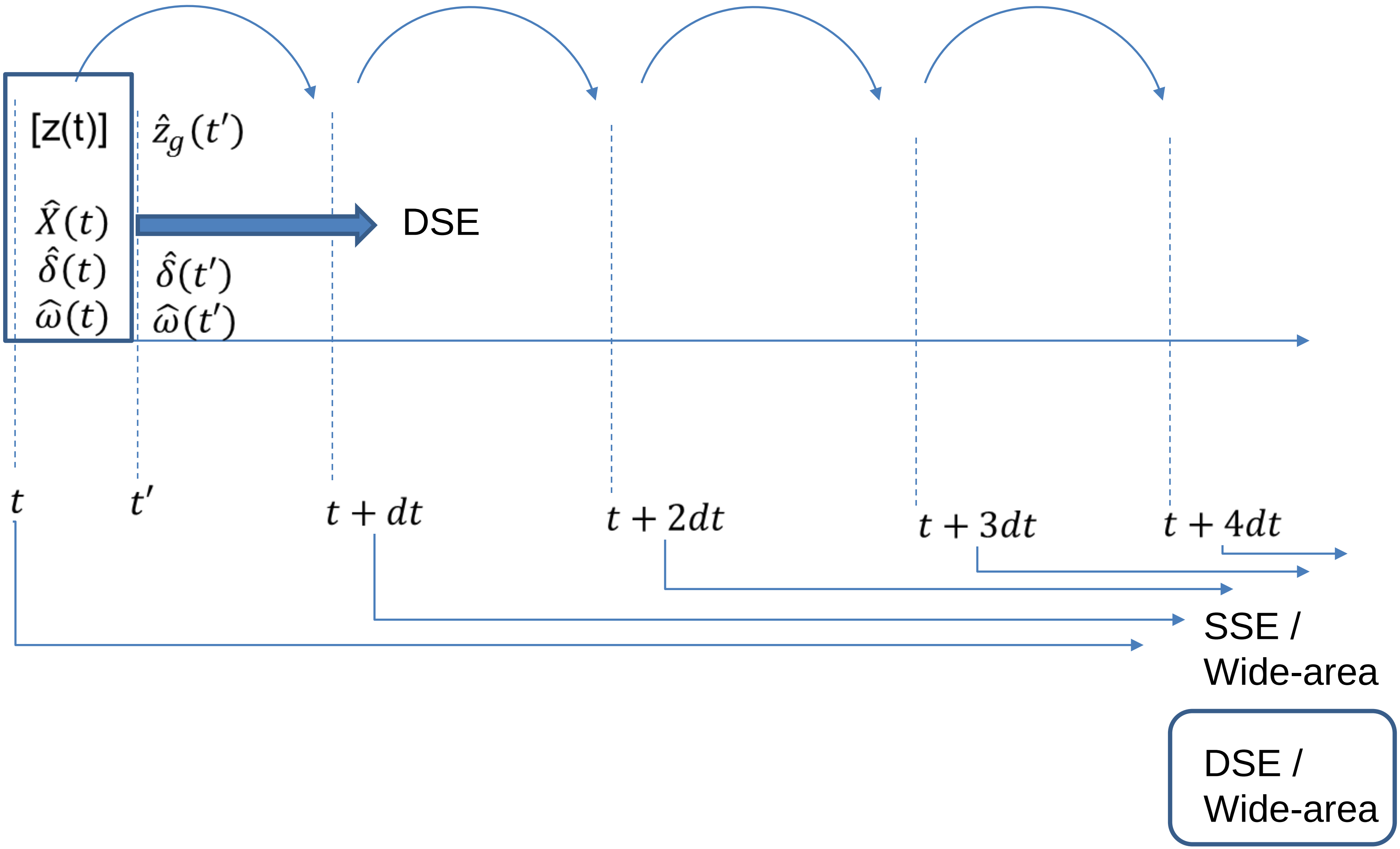
DIRECT



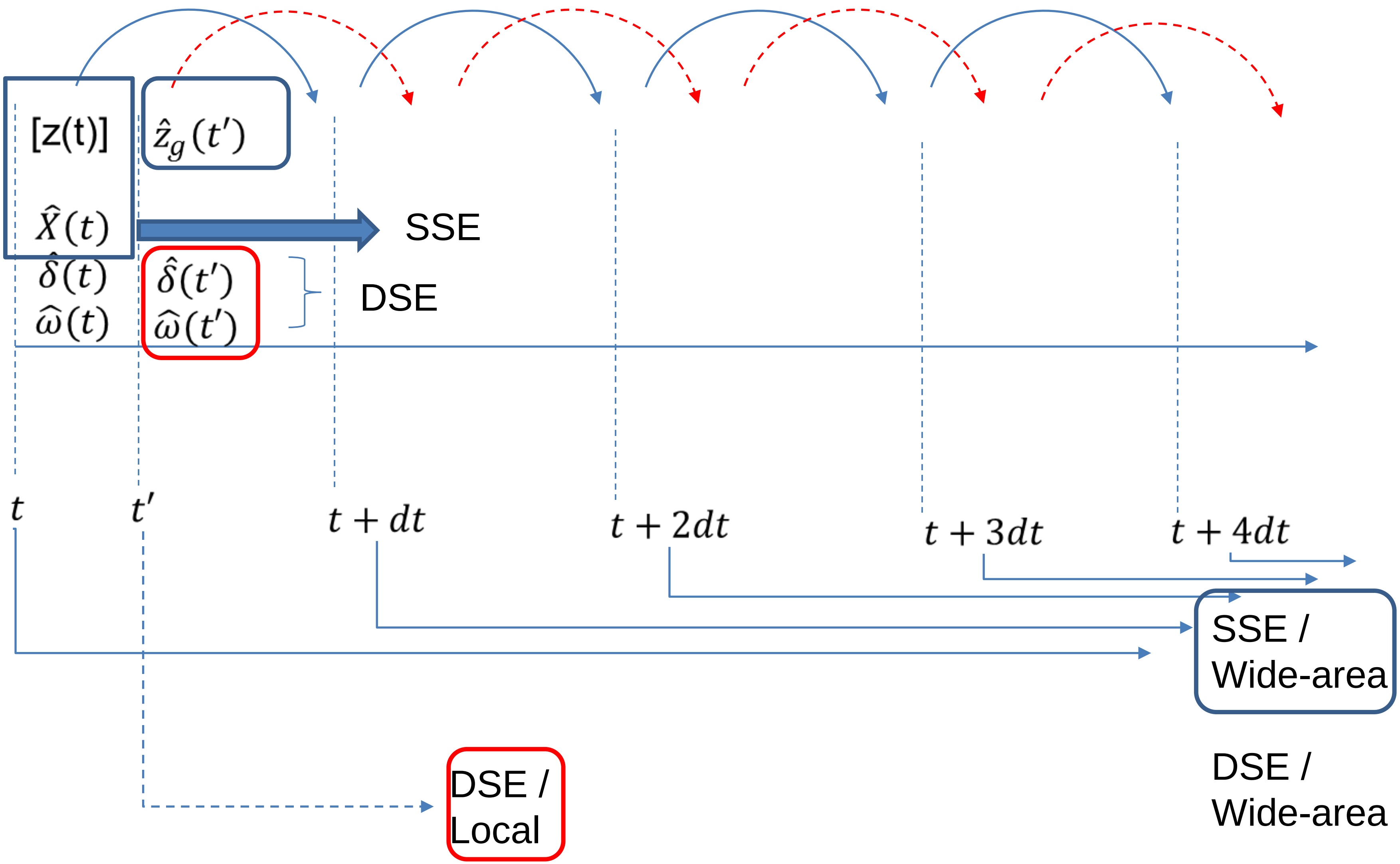
TWO-STAGE



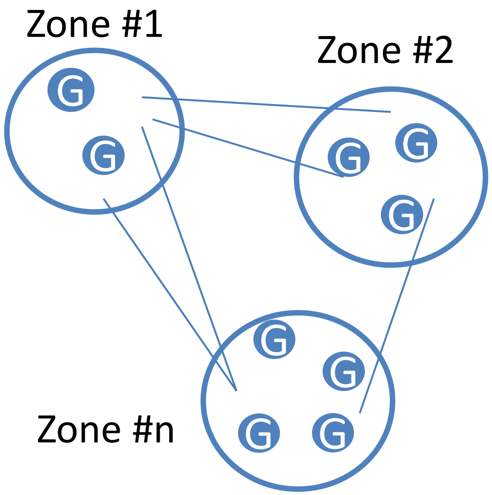
Timeline: Centralized/Direct DSE

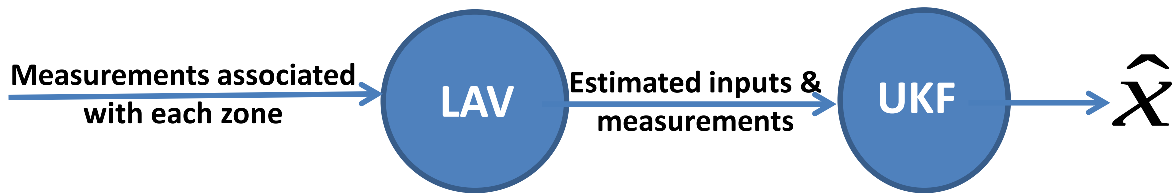


Timeline: Local/Two-Stage DSE

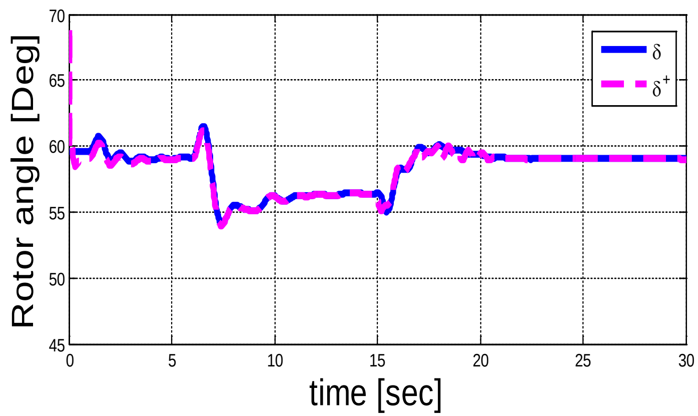


Robust Dynamic State Estimation

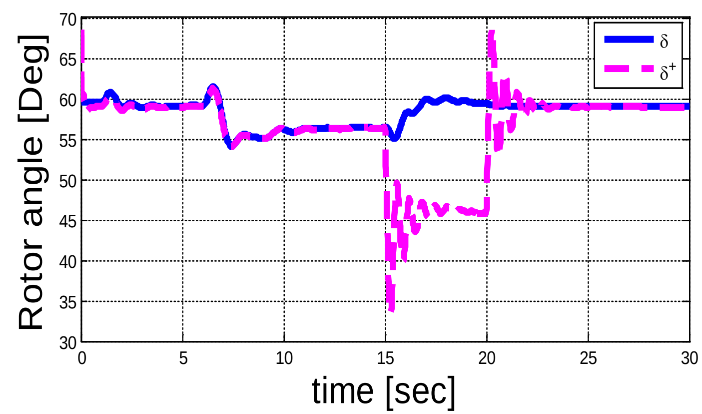




Bad data present starting at t=15 sec. until y=20 sec.



Robust DSE Using LAV estimates



DSE Using Raw Measurements

Observability Analysis for Time Varying Systems

For a time-varying dynamic system, the outcome of observability analysis will also be time dependent.

Outcome of observability analysis will no longer be binary, but the degree (or strength) of observability for a given measurement set at a given time instant will be of interest.

One metric to quantify this strength is the smallest singular value of the approximated observability matrix.

Approach

Two Alternative Observability Analysis Methods:

1. Use of small signal approximation and compute observability matrix for linear dynamic systems.

2. Use Lie derivatives to compute the observability matrix and its smallest singular value.

Linear Time-Invariant Discrete-Time System

$$X(k+1) = A\,X(k) + B\,U(k)$$
$$Z(k) = C\,X(k) + D\,U(k)$$

Observability matrix:

$$\tilde{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Dynamic system will be observable if the row rank of $\tilde{O}$ is equal to *n* (dimension of the state vector).

Small Signal Approximation

First order approximation for matrices *A* and *C* can be calculated at discrete time step *k*:

$$A_k = \frac{\partial f}{\partial x}\Big|_{x=x_k}\ ; \quad C_k = \frac{\partial h}{\partial x}\Big|_{x=x_k}$$

First order approximation for matrices *A* and *C* can be calculated at discrete time step *k*, yielding the approximate observability matrix ($\tilde{O}_k$) :

$$\tilde{O}_k = \begin{bmatrix} C_k \\ C_k A_k \\ \vdots \\ C_k A_k^{n-1} \end{bmatrix}$$

Results

MEAN AND STANDARD DEVIATION OF THE SMALLEST SINGULAR VALUE OF THE APPROXIMATED OBSERVABILITY MATRIX

Measurement	δ	ω	δ and ω	E'_q
Mean	**0.135**	**0.052**	**0.205**	**0.158**
STD	**0.006**	**0.002**	**0.009**	**0.001**

Measurement	E'_d	E'_{fd}	V_f	V_R
Mean	**0.062**	**0**	**0**	**0**
STD	**0.001**	**0**	**0**	**0**

Measurement	P_e	Q_e	All of the state variables
Mean	**0.003**	**2.077**	**1.067**
STD	**0.0005**	**0.041**	**0.009**

Observability Analysis for Time Varying Systems

Pro:
The main advantage of the linear approximation based approach is its computational simplicity.

Con:
Linear approximation based results may occasionally be highly inaccurate in particular under highly nonlinear operating conditions.

Observability Analysis: NL Systems

In case of nonlinear systems, observability will no longer be a global property but will be determined locally around a given operating state or equilibrium point.

This can be done via the use of Lie derivatives of the nonlinear measurement function *h* with respect to the nonlinear function describing system dynamics [*].

[*] K. Muske and T. Edgar, Nonlinear State Estimation, Prentice-Hall, 1997.

Observability Analysis: NL Systems

$$\dot{X} = f\big(x(t)\big) + u(x(t))$$

$$Z = h\big(x(t)\big)$$

Lie derivative of *h* with respect to *f* will be given by:

$$L_f h = \nabla h \cdot f$$

By definition:

$$L_f^0 h = h$$

$$L_f^k h = \frac{\partial (L_f^{k-1} h)}{\partial X} \cdot f$$

Observability Analysis: NL Systems

Defining Ω as:

$$\mathbf{\Omega} = \begin{bmatrix} L_f^0(h_1) & \cdots & L_f^0(h_m) \\ L_f^1(h_1) & \cdots & L_f^1(h_m) \\ \vdots & \cdots & \vdots \\ L_f^{n-1}(h_1) & \cdots & L_f^{n-1}(h_m) \end{bmatrix}$$

and a gradient operator as:

$$\mathrm{O} = d\Omega = \begin{bmatrix} dL_f^0(h_1) & \cdots & dL_f^0(h_m) \\ dL_f^1(h_1) & \cdots & dL_f^1(h_m) \\ \vdots & \cdots & \vdots \\ dL_f^{n-1}(h_1) & \cdots & dL_f^{n-1}(h_m) \end{bmatrix}$$

The observability matrix "*O*" defined above must have full rank in order for the system to be observable.

Results:

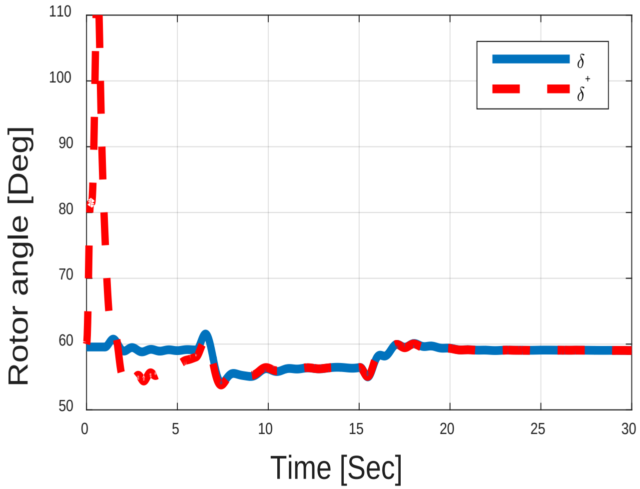
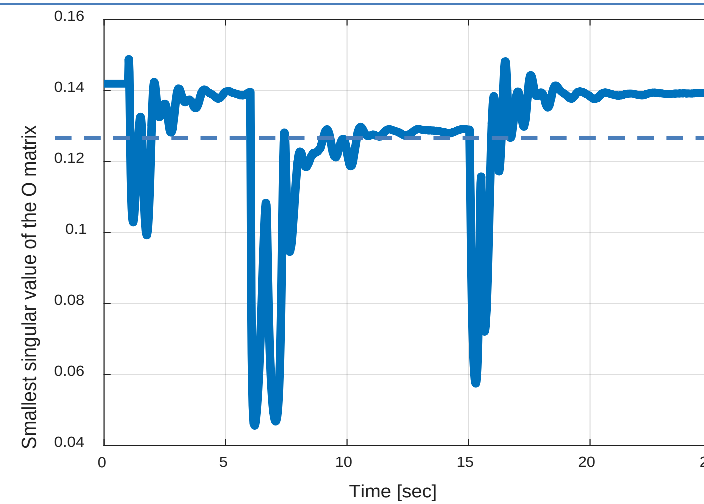
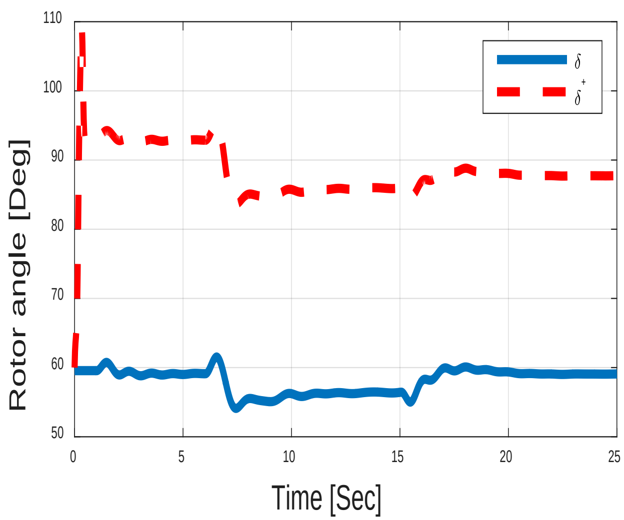
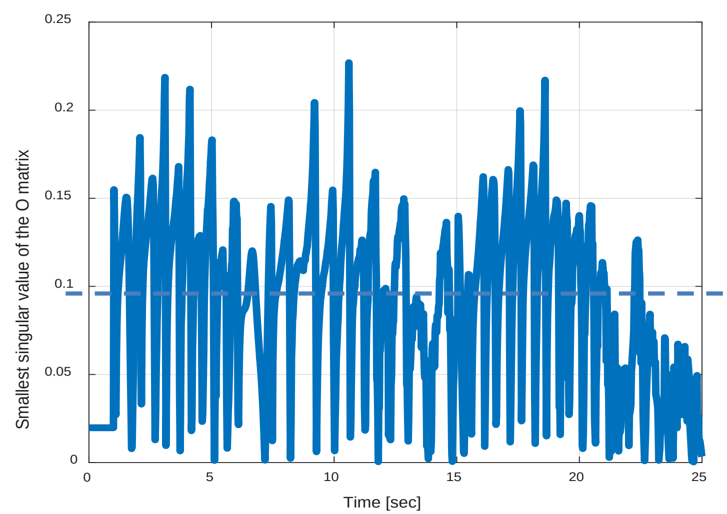
MEAN AND STANDARD DEVIATION OF THE SMALLEST SINGULAR VALUE OF THE OBSERVABILITY MATRIX- USING LIE-DERIVATIVES

Measurement	δ	ω	δ and ω	E'_q
Mean	0.127	0.09	0.191	0.158
STD	0.019	0.047	0.038	0.001

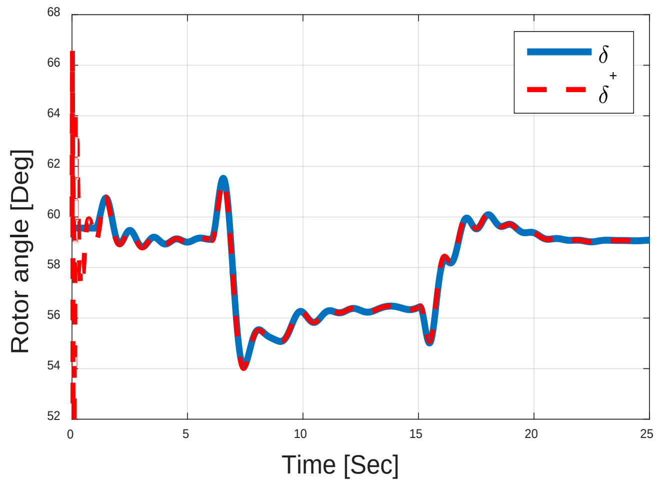
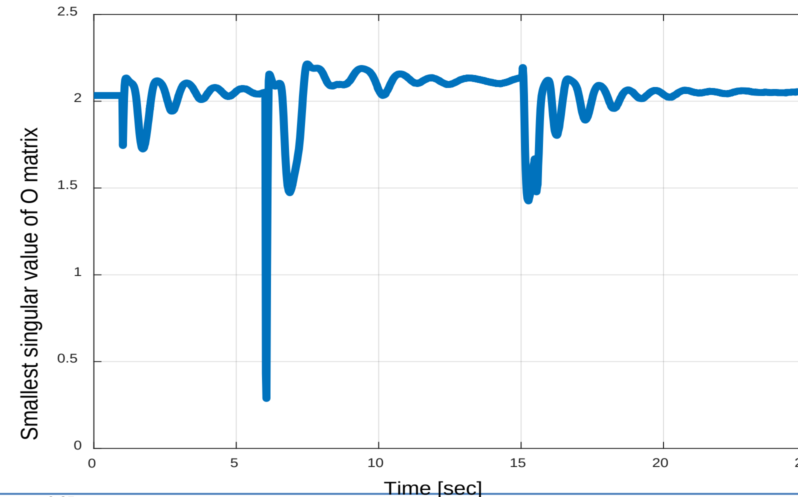
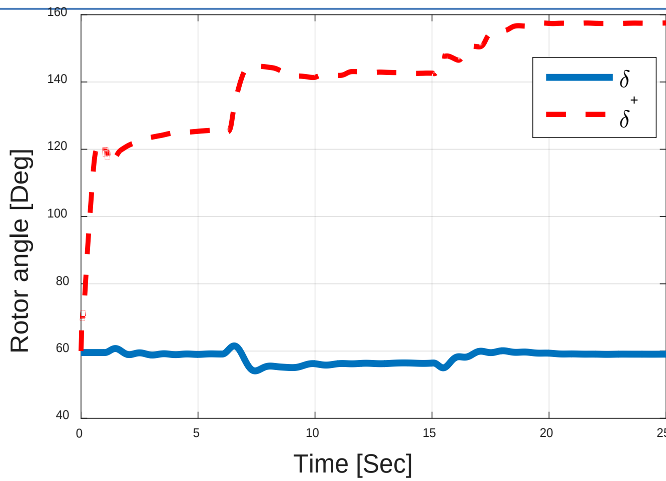
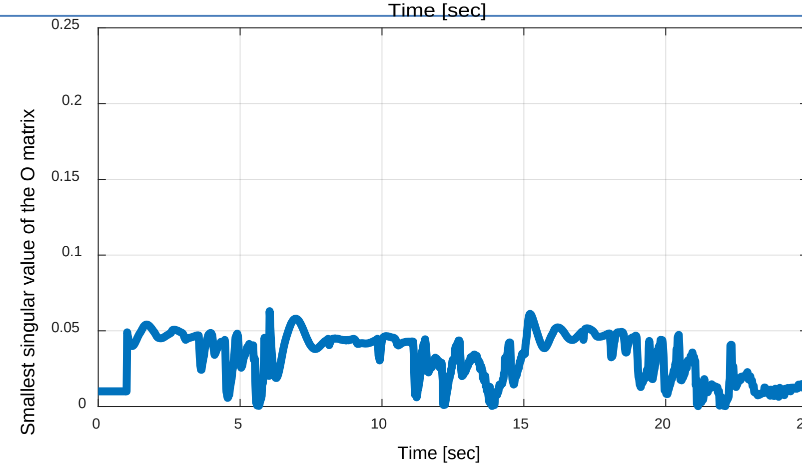
Measurement	E'_d	E'_{fd}	V_f	V_R
Mean	0.062	0	0	0
STD	0.003	0	0	0

Measurement	P_e	Q_e	All of the state variables
Mean	0.032	2.041	5.09
STD	0.015	0.14	2.07

Validation Via Simulations:

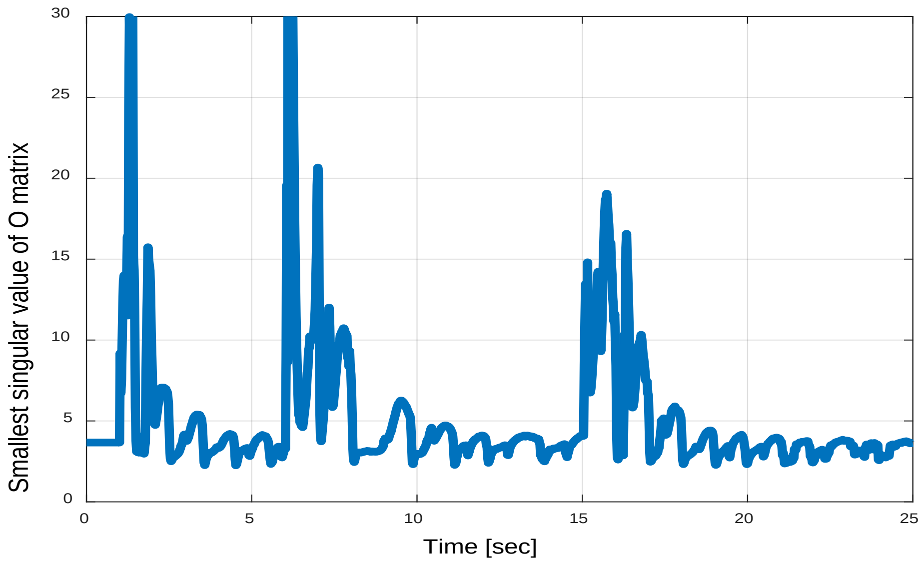
Meas.	Rotor Speed / Syn. speed	Smallest Singular Values	
δ			0.127
ω			0.09

Validation Via Simulations:

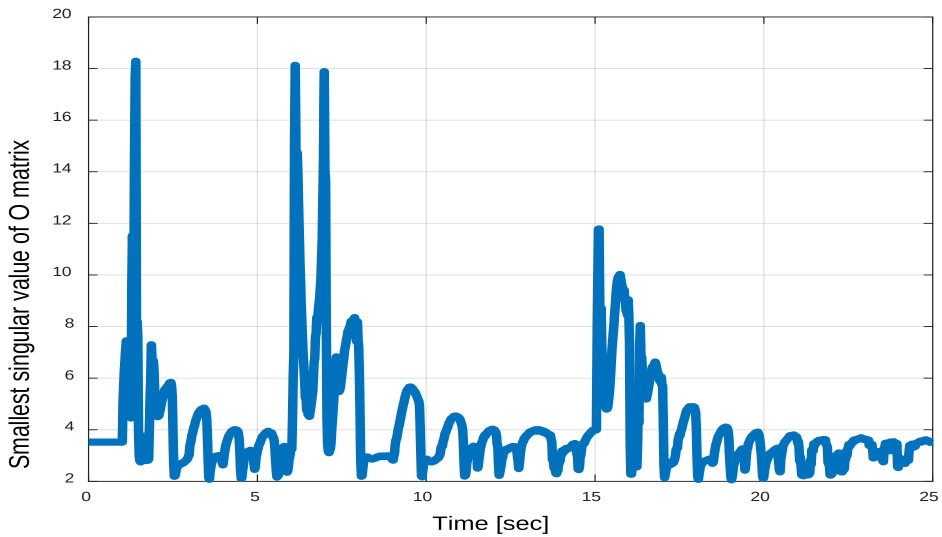
Meas.	Rotor Speed / Syn. speed	Smallest Singular Values	
Q_e			2.0
P_e			0.03

Smallest Singular Value Plots

Using all state variables



Using P_e and Q_e



Remarks and Conclusions

- Use of only phasor measurements simplifies the problem formulation and enables direct (non-iterative) solution.
- Hybrid SE can be beneficial in tracking system states during slow moving emergencies.
- LAV-estimator becomes a computationally competitive and robust alternative to WLS when using PMUs.
- Strength of observability for different measurement configurations appears to be consistent with the ability of the DSE to track the true trajectory of the dynamic states.
- Observability analysis can facilitate sensor selection for optimal tracking of dynamic states.

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Thank You

Any Questions?