



REACTIVE POWER AND VOLTAGE CONTROL

Salvador Acha Daza, Ph. D.

IEEE Distinguished Power Lecturer

June 2014

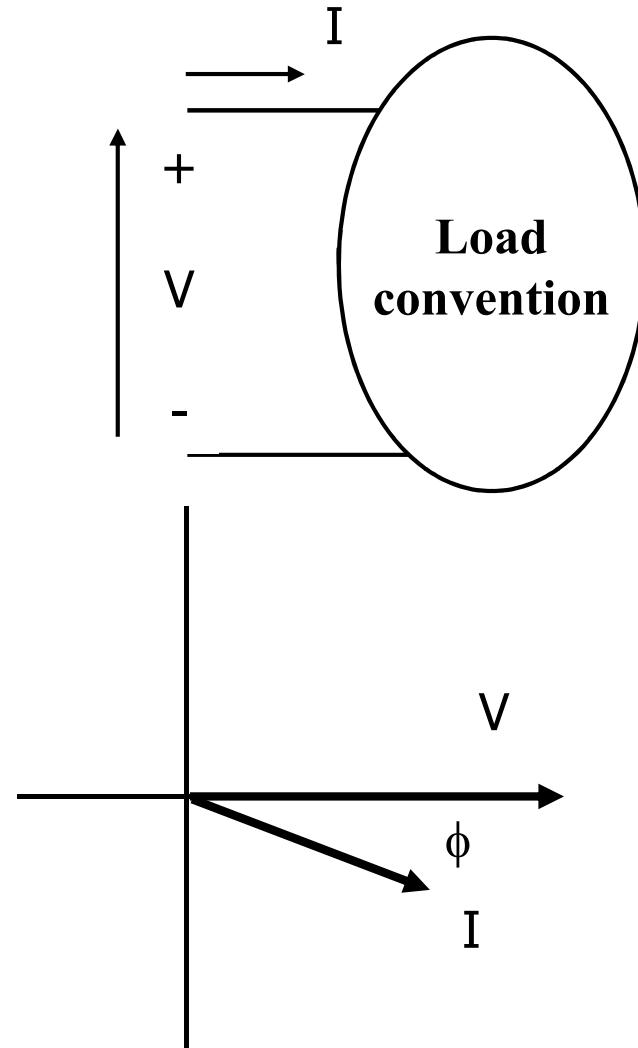
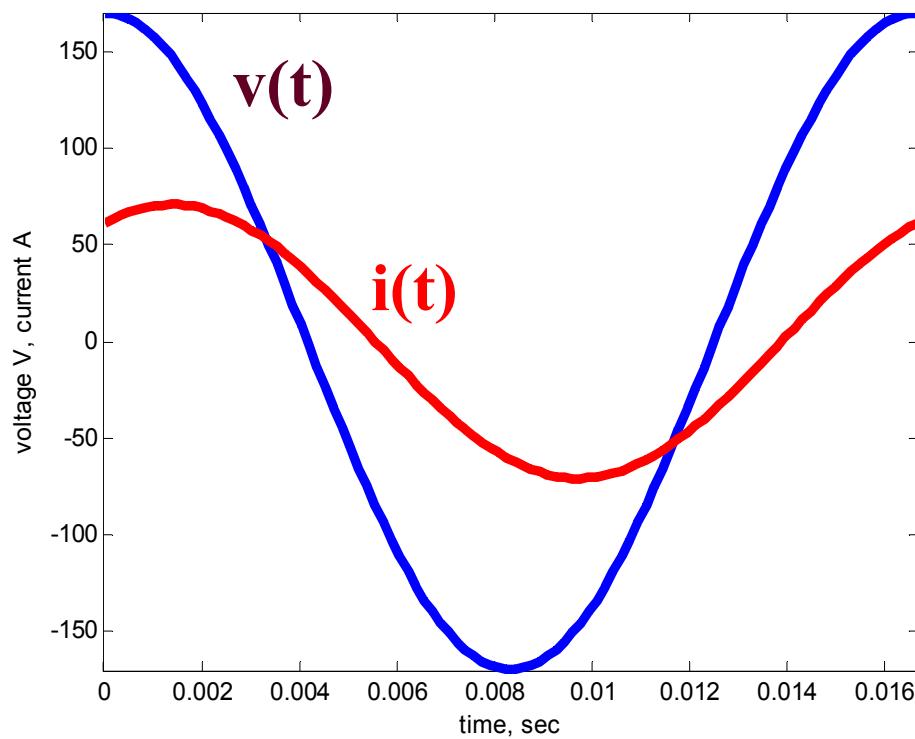
Guide

- 1 MODELING
 - Assumptions and V, I, P, Q
 - Transformers, Transmission Lines, Generators
 - SVC's (static VAR compensators)
- 2 BASICS ABOUT VOLTAGE CONTROL
 - Q-V relation
 - Reactive power flow and incremental model
 - Decoupled Load-Flow
 - Tap's control and generalized control
- 3 SENSITIVITY Q-V AND COORDINATED CONTROL
 - Sensitivity coefficients
 - Application and **coordinated** control
 - Q-V Congestion
 - Radial networks
- 4 VOLTAGE COLLAPSE
 - Angle stability
 - Voltage collapse

Modeling

Single phase, steady state: Voltage, Current

- AC voltage and current, $f = 60 \text{ Hz}$
- rms values: 120 V, 50 A
- If current lags voltage by 30°

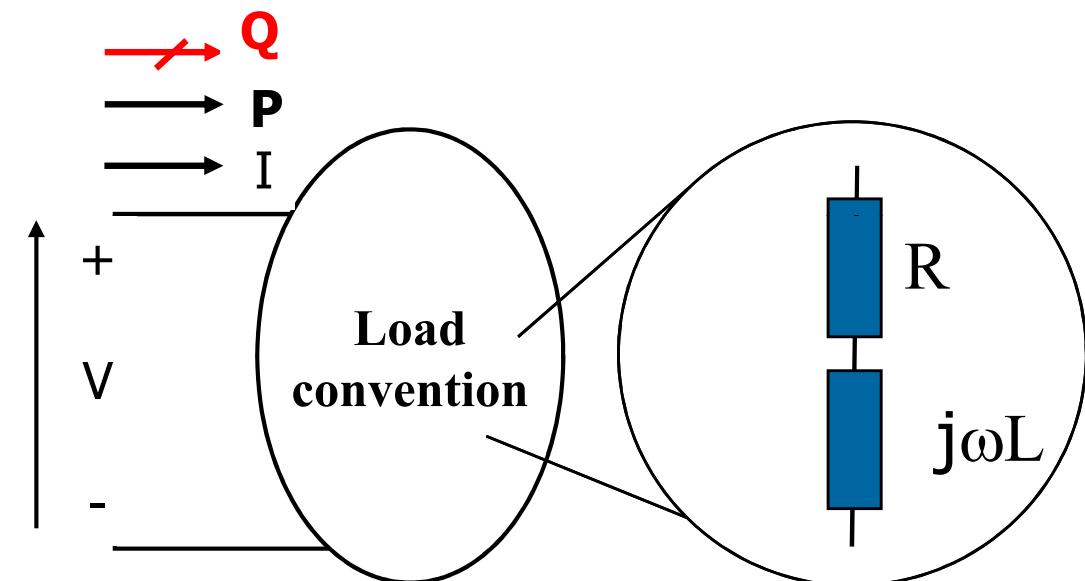
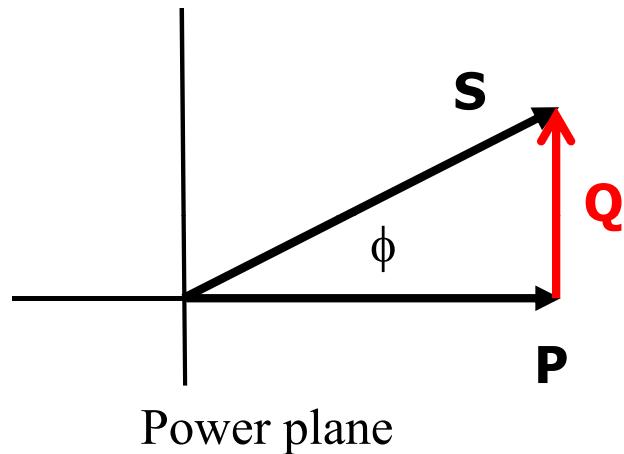


Phasor plane

Complex power S, P and Q

$$S = VI^* = P + jQ$$

$$S = (120\angle 0^\circ)(50\angle -30^\circ)^* = 5,196.1 + j3,000 \text{ VA}$$



- $\cos \phi$ is the **power factor**
- The case shows a “lagging power factor”

What is P and Q?

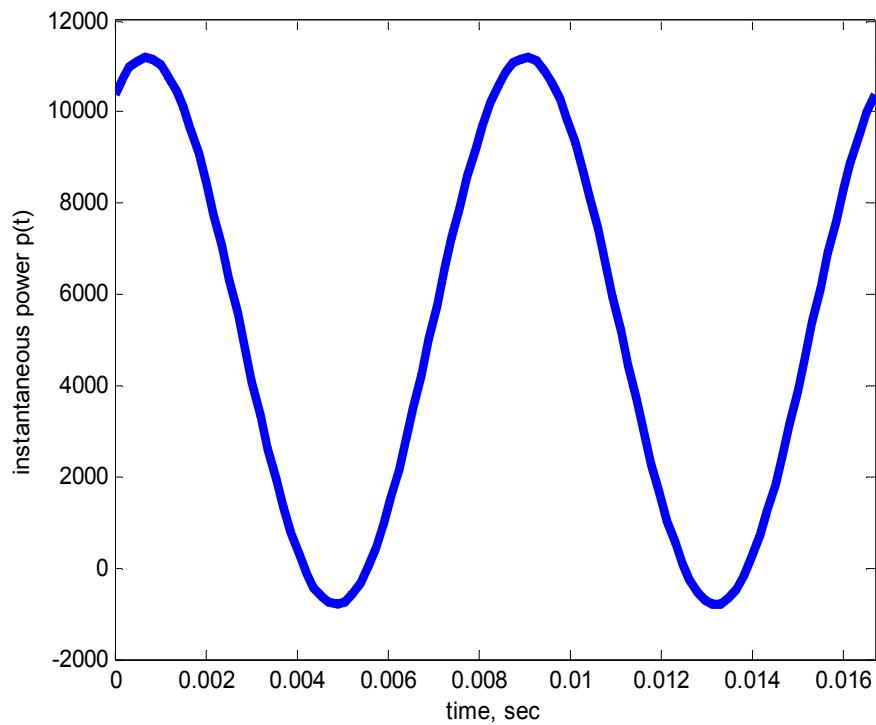
Instantaneous power $p(t)$:

$$p(t) = v(t) i(t) = V_m \cos(\omega t) I_m \cos(\omega t + \phi)$$

$$p(t) = VI \cos(\phi) (1 - \cos(2\omega t)) - VI \sin(\phi) \sin(2\omega t)$$

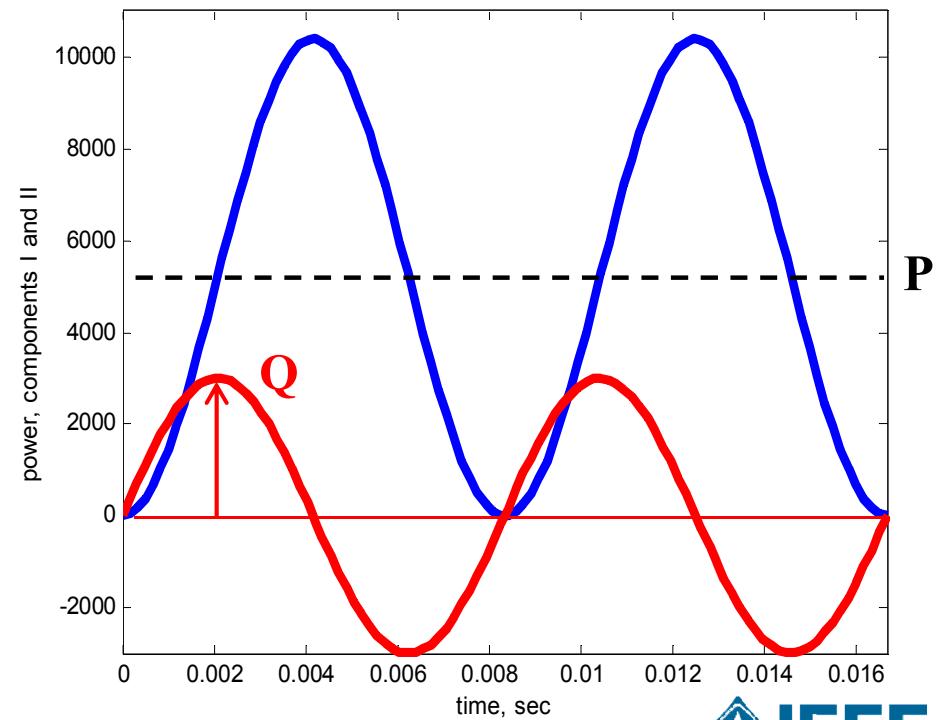
$$P = +VI \cos(\phi)$$

$$Q = -VI \sin(\phi)$$

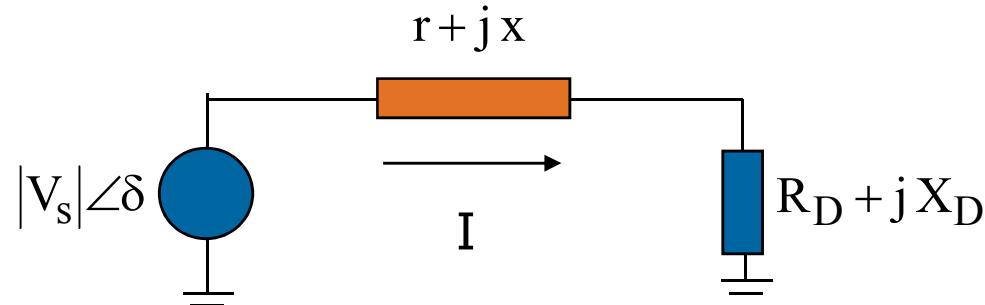


rms values

$$V = \frac{V_m}{\sqrt{2}}, \quad I = \frac{I_m}{\sqrt{2}}$$



Voltage and current, steady state



Positive sequence

$$I = \frac{|V_s| \angle \delta}{(r + R_D) + j(x + X_D)}$$

Assuming load and voltage as a fixed value (no use of load flow), $Z_{\text{Line}} = 0.05 + j0.5 \Omega$

Power factor effect on voltage, angle and losses

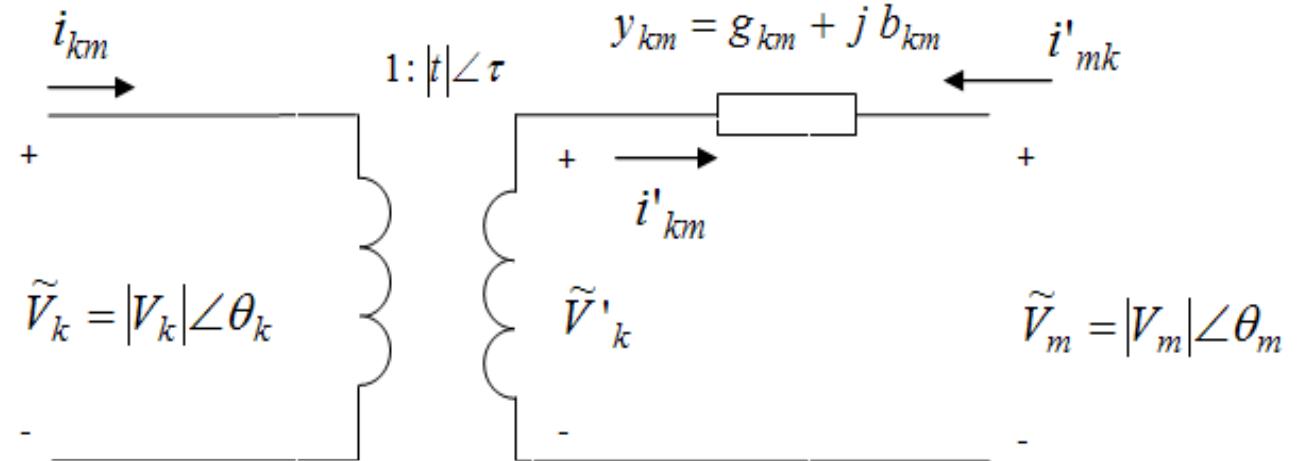
Nominal load voltage and current (rms values)

120.00 50.00

Voltage source	angle	PS	QS	PL	QL	pf(load)	Losses %
136.20	8.61	5321.15	4250.00	5196.15	3000.00	-0.87	2.35
131.01	10.33	5920.55	2802.91	5795.55	1552.91	-0.97	2.11
125.02	11.53	6125.00	1250.00	6000.00	0.00	+1.00	2.04
118.57	12.07	5920.55	-302.91	5795.55	-1552.91	+0.97	2.11
112.03	11.80	5321.15	-1750.00	5196.15	-3000.00	+0.87	2.35

Transformer Model

For an ideal transformer with complex relation



$$\frac{\tilde{V}_k}{\tilde{V}'_k} = \frac{1}{|t| \angle \tau}$$

$$\tilde{V}_k i_{km}^* = \tilde{V}'_k (i'_{km})^*$$

$$\frac{i_{km}^*}{(i'_{km})^*} = \frac{\tilde{V}'_k}{\tilde{V}_k} = |t| \angle \tau$$

Complex, real and reactive power, from k to m:

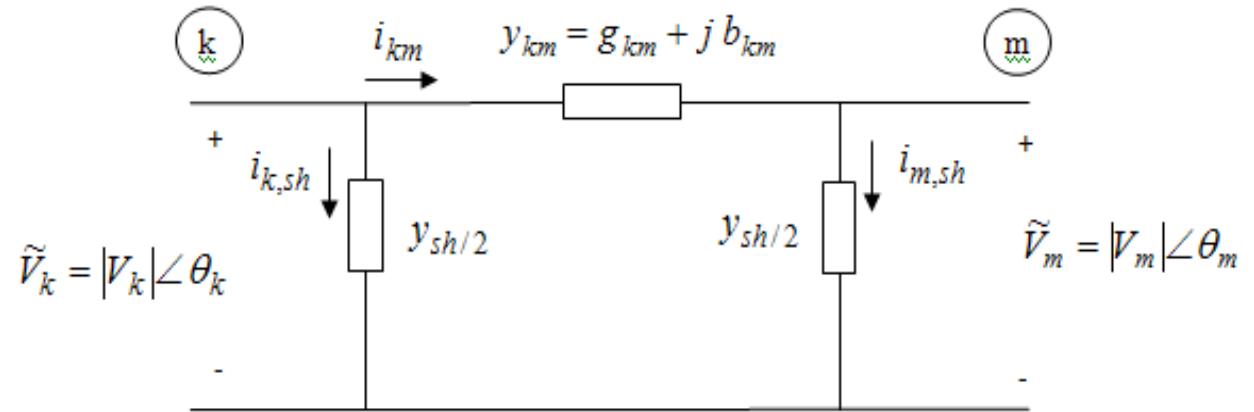
$$s_{km} = \tilde{V}_k i_{km}^* = \tilde{V}_k |t| \angle \tau (i'_{km})^* = \tilde{V}_k |t| \angle \tau (\tilde{V}'_k - \tilde{V}'_m)^* y_{km}^*$$

$$p_{km} = +|V_k|^2 |t|^2 g_{km} - |V_k| |V_m| |t| g_{km} \cos(\theta_k - \theta_m + \tau) - |V_k| |V_m| |t| b_{km} \sin(\theta_k - \theta_m + \tau)$$

$$q_{km} = -|V_k|^2 |t|^2 b_{km} - |V_k| |V_m| |t| g_{km} \sin(\theta_k - \theta_m + \tau) + |V_k| |V_m| |t| b_{km} \cos(\theta_k - \theta_m + \tau)$$

Transmission line model

A symmetric arrangement for a balanced transmission line, positive sequence values, using π equivalent



$$\tilde{V}_k = |V_k| e^{j\theta_k}$$

$$\tilde{V}_m = |V_m| e^{j\theta_m}$$

$$y_{km} = g_{km} + j b_{km}$$

$$y_{sh/2} = \frac{1}{-jX_{C/2}} = jB_{sh/2}$$

Complex power flow from k to m

$$\theta_{km} = \theta_k - \theta_m$$

$$s_{km} = \tilde{V}_k (i_{km} + i_{k,sh})^*$$

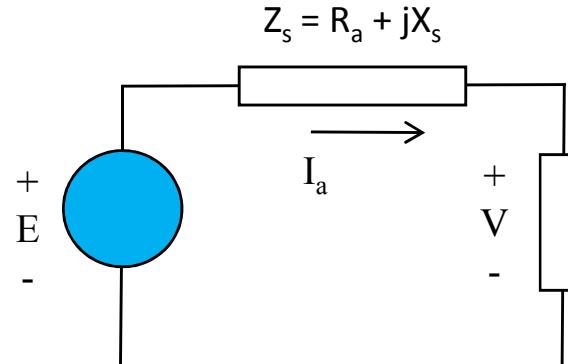
$$p_{km} = |V_k|^2 g_{km} - |V_k| |V_m| g_{km} \cos \theta_{km} - |V_k| |V_m| b_{km} \sin \theta_{km}$$

$$q_{km} = -|V_k|^2 (b_{km} + B_{sh/2}) - |V_k| |V_m| g_{km} \sin \theta_{km} + |V_k| |V_m| b_{km} \cos \theta_{km}$$

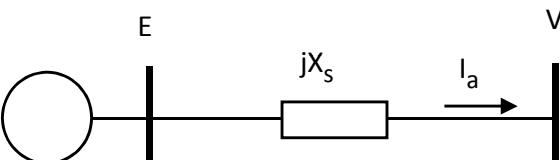
Simple synchronous machine model

Single phase model,
synchronous machine no
saliency:

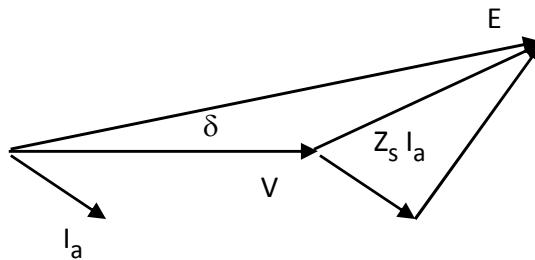
$$E = V + j X_s I_a$$



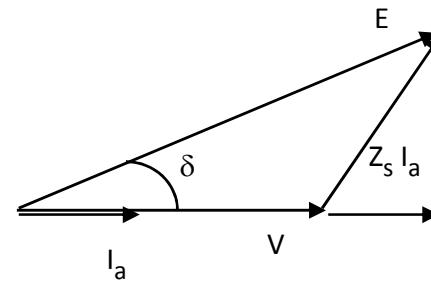
Equivalent circuit for
synchronous machine



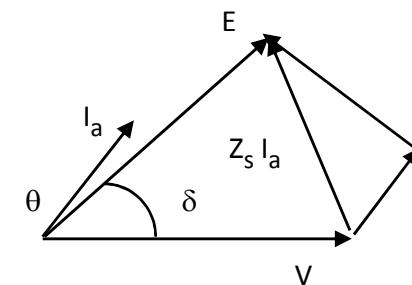
Synchronous machine
connected to an infinite bus



Load with lagging power
factor

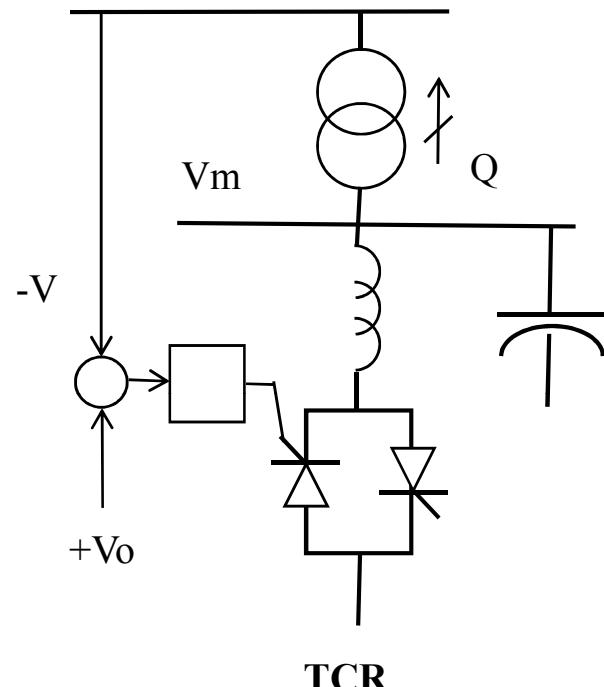
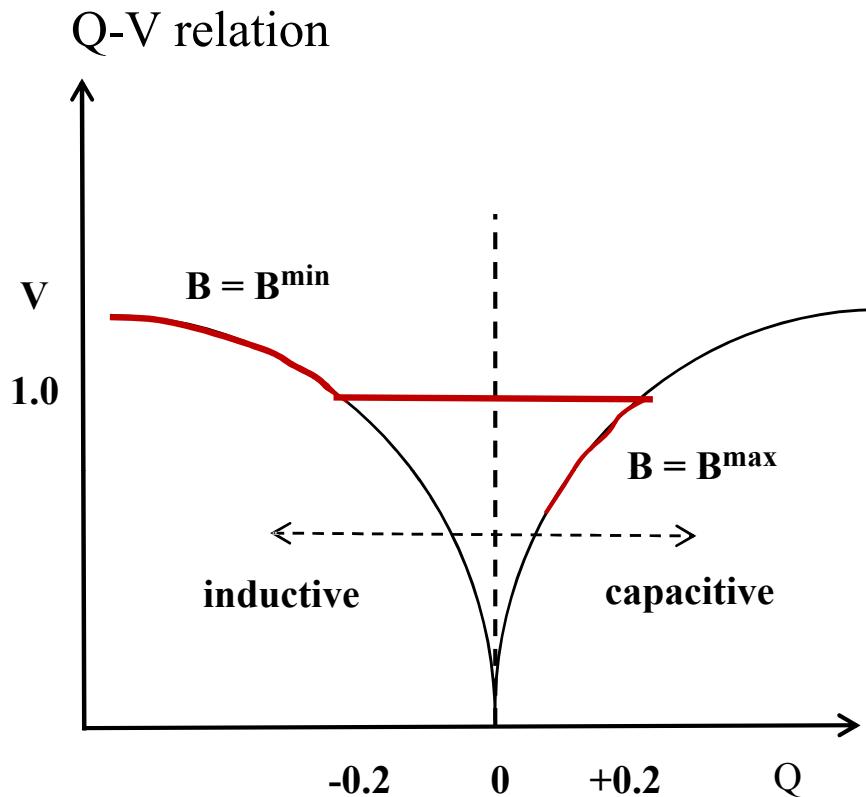


Load with unity pf



Load with leading pf

SVC characteristics

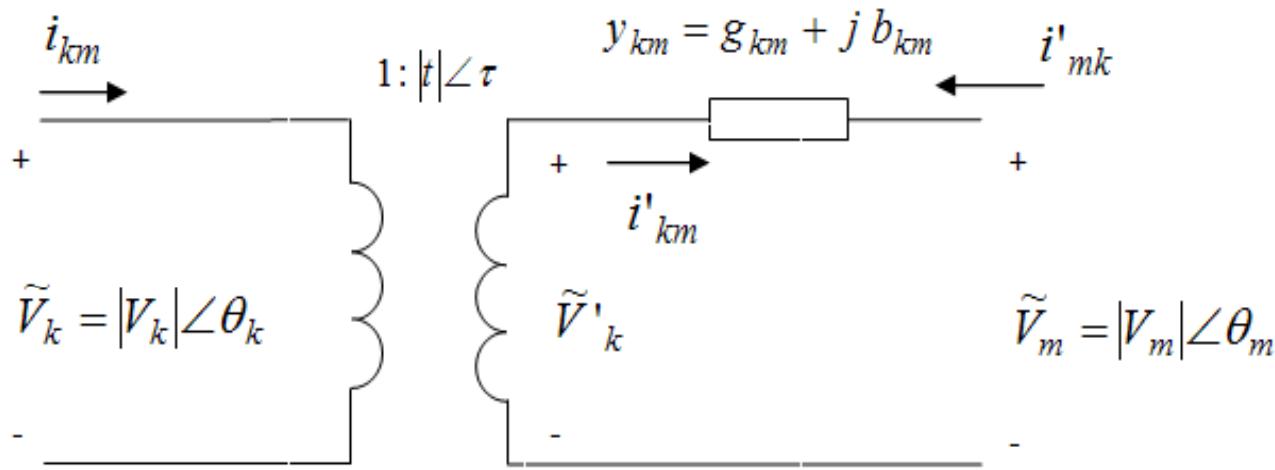


TSC (Thyristor Switched Capacitor)
TCR (Thyristor Controlled Reactors)

Basics about Voltage Control

Reactive power flow, Incremental Model

For a complex transformer in a balanced three phase power system, reactive power:



$$q_{km} = -|V_k|^2 |t|^2 b_{km} - |V_k| |V_m| |t| g_{km} \sin(\theta_k - \theta_m + \tau) + |V_k| |V_m| |t| b_{km} \cos(\theta_k - \theta_m + \tau)$$

Incremental reactive power Δq_{km} :

$$\Delta q_{km} = \frac{\partial q_{km}}{\partial \theta_k} \Delta \theta_k + \frac{\partial q_{km}}{\partial \theta_m} \Delta \theta_m + \frac{\partial q_{km}}{\partial \tau} \Delta \tau + \frac{\partial q_{km}}{\partial |V_k|} |V_k| \frac{\Delta |V_k|}{|V_k|} + \frac{\partial q_{km}}{\partial |V_m|} |V_m| \frac{\Delta |V_m|}{|V_m|} + \frac{\partial q_{km}}{\partial |t|} |t| \frac{\Delta |t|}{|t|}$$

Approximations

Reasonable assumptions will simplify the incremental reactive power flow, like:

$$g_{km} \ll b_{km}$$

$$\sin(\theta_k - \theta_m + \tau) \approx \theta_k - \theta_m + \tau$$

$$\cos(\theta_k - \theta_m + \tau) \approx 1.0$$

$$\frac{\partial q_{km}}{\partial \theta_k} = -|V_k| b_{km} (\theta_k - \theta_m + \tau)$$

$$\frac{\partial q_{km}}{\partial \theta_m} = +|V_k| b_{km} (\theta_k - \theta_m + \tau)$$

$$\frac{\partial q_{km}}{\partial \tau} = -|V_k| b_{km} (\theta_k - \theta_m + \tau)$$

$$\frac{\partial q_{km}}{\partial |V_k|} = -|V_k| b_{km}$$

$$\frac{\partial q_{km}}{\partial |V_m|} = +|V_k| b_{km}$$

$$\frac{\partial q_{km}}{\partial |t|} = -|V_k| b_{km}$$

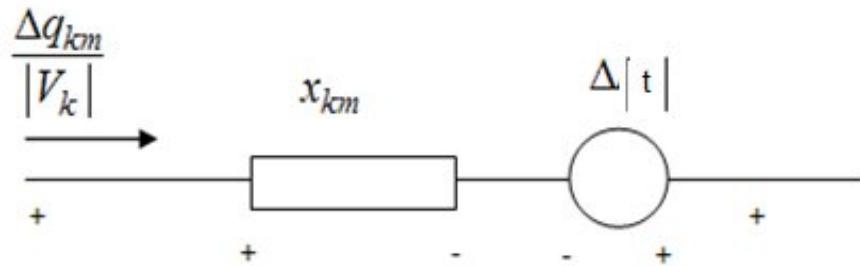
Substitution into the incremental form:

$$-x_{km} \frac{\Delta q_{km}}{|V_k|} + \Delta |V_k| - \Delta |V_m| = -\Delta |t|$$

Ohm's law and equivalent circuit

Ohm's circuit to study the incremental reactive power flow in a complex tap transformer:

$$-x_{km} \frac{\Delta q_{km}}{|V_k|} + \Delta |V_k| - \Delta |V_m| = -\Delta |t|$$



$$\Delta |V_k|$$

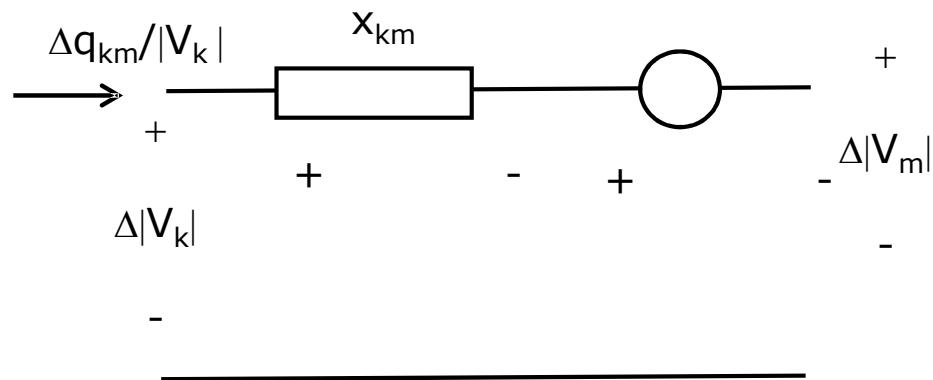
$$\Delta |V_m|$$

B. Stott, O. Alsac, Fast Decoupled Load Flow, IEEE Trans. On Power Apparatus and Systems, Vol. PAS-93, May-June, 1974.

Equivalent circuit for Δq in a transmission line

A similar procedure will give us an incremental circuit equivalent for a transmission line. The right hand side comes from the line charging susceptance.

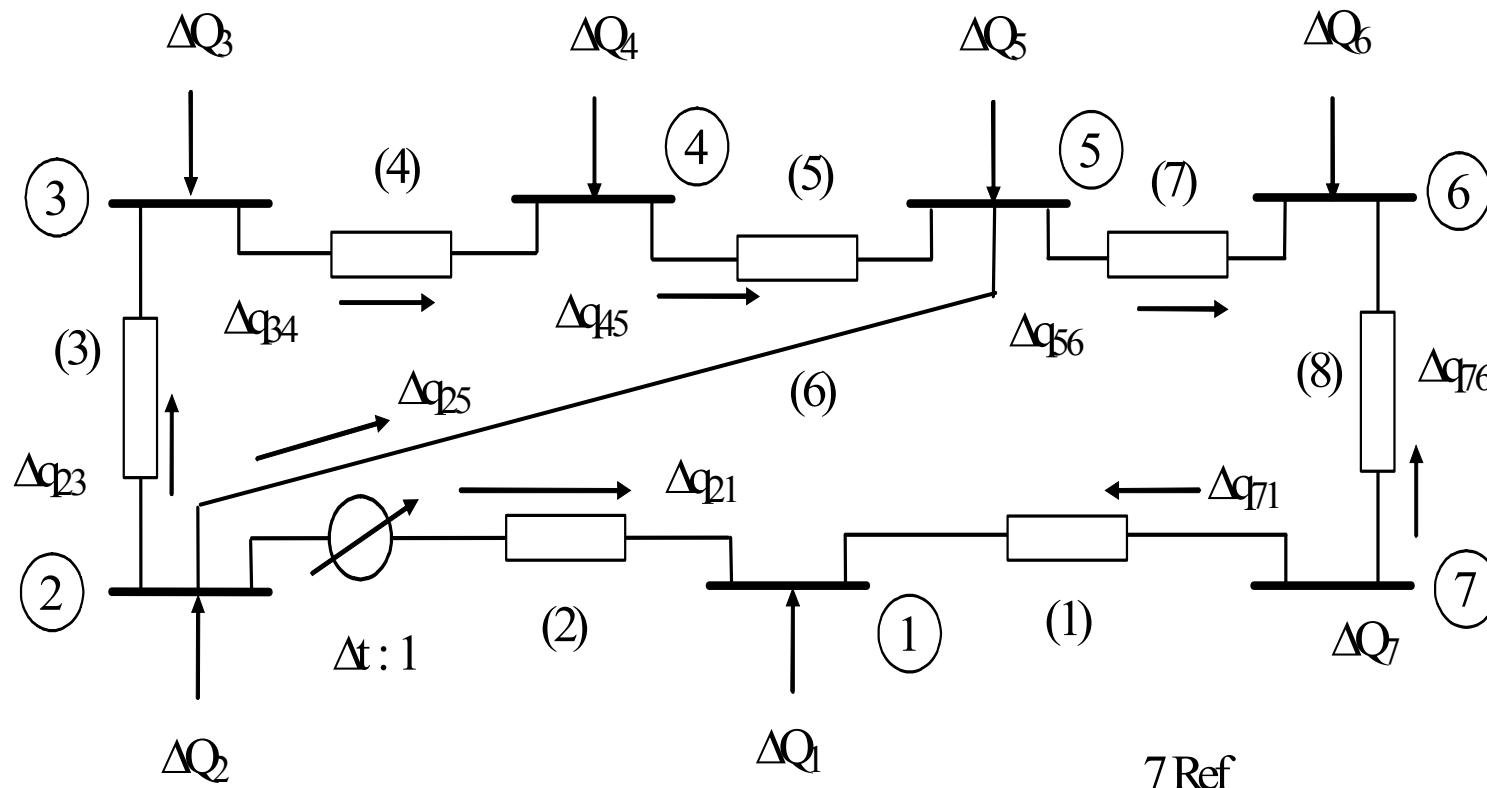
$$-x_{km} \frac{\Delta q_{km}}{|V_k|} + (\Delta|V_k| - \Delta|V_m|) = +2x_{km}Bsh/2\Delta|V_k|$$

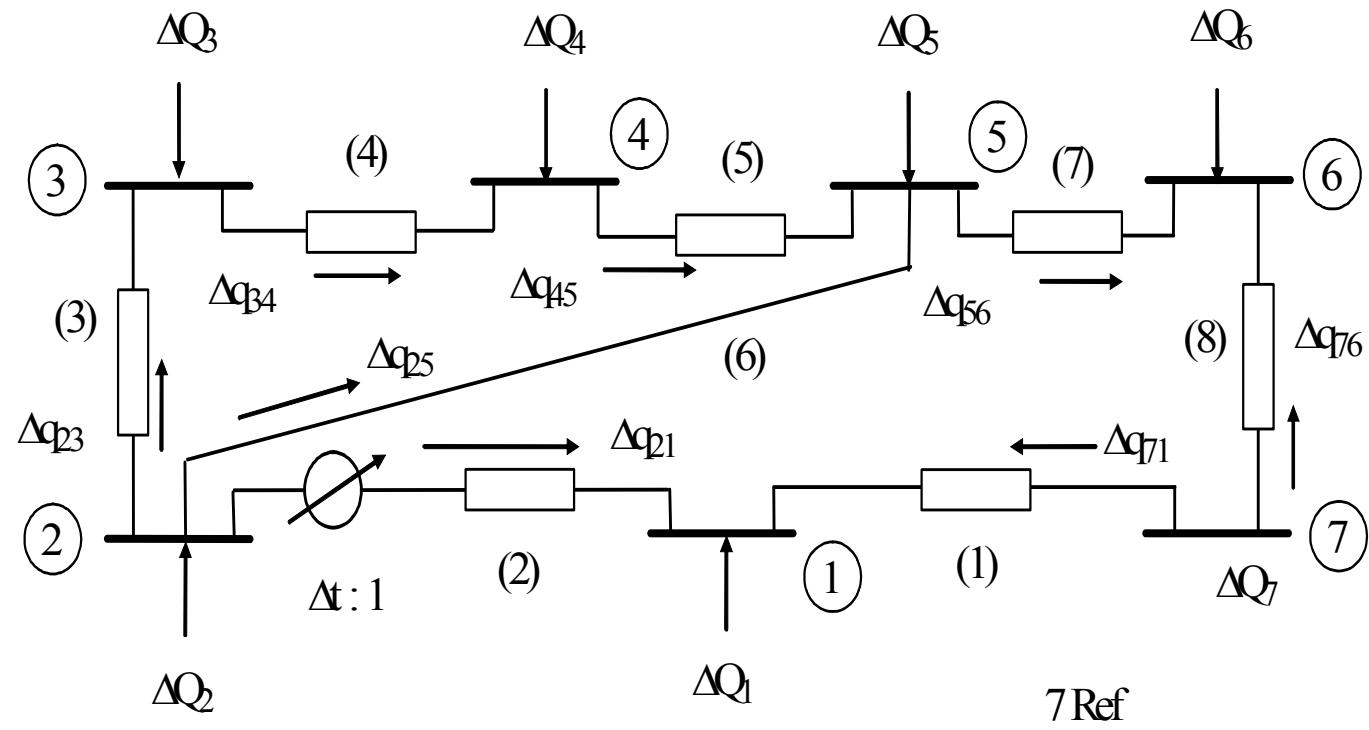


Voltage control by tap transformers

Apply an incremental model for each transformer and transmission line.

- Ohm's law and Kirchhoff's current law are used to set up a system of equations
- Solve for control actions on nodal variables and transmission elements.





Ohm's law for every element

- (1) $-x_{71}\Delta q_{71} + \Delta|V_7| - \Delta|V_1| = 0$
- (2) $-x_{21}\Delta q_{21} + \Delta|V_2| - \Delta|V_1| = -\Delta$
- (3) $-x_{23}\Delta q_{23} + \Delta|V_2| - \Delta|V_3| = 0$
- (4) $-x_{34}\Delta q_{34} + \Delta|V_3| - \Delta|V_4| = 0$
- (5) $-x_{45}\Delta q_{45} + \Delta|V_4| - \Delta|V_5| = 0$
- (6) $-x_{25}\Delta q_{25} + \Delta|V_2| - \Delta|V_5| = 0$
- (7) $-x_{56}\Delta q_{56} + \Delta|V_5| - \Delta|V_6| = 0$
- (8) $-x_{76}\Delta q_{76} + \Delta|V_7| - \Delta|V_6| = 0$

Node equations, Kirchoff's law

- | | |
|--------|---------------------------------------------------------------|
| node 1 | $-\Delta q_{71} - \Delta q_{21} = \Delta Q_1$ |
| node 2 | $+\Delta q_{21} + \Delta q_{23} + \Delta q_{25} = \Delta Q_2$ |
| node 3 | $-\Delta q_{23} + \Delta q_{34} = \Delta Q_3$ |
| node 4 | $-\Delta q_{34} + \Delta q_{45} = \Delta Q_4$ |
| node 5 | $-\Delta q_{25} - \Delta q_{45} + \Delta q_{56} = \Delta Q_5$ |
| node 6 | $-\Delta q_{56} - \Delta q_{76} = \Delta Q_6$ |

Sensitivity factors

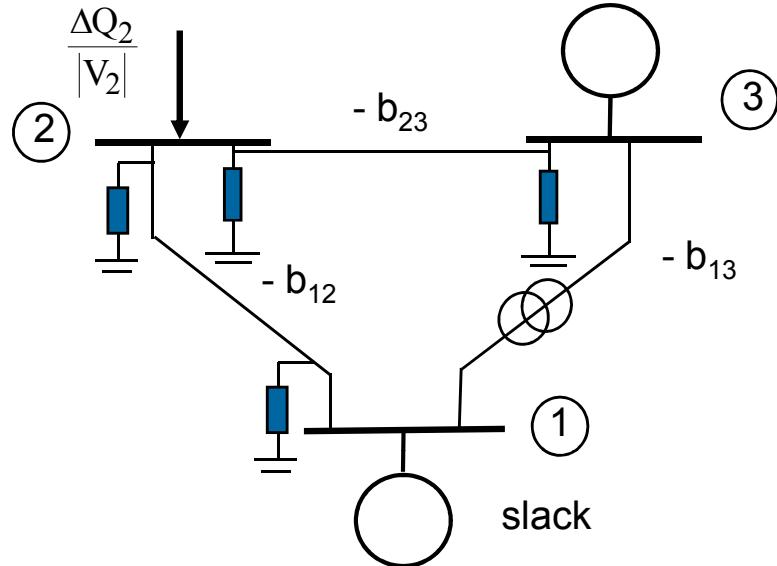
$$\left[\begin{array}{cccccccccccccc} -\frac{1}{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{15} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{8} & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{20} & 0 & 0 & 0 & +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{5} & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & +1 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Second column of the inverse

- (1) + 2.10117
- (2) - 2.10117
- (3) + 0.31128
- (4) + 0.31128
- (5) + 0.31128
- (6) + 1.78988
- (7) + 2.10117
- (8) - 2.10117
- 1 - 0.14008
- 2 + 0.71984
- 3 + 0.68872
- 4 + 0.64981
- 5 + 0.63035
- 6 + 0.21012

SENSITIVITY Q-V AND COORDINATED CONTROL

Network model and voltage-reactive power controls

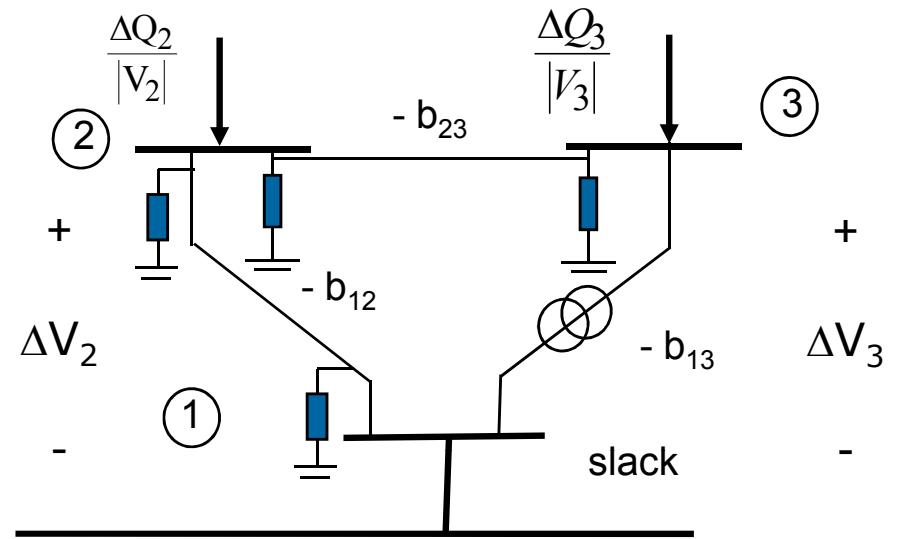


Series Elements

$$(1) \quad -x_{12} \frac{\Delta q_{12}}{|V_1|} + \Delta|V_1| - \Delta|V_2| = 0$$

$$(2) \quad -x_{13} \frac{\Delta q_{13}}{|V_1|} + \Delta|V_1| - \Delta|V_3| = -\Delta|t|$$

$$(3) \quad -x_{23} \frac{\Delta q_{23}}{|V_2|} + \Delta|V_2| - \Delta|V_3| = 0$$



Nodal Balance (including shunt currents)

$$\text{Node 1} \quad + \frac{\Delta q_{12}}{|V_1|} + \frac{\Delta q_{13}}{|V_1|} - 2B_{sht2}^{(1)} \frac{\Delta|V_1|}{|V_1|} = \frac{\Delta Q_1}{|V_1|}$$

$$\text{Node 2} \quad - \frac{\Delta q_{12}}{|V_1|} + \frac{\Delta q_{23}}{|V_2|} - 2(B_{sht2}^{(1)} + B_{sht2}^{(3)}) \frac{\Delta|V_2|}{|V_2|} = \frac{\Delta Q_2}{|V_2|}$$

$$\text{Node 3} \quad - \frac{\Delta q_{13}}{|V_1|} - \frac{\Delta q_{23}}{|V_2|} - 2B_{sht2}^{(3)} \frac{\Delta|V_3|}{|V_3|} = \frac{\Delta Q_3}{|V_3|}$$

Matrix equation and Data

$$\begin{bmatrix} -x_{12} & 0 & 0 & +1 & -1 & 0 \\ 0 & -x_{13} & 0 & +1 & 0 & -1 \\ 0 & 0 & -x_{23} & 0 & +1 & -1 \\ +1 & +1 & 0 & -2B_{sht2}^{(1)} & 0 & 0 \\ -1 & 0 & +1 & 0 & -2(B_{sht2}^{(1)} + B_{sht2}^{(3)}) & 0 \\ 0 & -1 & -1 & 0 & 0 & -2B_{sht2}^{(3)} \end{bmatrix} \begin{bmatrix} \Delta q_{12}/|V_1| \\ \Delta q_{13}/|V_1| \\ \Delta q_{23}/|V_2| \\ \Delta |V_1|/|V_1| \\ \Delta |V_2|/|V_2| \\ \Delta |V_3|/|V_3| \end{bmatrix} = \begin{bmatrix} 0 \\ -\Delta|t| \\ 0 \\ \Delta Q_1/|V_1| \\ \Delta Q_2/|V_2| \\ \Delta Q_3/|V_3| \end{bmatrix}$$

Nnods = 3 Elements = 3

Connectivity

Elements	Nsal	Nlleg	R	X	B/2	Code
1	1	2	0.0230	0.1380	0.2710	0
2	1	3	0.0012	0.0150	0.0000	1
3	2	3	0.0230	0.1380	0.2710	0

Node type: 0 comp, -1 load, +1 generator

Node type

1	0
2	-1
3	1

A change in power flow

If it is required to change the reactive power flow in a given line. Controls available are: a tap changer and power injection into nodes 2 and 3. Sensitivity values are used to solve for an optimal change to attain the reactive flow change.

$$\begin{bmatrix} \Delta q_{12}/|V_1| \\ \Delta q_{13}/|V_1| \\ \Delta q_{23}/|V_2| \\ \Delta |V_1|/|V_1| \\ \Delta |V_2|/|V_2| \\ \Delta |V_3|/|V_3| \end{bmatrix} = \begin{bmatrix} -3.2797 & +3.5910 & -3.5764 & 0 & -0.5474 & -0.0539 \\ +3.5910 & -3.0638 & +3.3224 & 0 & -0.4956 & -0.9540 \\ -3.5764 & +3.3224 & -3.3089 & 0 & +0.4935 & -0.0498 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.5474 & -0.4956 & +0.4935 & 0 & +0.0755 & +0.0074 \\ -0.0539 & -0.9540 & -0.0489 & 0 & +0.0074 & +0.0143 \end{bmatrix} \begin{bmatrix} 0 \\ \Delta|t| \\ 0 \\ \Delta Q_1/|V_1| \\ \Delta Q_2/|V_2| \\ \Delta Q_3/|V_3| \end{bmatrix}$$

A cost function and constraints are required. To minimize control effort to change the flow in line from node 2 to node 3. From sensitivity matrix:

$$\frac{\Delta q_{23}}{|V_2|} = S_{32} \Delta|t| + S_{35} \Delta Q_2/|V_2| + S_{36} \Delta Q_3/|V_3|$$

Cost function and constraints

$$\min \quad k_1(\Delta|t|)^2 + k_2(\Delta Q_2 / |V_2|)^2 + k_3(\Delta Q_3 / |V_3|)^2$$

$$s.a. \quad \Delta q_{23} / |V_2| - S_{32} \Delta|t| - S_{35} \Delta Q_2 / |V_2| - S_{36} \Delta Q_3 / |V_3| = 0$$

Lagrangian and its gradient:

$$\zeta(\Delta|t|, \frac{\Delta Q_2}{|V_2|}, \frac{\Delta Q_3}{|V_3|}) = k_1(\Delta|t|)^2 + k_2(\frac{\Delta Q_2}{|V_2|})^2 + k_3(\frac{\Delta Q_3}{|V_3|})^2 + \lambda(\frac{\Delta q_{23}}{|V_2|} - S_{32} \Delta|t| - S_{35} \frac{\Delta Q_2}{|V_2|} - S_{36} \frac{\Delta Q_3}{|V_3|})$$

$$\nabla \zeta = \begin{bmatrix} \frac{\partial \zeta}{\partial \Delta|t|} \\ \frac{\partial \zeta}{\partial \Delta Q_2 / |V_2|} \\ \frac{\partial \zeta}{\partial \Delta Q_3 / |V_3|} \\ \frac{\partial \zeta}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} 2k_1 \Delta|t| - \lambda S_{32} \\ 2k_2 \Delta Q_2 / |V_2| - \lambda S_{35} \\ 2k_3 \Delta Q_3 / |V_3| - \lambda S_{36} \\ \Delta q_{23} / |V_2| - S_{32} \Delta|t| - S_{35} \Delta Q_2 / |V_2| - S_{36} \Delta Q_3 / |V_3| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gradient must be zero for an optimal solution. We find a set of equations to be solved for the control actions required.

This case requires to solve a system of linear equations

$$\begin{bmatrix} 2k_1 & 0 & 0 & -S_{32} \\ 0 & 2k_2 & 0 & -S_{35} \\ 0 & 0 & 2k_3 & -S_{36} \\ -S_{32} & -S_{35} & -S_{36} & 0 \end{bmatrix} \begin{bmatrix} \Delta|t| \\ \Delta Q_2 / |V_2| \\ \Delta Q_3 / |V_3| \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\Delta q_{23} / |V_2| \end{bmatrix}$$

The change in controls to reduce the reactive power flow q_{23} in 0.2:

$$S_{min} = \begin{bmatrix} 2.0000 & 0 & 0 & -3.3224 \\ 0 & 2.0000 & 0 & -0.4935 \\ 0 & 0 & 2.0000 & 0.0498 \\ -3.3224 & -0.4935 & 0.0498 & 0 \end{bmatrix}$$

$$Dy = \begin{bmatrix} -0.0589 \\ -0.0087 \\ 0.0009 \\ -0.0354 \end{bmatrix} \quad \begin{bmatrix} \Delta|t| \\ \Delta Q_2 / |V_2| \\ \Delta Q_3 / |V_3| \\ \lambda \end{bmatrix} = \begin{bmatrix} -0.0589 \\ -0.0087 \\ +0.0009 \\ -0.0354 \end{bmatrix}$$

$$ans = -0.2000$$

The coordinated changes work to modify the required value, but control limits must be included (i. e. maximum/minimum setting must be observed).

Previous solution used k's as unit values.

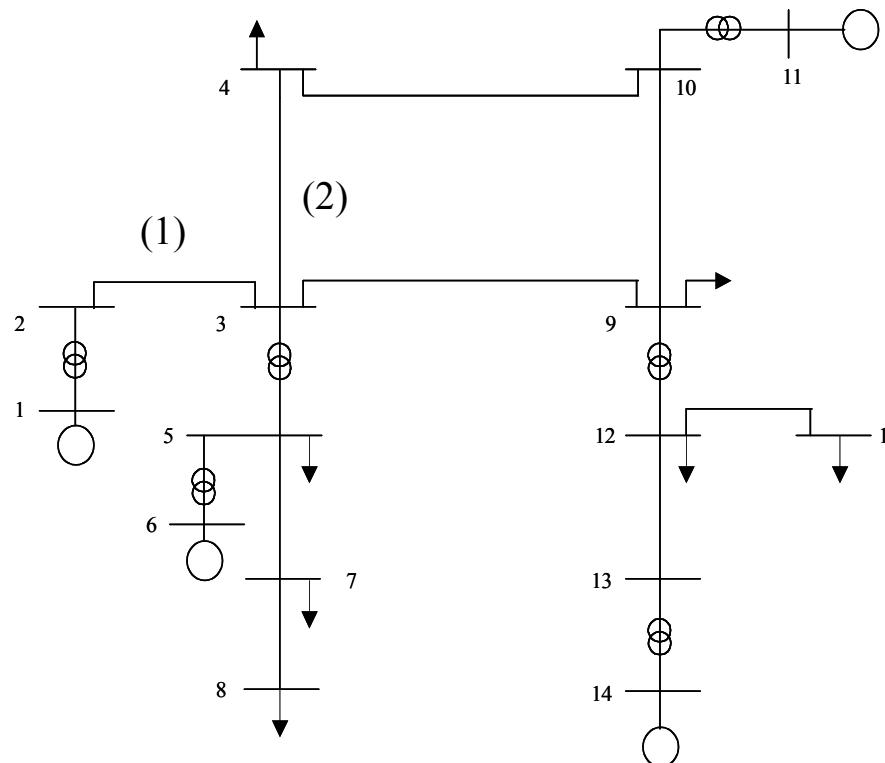
Different weight or importance in a control action can be handled through the k values. A change to reduce (or make larger) the control action of injected reactive power is shown by values $k_2 = 0.1$ and $k_3 = 0.1$

$$\begin{aligned} \text{Dy} &= \\ &\begin{matrix} -0.0492 \\ -0.0731 \\ 0.0074 \\ -0.0296 \end{matrix} \\ \text{ans} &= -0.2000 \end{aligned}$$
$$\begin{bmatrix} \Delta|t| \\ \Delta Q_2 / |V_2| \\ \Delta Q_3 / |V_3| \\ \lambda \end{bmatrix} = \begin{bmatrix} -0.0492 \\ -0.0731 \\ +0.0074 \\ -0.0296 \end{bmatrix}$$

Weak and Radial type networks

A real network might have a structure as shown, in which nodes and transmission elements show a radial structure; considered “longitudinal”; it is not a very robust network.

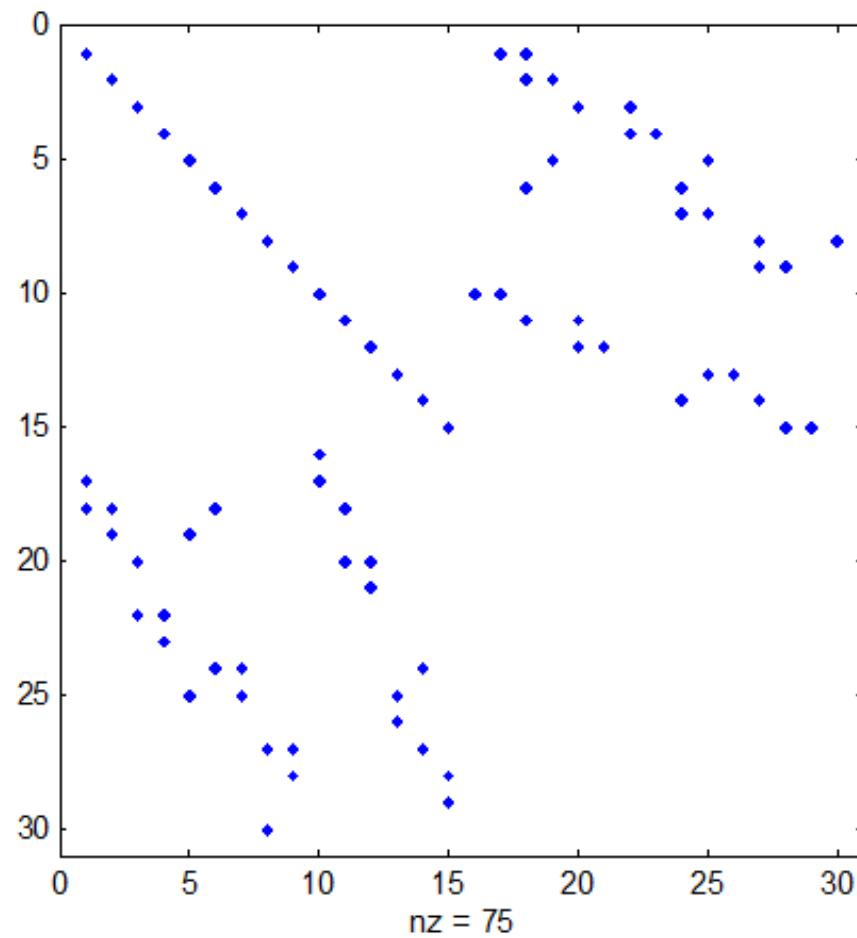
This type of network is very interesting to study control actions and magnitude of their influence.



15 nodes, 15 elements longitudinal network

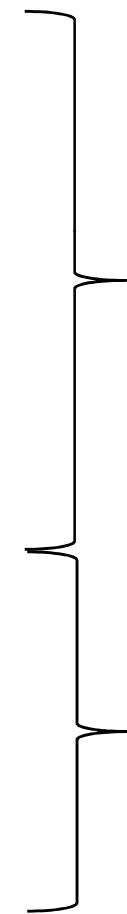
Figure shows the matrix structure for circuit study; elements and nodal information are in the tables.

Nodo	P _G (MW)	P _D (MW)
1	145	
2		
3		
4		52
5		71
6	81.5	
7		71.9
8		79.4
9		1.5
10		
11	88.7	
12		13.5
13		
14	26.6	
15		52.5



15 node system and elements, connectivity and reactance values.

Elemento	Nodo Salida - Nodo Llegada	x (pu)
(1)	2 - 3	0.1189
(2)	3 - 4	0.0568
(3)	5 - 7	0.0807
(4)	7 - 8	0.0451
(5)	4 - 10	0.1316
(6)	3 - 9	0.1458
(7)	10 - 9	0.0449
(8)	12 - 15	0.1699
(9)	12 - 13	0.0849
(10)	1 - 2	0.09303
(11)	3 - 5	0.1885
(12)	6 - 5	0.1330
(13)	11 - 10	0.0930
(14)	9 - 12	0.1885
(15)	14 - 13	0.1331



lines

transformers

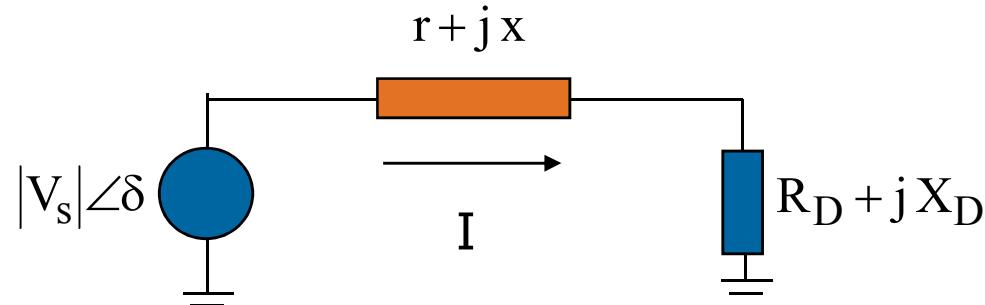
S =

Columns 1 through 8

-3.1237	-0.9899	0	0	-0.9899	-1.0824	0.6391	0
-0.9899	-3.4485	0	0	-3.4485	1.8060	-1.9376	-0.0000
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
-0.9899	-3.4485	0	0	-3.4485	1.8060	-1.9376	-0.0000
-1.0824	1.8060	0	0	1.8060	-3.6018	2.9980	0
0.6391	-1.9376	0	0	-1.9376	2.9980	-4.4065	0
0	0	0	0	0	0	0	0
-0.4433	-0.1316	0	0	-0.1316	-0.6039	-1.4085	0
-3.1237	-0.9899	0	0	-0.9899	-1.0824	0.6391	0
-1.0513	0.6526	0	0	0.6526	0.7135	-0.4213	0.0000
1.0513	-0.6526	0	0	-0.6526	-0.7135	0.4213	-0.0000
1.6290	1.5109	0	0	1.5109	1.1920	-2.4688	0
-0.4433	-0.1316	0	0	-0.1316	-0.6039	-1.4085	0
0.4433	0.1316	0	0	0.1316	0.6039	1.4085	0
0.0000	0.0000	0	0	0.0000	0.0000	-0.0000	-0.0000
0.2906	0.0921	0	0	0.0921	0.1007	-0.0595	0
-0.3380	0.2098	0	0	0.2098	0.2294	-0.1354	0
-0.2818	-0.5943	0	0	0.4057	0.1268	-0.0254	-0.0000
-0.1398	0.0868	0	0	0.0868	0.0949	-0.0560	0.0000
-0.0000	0.0000	0	0	0.0000	0.0000	-0.0000	0
-0.1398	0.0868	-1.0000	0	0.0868	0.0949	-0.0560	0.0000
-0.1398	0.0868	-1.0000	-1.0000	0.0868	0.0949	-0.0560	0.0000
-0.1802	-0.0535	0	0	-0.0535	-0.2455	-0.5725	0
-0.1515	-0.1405	0	0	-0.1405	-0.1109	0.2296	0
-0.0000	-0.0000	0	0	-0.0000	-0.0000	0.0000	0
-0.0966	-0.0287	0	0	-0.0287	-0.1316	-0.3070	0
-0.0590	-0.0175	0	0	-0.0175	-0.0804	-0.1875	0
-0.0000	-0.0000	0	0	-0.0000	-0.0000	-0.0000	-0.0000
-0.0966	-0.0287	0	0	-0.0287	-0.1316	-0.3070	-1.0000

Q-V RELATION AND VOLTAGE SUPPORT

Voltage and current, steady state



Positive sequence

$$I = \frac{|V_s| \angle \delta}{(r + R_D) + j(x + X_D)}$$

Assuming load and voltage as a fixed value (no use of load flow), $Z_{\text{Line}} = 0.05 + j0.5 \Omega$

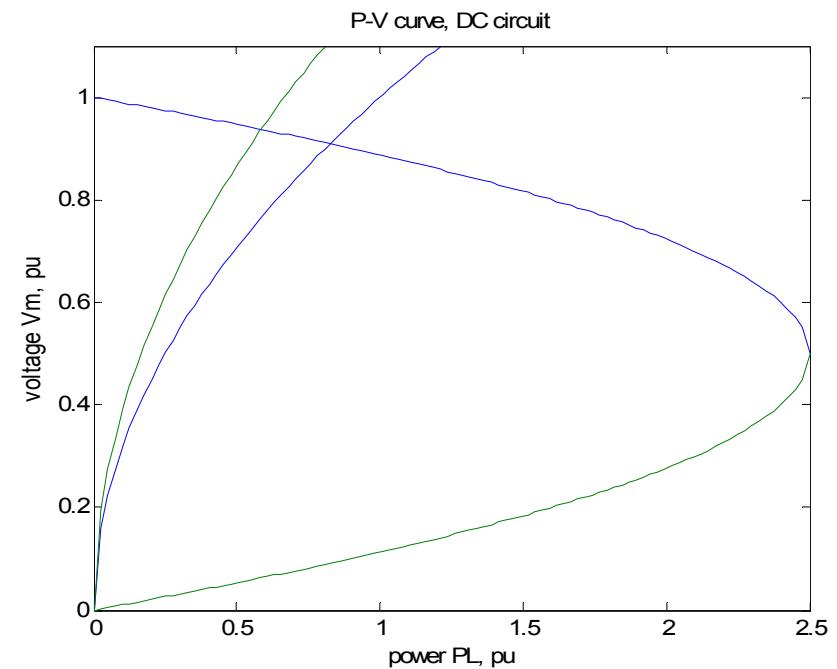
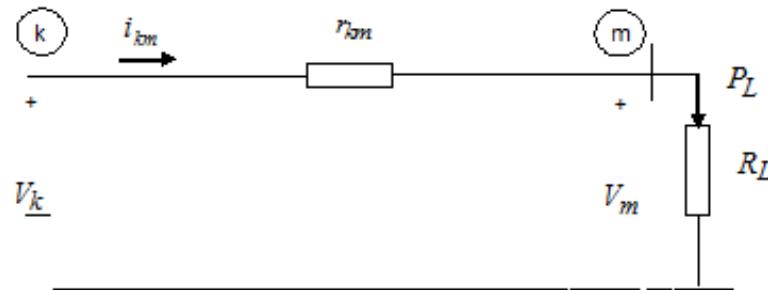
Power factor effect on voltage, angle and losses

Load voltage and current, rms values

120.00 50.00

Voltage source	angle	PS	QS	PL	QL	pf(load)	Efficiency
136.20	8.61	5321.15	4250.00	5196.15	3000.00	-0.87	2.35
131.01	10.33	5920.55	2802.91	5795.55	1552.91	-0.97	2.11
125.02	11.53	6125.00	1250.00	6000.00	0.00	+1.00	2.04
118.57	12.07	5920.55	-302.91	5795.55	-1552.91	+0.97	2.11
112.03	11.80	5321.15	-1750.00	5196.15	-3000.00	+0.87	2.35

PV curve and load characteristics



System's PV curve and for a resistive load, $R_L = 1.5, 1.1 \text{ pu}$.

By

Salvador Acha Daza, Ph. D.

dr.acha@niat.com.mx

(512) 623 9953

<http://www.niat.com.mx/>