Resource Allocation and User Grouping for Sum Rate and Fairness Optimization in NOMA and IoT

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***Abstract*—In this paper, we present the joint optimization of sum rate and fairness for contention based uplink multiple access with non-orthogonal multiple access (NOMA) communication system by resource allocation and user grouping. In particular, we study the cases of many users sharing the same resources that address application of the the internet of things (IoT). The key feature of contention based multiple access is to serve multiple users at the same time and frequency. With different power levels and user grouping, it can achieve better spectral efficiency over conventional orthogonal multiple access (OMA). However, unlike the OMA system, NOMA results in additional inter-user interference (IUI). It has also been shown that, without proper resource allocation for users in the uplink NOMA, the weak users can always be in outage. In this work, we have developed algorithms on subbands assignment, user grouping, and power allocation for joint optimization of sum rate and fairness. The algorithm allocates resources iteratively to handle the IUI in each iteration. Given a number of N_s subbands allocation to each user, we could prevent starvation of poor users, e.g. cell edge users. We have also compare and analyze the sum rate and fairness performance with different combination of L and N_s. We also find that, by properly limiting the maximum number of subbands each user can use, the system could better exploit multi-user diversity to improve the sum rate and hence the energy efficiency. The numerical simulations are also conducted to verify the results.**

***Index Terms*—Non-orthogonal multiple access; multiple radio access; resource allocation; sum rate optimization; fairness.**

I. Introduction

The concept of non-orthogonal multiple access (NOMA) has been proposed in order to support more users than the number of available orthogonal time, frequency, or code domain resources. The muti-user superposition transmission (MUST), i.e. the concept of power domain NOMA, has been included as radio access technique for 4G Long-Term Evolution (LTE) Release 14 and beyond. Since then, the general NOMA technology is envisioned to be an promising component of 5G mobile networks [1]. In [2], it is demanded that the next generation (5G) wireless communication networks should offer a significant improvement in coverage, throughputs, spectrum efficiency, massive connectivity in IoT service, and user experience. In uplink NOMA, multiple users are grouped and can share the same subbands simultaneously [3]. At the transmitter side, power domain [4] and code domain [5] strategies are developed for the grouped non-orthogonal users. However, Inter User Interference (IUI) is inevitable. At the receiver side, more complicated multi-user detection (MUD) techniques are required in each subband to detect the signals transmitted from users. Compared to the orthogonal multiple access (OMA) system, the NOMA has been shown to achieve better capacity region and enhanced spectrum efficiency with challenges and opportunities [6] [7].

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System sum rate and fairness are subject to resource allocation and receiver architecture. Most papers focus on downlink NOMA resource allocation. However, in uplink NOMA, the resource allocation strategy is less addressed. It has been studied that, without proper allocation of target data rate for each user in uplink NOMA, a user can always be in outage [8]. In receiver side, most papers in NOMA focus on zero forcing (ZF) successive interference cancellation (SIC) receiver which need only one antenna in receiver. Note that ZF-SIC receiver is often called as SIC receiver. In [9], the uplink non-orthogonal multiple access applies minimum mean square error (MMSE) SIC receiver. Sum rate and fairness are subject to both ZF-SIC and MMSE-SIC receiver. With total N subbands, NOMA can support massive connectivity with total number of resources $N \times L$. With more number of users K in the system, we should take advantage of multi-user diversity. In [3], based on greedy principle, the algorithm with Local Rate Maximization (LRM) and Global Objective Maximization (GOM) method aim to maximize sum rate. Complexity of algorithms is high, it grows with K and N. Water-filling range varies with current subbands allocation. With greedy principle, strong users own more subbands, fairness performance decreases. NOMA can accommodates to more users. With massive connectivity, cell throughput increases as K increases [10] since the spatial diversity is more. Multi-user gain is more than less users.

Compared with the majority of NOMA research works, the main difference of this work is that we address the scenarios that the number of users is much greater than that of the subbands to support massive connectivity demand in IoT. In uplink NOMA, the system sum rate and fairness are subject to resource allocation policy and receiver architecture. With fixed power, the energy efficiency is optimized in term of sum rate maximization.

In [3], based on greedy principle, the cell-edge users with poor channel condition are not guaranteed to win the subband resources. In this work, we present $\mathbf{Algorithm\ 1}$ [11] with minimum number of subbands that each user could use, the system can prevent the poor users from starvation and enhance fairness. The advantage of **Algorithm 1** becomes more significant with larger K. Especially when cell-center users usually occupy majority of resources and cell-edge users will not be guaranteed to win resource. For intra-cell uplink NOMA, power allocation is subject to IUI. IUI dynamically varies with power allocation. In **Algorithm 2**, we are going to perform the power allocation. To deal with the varying IUI in power allocation process, **Algorithm 2** is performed iteratively. Sum rate will increase with **Algorithm 2** iteratively. The optimal combination of L and N_s in terms of spectral efficiency is evaluated numerically. The numerical simulation on sum rate and fairness are conducted to validate the algorithms and compared with different receiver architecture.

II. Uplink NOMA Scheme and Analysis

A. System Model

Assume K users in a cellular uplink transmission system and N subbands are available for the multiuser system, $K \geq N$. Each user is equipped with one antenna, $N_t = 1$, and the BS has N_r receive antennas. Let $\mathbf{H_k} \in C^{N_r \times N}$ be the channel matrix of user k, with the channel element $h_{k,n}^{(n_r)}$ from the n_r-th antenna at the BS on subband $n, n = 1, ..., N$, of user k. Assume that we apply full CSI system. To improve spectral efficiency, a subband can be allocated to the L users that have better sum channel coefficient $g_{k,n}$. The detection complexity also increases with the number of users L on each subband.

As the L users interfere with each other in the same subband, the BS receiver requires to separate and detect the signals from the L users. Each subband at BS exercies L-multiuser detection independently.

1) Base station with ZF-SIC detector : Let the received signal be y with L streams. When performing ZF-SIC detection, we pass y to the linear ZF filter, $1/h_{j,n}^{(1)}$, then decode the stream, $p_{j,n}h_{j,n}^{(1)}$, with best SINR. Then we substract it from y get $\tilde{y}$, passing $\tilde{y}$ with total $L-1$ streams to the linear ZF filter and decode the stream with best SINR. The above process repeat until each stream is decoded.

2) Base station with MMSE-SIC detector : With N_r antennas in BS, the MMSE-SIC receiver is applied in each subband. The MMSE-SIC can achieve the MIMO capacity [12]. The constraint on MMSE-SIC receiver is that the BS should offer the number of antennas N_r not less than the number of users L in each subband.

Let the received signal vector be $\mathbf{y}$ with the dimension N_r by 1, $\mathbf{H}$ be the channel matrix with dimension N_r by $L \times N_t$. We assume that $N_t = 1$, therefore, $\mathbf{H}$ be the channel matrix with dimension N_r by L. σ^2 is noise power, $\mathbf{I}_{N_r}$ is identity matrix with dimension N_r by N_r. When performing MMSE-SIC detection, we pass $\mathbf{y}$ to the linear MMSE filter, $(\mathbf{H}^*\mathbf{H} + \sigma^\mathbf{2}\mathbf{I}_{N_r})^{-\mathbf{1}}\mathbf{H}^*$, and the stream with best SINR is decoded. Then we substract stream from $\mathbf{y}$ with total L streams and get $\tilde{\mathbf{y}}$, passing $\tilde{\mathbf{y}}$ with total $L-1$ streams to the linear MMSE filter and decode the stream with best SINR. The above process repeat until each stream is decoded.

B. Problem formulation

The objective of the resource allocation is to maximize the maximum sum rate, which is subject to the constraint on L, N_s, receiver architecture and total transmit power for each user. The throughput rate after resource allocation is evaluated based on MMSE-SIC receiver architectures.

Let the sum channel coefficient for user k on subband n be $g_{k,n}$, and assume homogeneous channels across the receive antennas, i.e.

$$g_{k,n} = |h_{k,n}^{(1)}|^2 + |h_{k,n}^{(2)}|^2 + \cdots + |h_{k,n}^{(N_r)}|^2. \tag{1}$$

With the user interference in the same subband, the sum rate of user k can be expressed as,

$$R_k = \sum_{n=1}^{N} \log_2(1 + p_{k,n}\mathbf{h}_{k,n}^*\mathbf{K}_z^{-\mathbf{1}}\mathbf{h}_{k,n}), \tag{2}$$

where $\mathbf{h}_{k,n}$ is the channel vector $[h_{k,n}^{(1)}, h_{k,n}^{(2)}, ..., h_{k,n}^{(N_r)}]^{\mathrm{T}}$ and $\mathbf{h}_{k,n}^*$ denotes conjugate transpose of $\mathbf{h}_{k,n}$. The $\mathbf{K}_z$ denotes the covariance matrix of colored noise, with $\mathbf{K}_z = (\sigma^\mathbf{2}\mathbf{I}_{N_r} + \mathbf{I}_{k,n})$, where σ^2 represents the i.i.d. noise power of each subband, $\mathbf{I}_{N_r}$ is the identity matrix, and $\mathbf{I}_{k,n}$ represents the interference matrix that the kth user receives on the subband n from other users. Assume that decoding in the order of signal-to-interference and noise ratio (SINR), $\rho_{k,n}$, in each subband, where $\rho_{k,n} = p_{k,n}\mathbf{h}_{k,n}^*(\sigma^\mathbf{2}\mathbf{I}_{N_r} + \sum\limits_{j \in j \neq k} x_{j,n}p_{j,n}\mathbf{h}_{j,n}\mathbf{h}_{j,n}^*)^{-\mathbf{1}}\mathbf{h}_{k,n}$, $\mathbf{I}_{k,n}$ can be represented as

$$\mathbf{I}_{k,n} = \sum_{j \in \rho_{j,n} < \rho_{k,n}} x_{j,n}p_{j,n}\mathbf{h}_{j,n}\mathbf{h}_{j,n}^* \tag{3}$$

where $x_{k,n} \in \{0,1\}$. $x_{k,n} = 1$ represents that user k can use subband n, and $x_{k,n} = 0$ represents user k can not use subband n. The $p_{k,n}, \forall k$ represents the transmission power for user k on subband n. Here, besides the problem formulation in [3], we add one new constraint that the number of subbands each user can use simultaneously is fixed to N_s, and try to maximize the sum rate of the system. The problem is formulated as,

$$\max_{x_{k,n}, p_{k,n}} \quad \sum_{n=1}^{N}\sum_{k=1}^{K} \log_2(1 + x_{k,n}p_{k,n}\mathbf{h}_{k,n}^*\mathbf{K}_z^{-\mathbf{1}}\mathbf{h}_{k,n}) \tag{4a}$$

$$s.t. \quad \sum_{k=1}^{K} x_{k,n} = L, \forall n \tag{4b}$$

$$\sum_{n=1}^{N} x_{k,n} = N_s, \forall k \tag{4c}$$

$$\sum_{n=1}^{N} x_{k,n}p_{k,n} = P_k, \forall k \tag{4d}$$

$$x_{k,n} \in \{0,1\}, \forall k, n. \quad k \in \{1, 2, ..., K\} \tag{4e}$$

where $p_{k,n}$ represents the transmit power for user k on subband n. If $x_{k,n} = 0$, we let $p_{k,n} = 0$, and if $x_{k,n} = 1$, user

k allocate power $p_{k,n}$ on subband n. P_k represents the total transmit power of the user k on the allocated N_s subbands.

The formula (4a) can be written as,

$$\max_{x_{k,n},p_{k,n}} \quad \sum_{n=1}^{N} \log_2 \det(\mathbf{I}_{N_r} + \frac{\mathbf{1}}{\sigma^2} \sum_{k=1}^{K} x_{k,n} p_{k,n} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^{*}) \quad (5)$$

Given N subbands in a NOMA system, each subband is shared by L users simultaneously, and each user can use fixed number of N_s subbands. Then the number of users in the NOMA system can be expressed as,

$$K = \frac{N \times L}{N_s} \quad (6)$$

In a contention based uplink NOMA system, the sum rate varies as N_s changes.

Proposition 1. *Assume that the users are uniformly random distributed over geometry of the cellular network. With fixed number of resource elements $N \times L$ and by varying the number of subbands N_s that each user can use simultaneously, the total number of users would be $K = \frac{N \times L}{N_s}$. Then, the expected sum rate*

$$E[R(N_s)] = E[\sum_{n=1}^{N} \log_2 \det(\mathbf{I}_{N_r} + \frac{\mathbf{1}}{\sigma^2} \sum_{k=1}^{K} x_{k,n} p_{k,n} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^{*})] \quad (7)$$

can be maximized when $N_s = 1$. That is, each user is allocated to one subband only, though many users share the same subband. When $N_s = 1$, the number of users K to the system is maximized.

The numerical validation of $\mathbf{Proposition\ 1}$ will be presented in the Sec. IV.

III. Proposed Resource Allocation Method

In this paper, the resource allocation for uplink NOMA system consists of two phases. In the first phase, the subband allocation and user grouping are determined with $\mathbf{Algorithm\ 1}$. In the second phase, the power to the multiband of each user is determined based on $\mathbf{Algorithm\ 2}$.

A. *Joint Subband Allocation and User Grouping Algorithm*

A joint subband allocation and user grouping algorithm with constrained numbers of N, L and N_s is first developed. We propose a low complexity $\mathbf{Algorithm\ 1}$ which depends on channel gain $g_{k,n}$. Since $g_{k,n}$ depends on channel condition, $\mathbf{Algorithm\ 1}$ can derive the multi-user diversity. Based on the greedy principle in each subband, the algorithm is designed to achieve the maximum sum rate. With the constraint on N_s, the algorithm increase fairness compared to prior arts that cell-edge users may not guarantee to win subbands. The algorithm consists of two main stages.

Stage 1: User Grouping. Each subband identifies the preferred user with the best sum channel coefficient, $b_n = \arg\max_{k \in \mathbb{U}} g_{k,n}$. The subbands select the same user b_n are contained in set $\mathbb{B}_{b_n}$.

Stage 2: Subband Allocation. In the same set $\mathbb{B}_{b_n}$, the subband i^* with best sum channel coefficient is assigned to the user b_n. Then the resource, subband i^* of user b_n, becomes unavailable for the rest of allocation process.

The process repeat until each subband is allocated with L users and each user uses N_s subbands . In each round, one subband resource is allocated to one user.

Algorithm 1 Joint subband assignment and users grouping algorithm

Input: K=number of users in a cell, N =number of subbands L=number of users on each subband, N_s=number of subbands per user, $g_{k,n}$= channel coefficient of user k at subband n

Output: Find $x_{k,n}, \forall k, n.$, user k using the subband n or not.

1: **Initialization:** $\mathbb{B}^u = \{1, ..., N\}$, $\mathbb{U} = \{1, ..., K\}$, $\mathbb{B}^a = \{\emptyset\}$, $\mathbb{B}_k = \{\emptyset\}, \forall k$, $c_n = 0, \forall n$
2: **for** $l = 1$; $l <= L$; $l++$ **do**
3:     **User Grouping : Subband select the best user :** $b_n = \arg\max_{k \in \mathbb{U}} g_{k,n}$, $n \in (c_n \neq l)$
4:     **Subbands of the same b_n are grouped to the set** $\mathbb{B}_{b_n}$
5:     **for** $n = 1$; $n <= N$; $n++$ **do**
6:         **Subband Allocation :**
7:         $i^* = \arg\max_{i \in \mathbb{B}_{b_n}} g_{b_n,i}, \forall b_n$, $i \in (c_n \neq l)$
8:         **Assign user b_n to subband i^* : Set** $x_{b_n,i^*} = 1$,
9:         $g_{b_n,i^*} = 0$, $c_{i^*} = c_{i^*} + 1, \mathbb{B}_{b_n} = \mathbb{B}_{b_n} - \{i^*\}$
10:         **if** $\sum_{t=1}^{N} x_{b_n,t} = N_s$ **then**
11:             **Set** $\mathbb{U} = \mathbb{U} - \{b_n\}$, $\mathbb{B}_{b_n} = \{\emptyset\}$, $g_{b_n,t} = 0, \forall t$
12:         **end if**
13:         **if** $\sum_{k=1}^{K} x_{k,n} = L$ **then**
14:             **Set** $\mathbb{B}^u = \mathbb{B}^u - \{n\}$, $\mathbb{B}^a = \mathbb{B}^a + \{n\}$
15:         **end if**
16:     **end for**
17:     **For subbands without any user this round (i.e. $n \in (c_n \neq l)$), back to step 3 and redo user grouping and subband grouping for the remaining subbnads.**
18: **end for**

B. *Multi-band Iterative Water Filling Algorithm*

In OMA, each subband is allocated one user, IUI does not exist. In NOMA, since each user suffers IUI in the same subband and IUI will change dynamically with $\mathbf{Algorithm\ 2}$, power allocation should be performed iteratively [13], it is generally a NP-hard problem [14] to solve the power allocation for all the users. When $N_s > 1$, each user is allocated to more than one subband, power allocation problem over multiband needs to be resolved for each user. To resolve the multiband power allocation, borrowed from the concept in [13], the multi-band water-filling algorithm should be carried out iteratively which is summarized in $\mathbf{Algorithm\ 2}$. At each iteration, every user performs multi-band water-filling based on IUI. After each user performs multi-band water-filling, we call it one iteration. The algorithm converges after few iteration times with any initial power $p_{k,n}, \forall k, n$. Multi-band water-filling is aim to find the maximum sum rate with allocated N_s subbands for user k.

Let $\mathbb{S}_k$ be the set of subbands that are allocated to user k. The maximum sum rate problem over the set $\mathbb{S}_k$ can be expressed as

$$\max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 \det(\mathbf{I}_{N_r} + \frac{\mathbf{1}}{\sigma^2} \sum_{j=1}^{K} x_{j,n} p_{j,n} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^*) \tag{8a}$$

$$= \max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 \det(\sigma^2 \mathbf{I}_{N_r} + \sum_{j=1}^{K} x_{j,n} p_{j,n} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^*) \tag{8b}$$

$$= \max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 \det(x_{k,n} p_{k,n} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^* + \mathbf{S}_{z_{k,n}}) \tag{8c}$$

where

$$\mathbf{S}_{z_{k,n}} = \sigma^2 \mathbf{I}_{N_r} + \sum_{j=1, j\neq k}^{K} x_{j,n} p_{j,n} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^* \tag{9}$$

where $\mathbf{I}_{N_r}$ is and identity matrix with dimension N_r by N_r. $\mathbf{S}_{z_{k,n}}$ is equivalent to the colored noise covariance matrix to user k. The colored noise covariance matrix $\mathbf{S}_{z_{k,n}}$ is whitened before single user power allocation [13]. The process consists of two main stages.

Stage 1: Let the eigen decomposition of $\mathbf{S}_{z_{k,n}}$ be $\mathbf{S}_{z_{k,n}} = \mathbf{Q}_{k,n} \mathbf{\Delta}_{k,n} \mathbf{Q}_{k,n}^*$, where $\mathbf{Q}_{k,n}$ is orthogonal matrix and $\mathbf{\Delta}_{k,n}$ is diagonal matrix of eigenvalues. Define $\hat{\mathbf{h}}_{k,n} = \mathbf{\Delta}_{k,n}^{-\frac{1}{2}} \mathbf{Q}_{k,n}^* \mathbf{h}_{k,n}$, the formulation (8c) can be rewritten as

$$\max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 \det(p_{k,n} \hat{\mathbf{h}}_{k,n} \hat{\mathbf{h}}_{k,n}^* + \mathbf{I}_{N_r}) \tag{10}$$

Stage 2: The $\hat{\mathbf{h}}_{k,n}$ can be decomposed such that $\hat{\mathbf{h}}_{k,n} = \mathbf{F}_{k,n} \mathbf{\Sigma}_{k,n} \mathbf{M}_{k,n}^{\mathbf{T}}$ by singular value decomposition, where $\mathbf{F}_{k,n}$ is an orthornomal matrix, $\mathbf{M}_{k,n} = 1$, $\mathbf{\Sigma}_{k,n}$ is vector $[\tilde{h}_{k,n}, 0, 0, \cdots, 0]^{\mathrm{T}}$ with dimension N_r by 1. The $\tilde{h}_{k,n}$ is the equivalent channel for user k on subband n, the formulation (10) can be rewritten as

$$p_{k,n}^* = \arg\max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 \det(p_{k,n} \mathbf{\Sigma}_{k,n} \mathbf{\Sigma}_{k,n}^* + \mathbf{I}_{N_r}) \tag{11a}$$

$$= \arg\max_{p_{k,n}} \sum_{n\in\mathbb{S}_k} \log_2 (1 + p_{k,n} |\tilde{h}_{k,n}|^2), \forall n \in \mathbb{S}_k \tag{11b}$$

The problem becomes a conventional single user power allocation for user k over N_s subbands with equivalent channel $\tilde{h}_{k,n}$ as

$$\mu_k = p_{k,n}^* + \frac{1}{|\tilde{h}_{k,n}|^2}, \forall n \in \mathbb{S}_k \tag{12}$$

where μ_k is power level for user k. We show that finding maximum sum rate over N_s subbands which user k allocated to is equivalent to find optimal power allocation for user k. Note that, in Stage 2, we need to find $|\tilde{h}_{k,n}|^2, \forall n \in \mathbb{S}_k$. Since each user is equipped with one antenna, we can simplify the Stage 2 and find that by $|\tilde{h}_{k,n}|^2 = \hat{\mathbf{h}}_{k,n}^* \hat{\mathbf{h}}_{k,n}, \forall n \in \mathbb{S}_k$.

Algorithm 2 Multi-band iterative power allocation algorithm

Input: Number of users=K, Number of subbands=N
Number of users on each subband=L, Number of subbands per user=N_s, Iteration times=$iternum$, User k using the subband n or not $x_{k,n}, \forall k, n.$, Initial power $p_{k,n}, \forall k, n.$

Output: Find $p_{k,n}, \forall k, n.$ after iteration times

```
for l = 1; l <= iternum; l++ do
    for k = 1; k <= K; k++ do
        let S = {∅}
        for n = 1; n <= N; n++ do
            if x_{k,n} = 1 then
                Let S_{z_{k,n}} = I_{k,n} + σ^2 I_{N_r}
                where I_{k,n} = Σ_{j=1,j≠k}^K x_{j,n} p_{j,n} h_{j,n} h*_{j,n}
                Do eigenvalue decomposition for S_{z_{k,n}}
                get S_{z_{k,n}} = Q_{k,n} Δ_{k,n} Q*_{k,n}
                Define ĥ_{k,n} = Δ_{k,n}^{-1/2} Q_{k,n}^* h_{k,n}
                get equivalent channel gain |h̃_{k,n}|^2
                |h̃_{k,n}|^2 = ĥ*_{k,n} ĥ_{k,n}
                S = S + {n}
            end if
        end for
        Do single user PA over S with equivalent
        channel gain |h̃_{k,n}|^2, ∀n ∈ S
        get optimal p*_{k,n}, ∀n ∈ S
    end for
end for
```

IV. Simulations

Assume each user is equipped with single antenna and the BS is equipped with N_r antennas. The users are randomly, independently, and uniformly scattered over the cellular network. The Poisson point process is applied for the geometric distribution of the users. The channel element is $h_{k,n}^{(m)} \sim CN(0, \frac{1}{1+\alpha^2})$. The α represents the distance between user and cell-center which is bounded between 0 and 10. The smaller the value of α represents that the user is nearer to cell center and vice versa. The maximum transmit power of each user is 23 dBm, and the total bandwidth is 1 MHz. σ_{PSD}^2 represents the noise power spectrum density. When $N_s = 1$, we apply **Algorithm 1** and equal power allocation. While $N_s > 1$, we apply **Algorithm 1** and **2** with different iteration times. When we apply the **Algorithm 2**, we let initial power $p_{k,n} = 0, \forall k, n$. Note that with equal power allocation, the power allocated to each subband is $\frac{P_k}{N_s}, \forall k$. The fairness in simulation can be evaluated by Jain's fairness index J, [15], and $J = \frac{(\sum_{k=1}^{K} R_k)^2}{K \sum_{k=1}^{K} R_k^2}$, which is bounded between 0 and 1.

A. Sum rate analysis with fixed resources number $N \times L$

Given $N = 30$, $L = 4$ in an uplink NOMA system, with fixed number of resources $N \times L$ and noise power spectral density $\sigma_{PSD}^2 = -70$(dB/Hz). Further, we constrain the total transmit power $\sum_{k=1}^{K} P_k$ for the system is fixed, where $P_k, \forall k$

is equal. Note that $P_k, \forall k$ and K varies with N_s. When the iteration is equal to zero in **Algorithm 2** or N_s is equal to one, the equal power allocation is applied. In Fig. 1, with MMSE-SIC ($N_r = 4$) and ZF-SIC ($N_r = 1$) receiver, it shows the sum rate in **Algorithm 2** with different number of iterations. We observe that sum rate for both MMSE-SIC and ZF-SIC receiver converge fast after few iteration times. When N_s is equal to one, the system can achieve maximum sum rate which verifies the **Proposition1**.

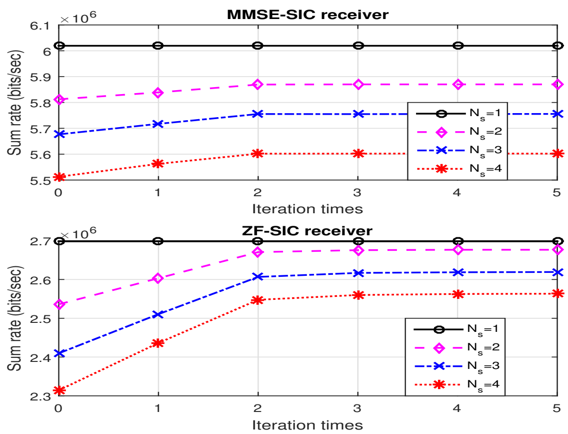

Fig. 1: Sum rate with MMSE-SIC and ZF-SIC receiver

B. Sum rate analysis for Fixed number of users K

Given K users in the system, we consider the fixed ratio of $\frac{L}{N_s}$ for simplicity but different combination of L and N_s can be applied. Assume that $\frac{L}{N_s} = 1$, $K = N = 30$. In Fig.2, we compare the sum rate with MMSE-SIC ($N_r = 4$) and ZF-SIC ($N_r = 1$) receiver. Iterations for **Algorithm 2** is equal to three. We observe that if L grows, the sum rate increases. While the sum rate improvement decrease due to the increasing of interference in each subband. Sum rate performance with MMSE-SIC receiver is obviously better than sum rate performance with ZF-SIC receiver, while the complexity for MMSE-SIC receiver also increase. We observe the tradeoff between receiver architecture and sum rate. Note that $L = 1$ actually represents the OMA system.

C. Sum rate and fairness comparison with fixed resources number $N \times L$

Given $N = 30$, $L = 4$ in an uplink NOMA system, with fixed resource number $N \times L$. The transmit power of each user $P_k, \forall k$, is fixed in 23 dBm , $\sigma^2_{PSD} = -70$(dB/Hz). In our system model, we apply the algorithm with method LRM and GOM in [3] and compare the sum rate and fairness with our algorithm and ZF-SIC receiver. We also compare the fairness between MMSE-SIC and ZF-SIC receiver. In Fig. 3, the simulation result shows that with our **Algorithm 1** and **Algorithm 2**, the sum rate performance is better than either LRM or GOM. Sum rate performance for NOMA is better

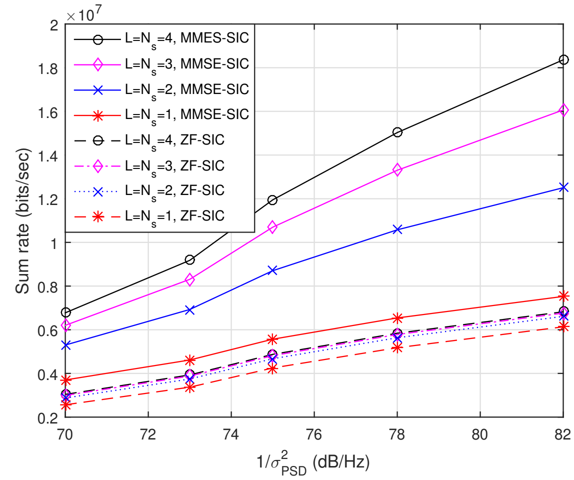

Fig. 2: Sum rate with MMSE-SIC and ZF-SIC receiver

than OMA's due to L users in each subband. In Fig. 4, the simulation result shows that with **Algorithm 1** and equal power allocation, fairness performance is better than either LRM or GOM due to the constraint on N_s. When $N_s > 1$ with **Algorithm 1** and **Algorithm 2**, though the sum rate performance is better than either LRM or GOM in Fig. 3, the fairness performance decrease. We observe the tradeoff between sum rate and fairness. For **Algorithm 1** and equal power allocation, the system achieve optimal fairness when $N_s = 1$. With N_s decreasing, user diversity increases since K increases. Fairness performance increases benefited from multi-user diversity since **Algorithm 1** depends on $g_{k,n}$. Opportunity for subbands to select the user with better $g_{k,n}$ increases. Fainess performance for NOMA is better than OMA's since users have more resource to occupy. In Fig. 5, fairness performance for MMSE-SIC receiver is better than ZF-SIC receiver. We observe the tradeoff between receiver architecture and fairness. The fairness for both MMSE-SIC and ZF-SIC receiver decrease with the iterations in **Algorithm 2** and obtain the optimal fairness when $N_s = 1$.

V. Conclusions

The NOMA accommodates multiple users in the same subband, which allows more users in system as compared with OMA. With proposed **Algorithm 1** that forcing non-zero number of subbands N_s for each user, the cell-edge users with poor channel condition are guaranteed to win the subbands and thus enhance fairness. Fairness performance is better than LRM and GOM [3]. Applying **Algorithm 1** and **2**, the system can achieve better sum rate performance than prior arts of LRM and GOM [3]. Given fixed resource blocks $(N \times L)$, usually the sum rate performance and the fairness trade off as shown in the prior arts. With massive connectivity, both the sum rate performance and the fairness increase as the number of users K grows, when $N_s = 1$, the system not only achieves maximum sum rate but also achieves the optimal fairness. As the maximum number of users is allowed

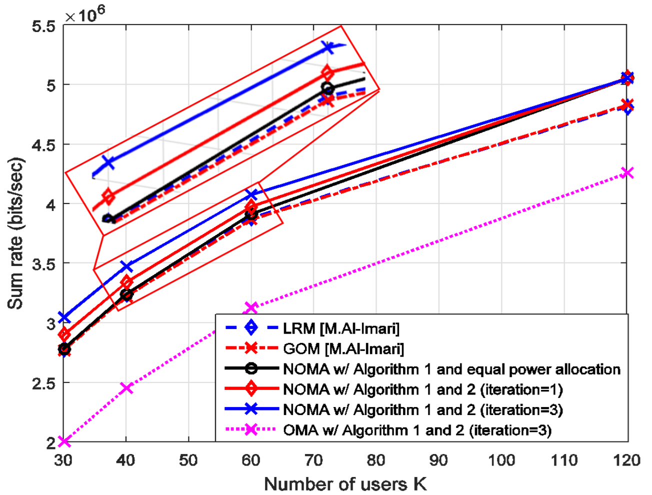

Fig. 3: Sum rate comparison with prior GOM and LRM [3] NOMA schemes using ZF-SIC receiver, and fixed $N \times L = 120$. For OMA, $L = 1$, $N_s \leq \frac{N \times 4}{K}$.

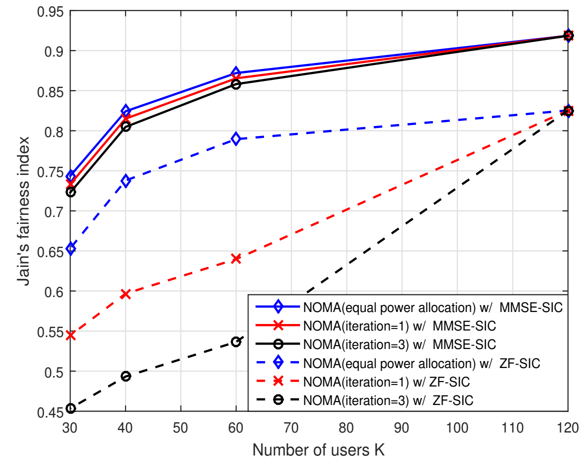

Fig. 5: Fairness evaluation as number of users increases for MMSE-SIC and ZF-SIC receiver.

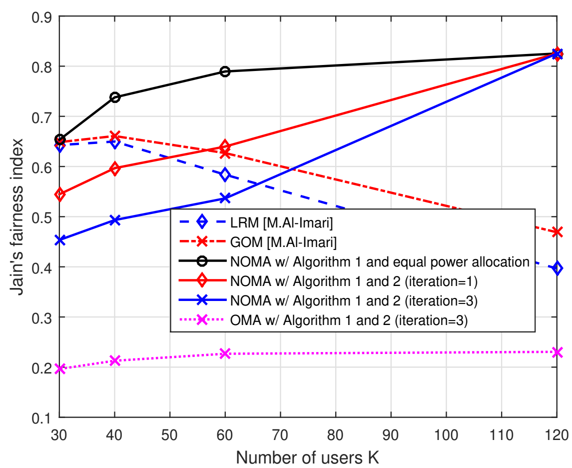

Fig. 4: Fairness comparison with prior GOM and LRM [3] NOMA schemes using ZF-SIC receiver, and fixed $N \times L = 120$. For OMA, $L = 1$, $N_s \leq \frac{N \times 4}{K}$.

to the system, better multi-user diversity can be exploited for sum rate and fairness. Tradeoff between sum rate, fairness and receiver architecture can also be observed with MMSE-SIC and ZF-SIC receiver.

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