Real-Time Demand Response with Uncertain Renewable Supply

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Caltech

CCW 2011

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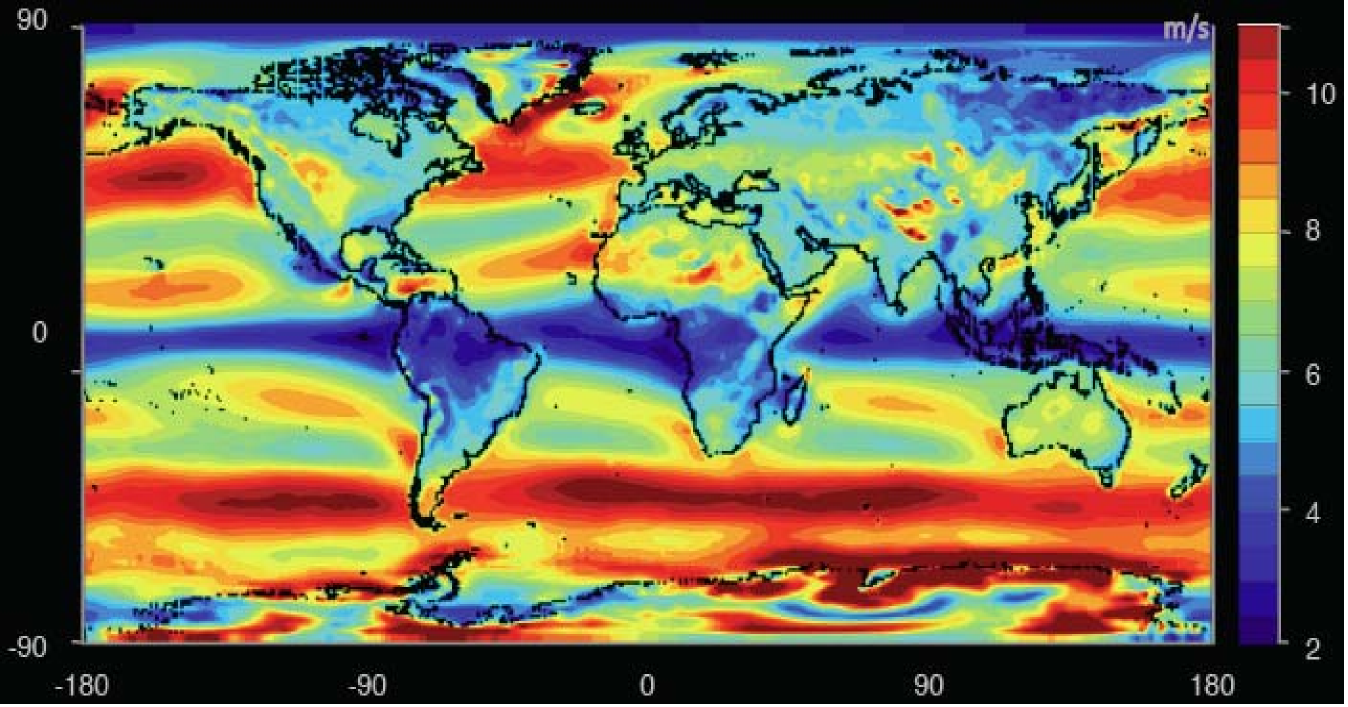
Outline

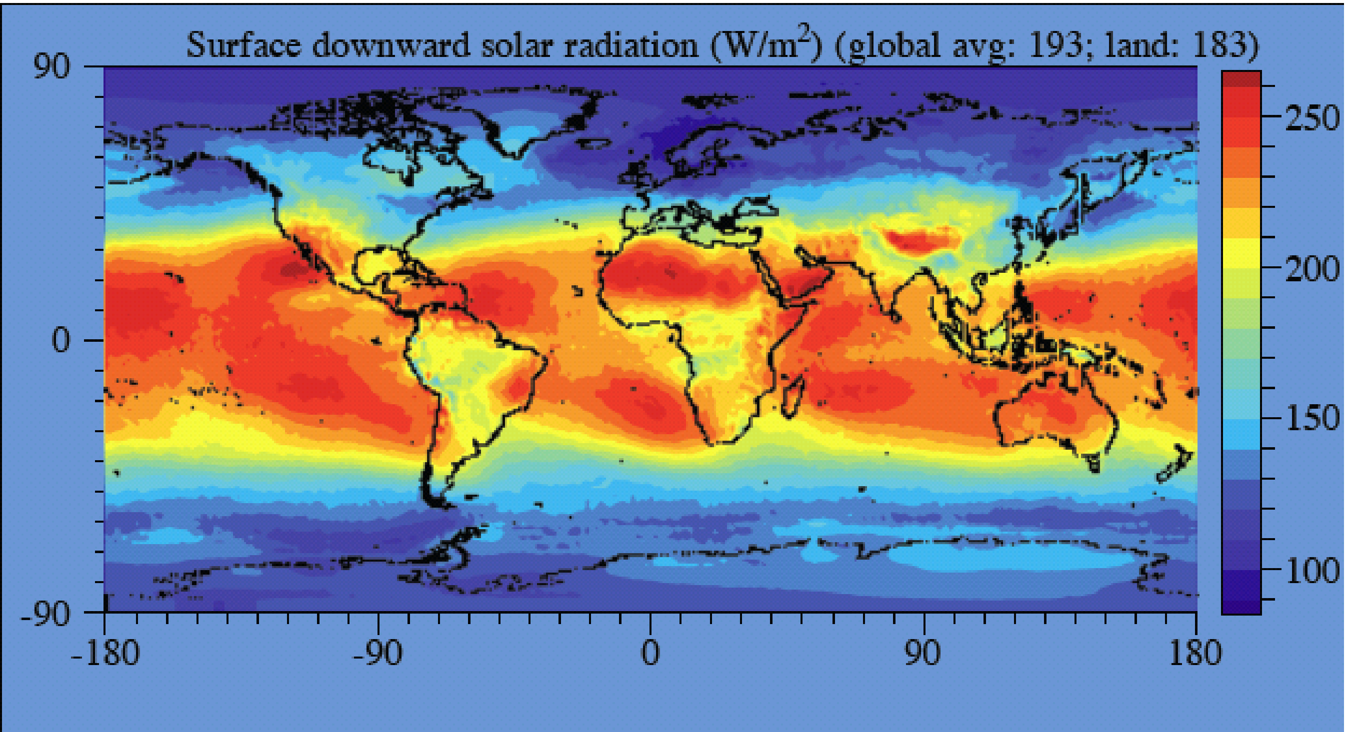
Renewable integration

DR model

Results



Wind power over land (outside Antartica):
70 – 170 TW

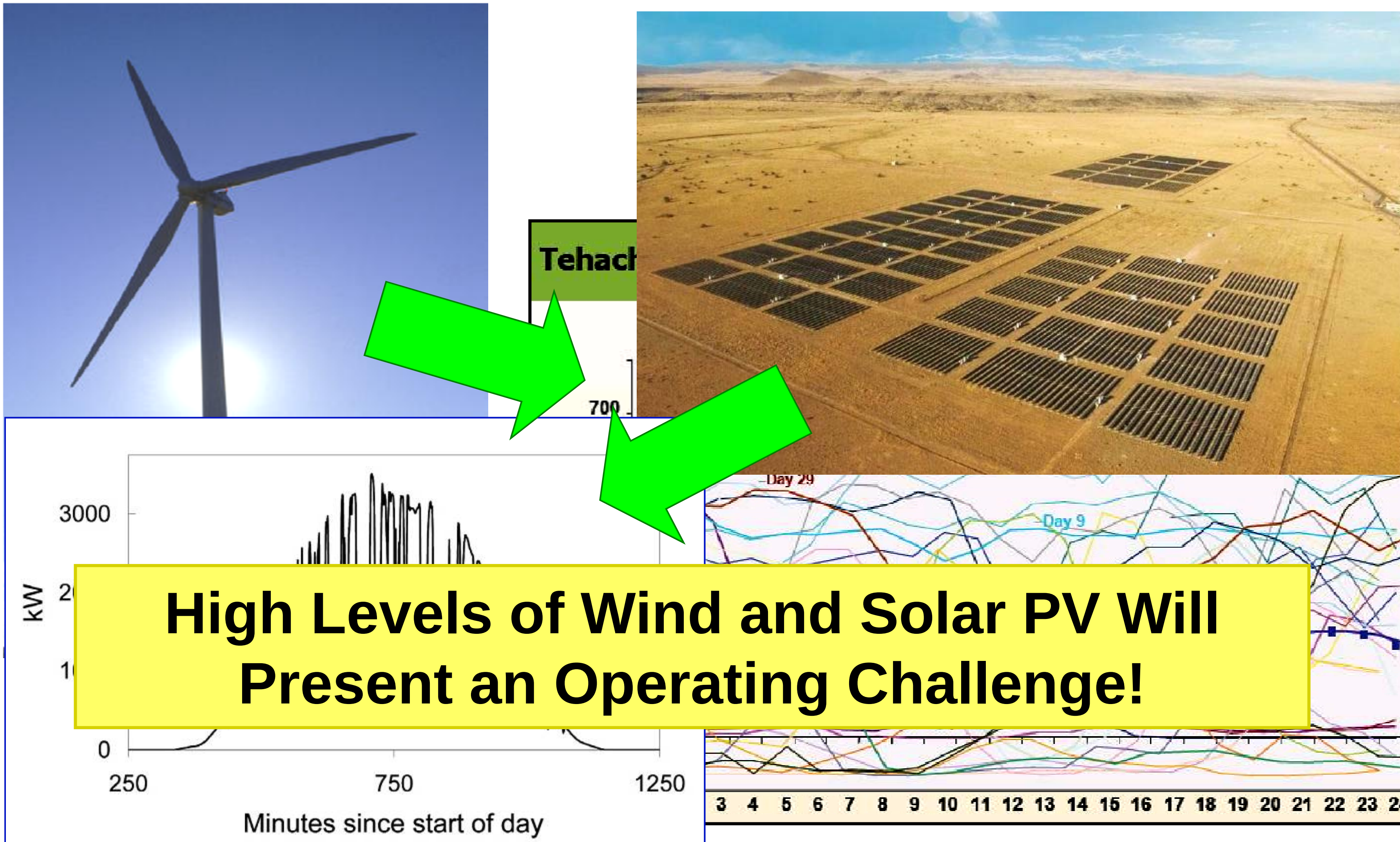


Solar power over land:
340 TW

World power demand:
16 TW
Electricity demand:
2.2TW
Installed wind capacity
128 GW

Source: M. Jacobson, 2011

Uncertainty of renewables



High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang

ELECTRIC POWER RESEARCH INSTITUTE

Challenges of uncertainty

- Matching supply and demand
  - Market as well as engineering challenges
- Demand response
  - Matching adaptable loads to uncertain supply

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Features to capture

Wholesale markets

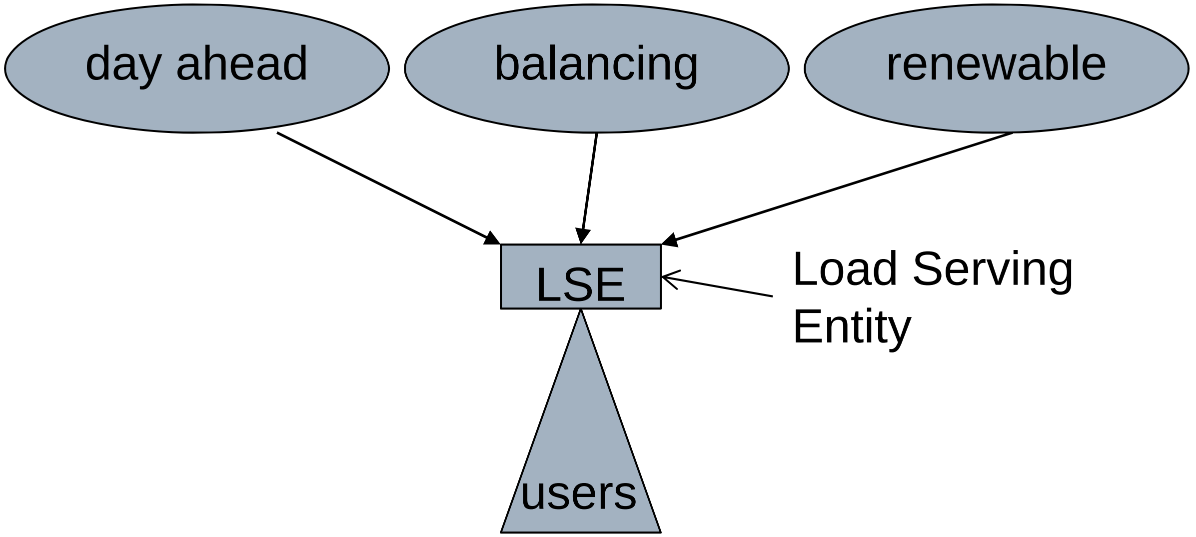
- Day ahead, real-time balancing

Renewable generation

- Random

Demand response

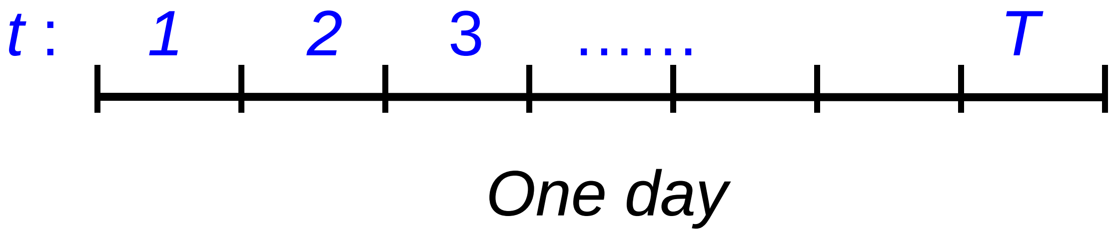
- Real-time control (through pricing)



Related works

- Demand side
  - Charging of Electric vehicles [Clement-Nyns et al. 2010], [Pang et al. 2011]
  - Coordinated scheduling of different appliances [Pedrasa et al. 2010], [Mohsenian-Rad et al. 2010]
  - No explicit consideration of supply, renewables
- Supply side
  - Unit-commitment problem with random generator and line outages [Bouffard-Galiana-Conejo, 2005]
  - Wholesale market with uncertainties [Pritchard-Philpott-Zakeri, 2010]

Model: user

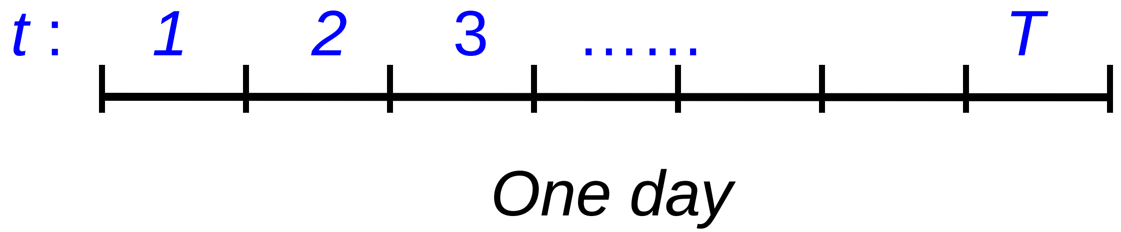


- User i (or appliance i) consumes $x_i(t)$ in period t
- Utility function: $U_i(\mathbf{x_i})$ where $\mathbf{x_i} = (x_i(t))$
- Consumption constraints:

$$A_i \mathbf{x_i} \leq b_i$$

- Models of appliances (AC, electric vehicle, entertainment system, battery, etc.) [Li-Chen-Low'11]

Model: user



- User i (or appliance i) consumes $x_i(t)$ in period t
- Utility function: $u_i(x_i(t))$ in period t
- Consumption constraints:

$$\underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t)$$

$$\sum_{t=1}^{T} x_i(t) \geq R_i$$

Model: LSE (load serving entity)

capacity

used

Power procurement

- Day-ahead power: $P_d(t)$; $c_d\left(P_d(t)\right),\ c_o\left(P_o(t)\right)$
  - $P_d(t)$ decided a day ahead
- Renewable power: $P_r(t)$; $c_r\left(P_r(t)\right) = 0$
  - Random variable, realized in real-time t-
- Real-time balancing power: $P_b(t)$; $c_b\left(P_b(t)\right)$

costs

$$P_o(t) + P_b(t) + P_r(t) \geq \sum_i x_i(t)$$

$t = 1,2,\ldots, T$

Day-ahead t-

$P_d(\tau), \forall \tau$

$P_r(t)$

decide $P_b(t), P_o(t)$

Model: LSE (load serving entity)

- The minimal cost in period t, given $P_d(t)$, $P_r(t)$, and $x(t):=(x_i(t), ;i)$, is

$$c_t(x(t), P_d(t); P_r(t)),$$

after optimizing over $P_o(t)$ and $P_b(t)$

Specifically,

$$c_t(x(t), P_d(t); P_r(t)) :=$$

$$\min_{P_o(t), P_b(t) \geq 0} \{c_d(P_d(t)) + c_o(P_o(t)) + c_r(P_r(t)) + c_b(P_b(t))\}$$

$$s.t. \;\; P_o(t) \leq P_d(t), P_o(t) + P_r(t) + P_b(t) \geq \sum_i x_i(t)$$

Model

Goal

- Choose supply P_d(t) (day-ahead) and demand x_i(t) (real-time) to maximize expected welfare

$$E[\sum_{t=1}^{T} W_t(x(t), P_d(t); P_r(t))]$$

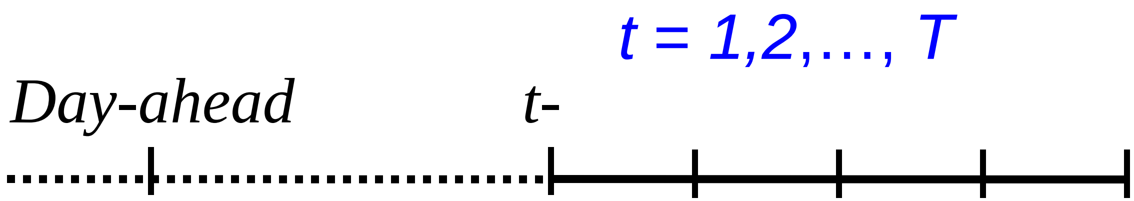
where

$$W_t(x(t), P_d(t); P_r(t)) := \sum_i u_i(x_i(t)) - c_t(x(t), P_d(t); P_r(t))$$

Features of the problem

- Multi-timescale decisions; uncertainty; requiring distributed algorithms.

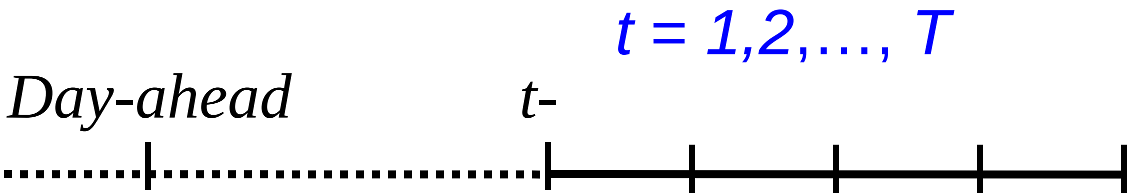
Dynamic-program formulation



$(T+1)$-stage DP

- Day-ahead: Choose $P_d := (P_d(\tau), \tau = 1, 2, \ldots, T)$
- Real-time: At t-, choose $x_i(t)$

Dynamic-program formulation



State: 0

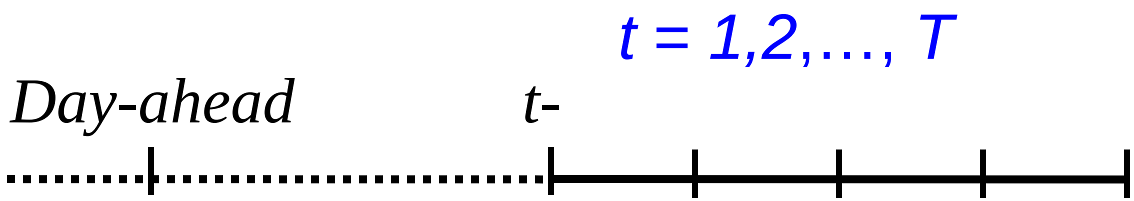
Input: P_d

Reward: 0

(T+1)-stage DP

- Day-ahead: Choose $P_d := (P_d(\tau), \tau = 1, 2, ..., T)$
- Real-time: At t-, choose $x_i(t)$

Dynamic-program formulation



State:	0	$(P_d, R_i(t), P_r(\tau), \tau \le t)$
Input:	P_d	$x_i(t), \forall i$
Reward:	0	$W_t(x(t), P_d(t); P_r(t))$

Remaining demand

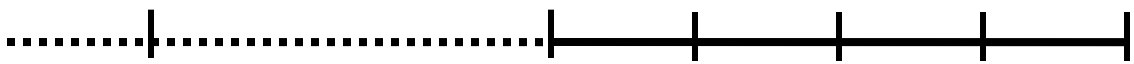
State evolution: $R_i(1) = R_i$

$$R_i(t+1) = R_i(t) - x_i(t)$$

(T+1)-stage DP

- Day-ahead: Choose $P_d := (P_d(\tau), \tau = 1, 2, ..., T)$
- Real-time: At t-, choose $x_i(t)$

Dynamic-program formulation



State: 0 $\quad (P_d, R_i(t), P_r(\tau), \tau \le t)$

Remaining demand

Input: P_d $\quad x_i(t), \forall i$

Reward: 0 $\quad W_t(x(t), P_d(t); P_r(t))$

Terminal reward:

$-\infty$ if $R_{i'}(T+1) > 0$

0 otherwise

State evolution: $R_i(1) = R_i$

$$R_i(t+1) = R_i(t) - x_i(t)$$

(T+1)-stage DP

- Day-ahead: Choose $P_d := (P_d(\tau), \tau = 1, 2, \ldots, T)$
- Real-time: At t-, choose $x_i(t)$

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Results

- Distributed algorithm
- Numerical results
- Effect of renewable on welfare

Distributed algorithm

- Algorithm 1
  - Main idea: solve a deterministic problem in each step, using the conditional expectation of $P_{r.}$ Apply the decision at the current step.
  - One day ahead, decide P_d^* by solving

$$\max_{P_d,x} \sum_{\tau=1}^{T} W_\tau \left(x(\tau), P_d(\tau); \overline{P}_r(\tau) \right) \quad s.t. \sum_{\tau=1}^{T} x_i(\tau) \geq R_i$$

    - Distributed implementation (primal-dual algorithm)

  - At time t- where $t=1,2,\ldots,T$, decide $x^*(t)$ by solving

$$\max_{x} \sum_{\tau=t}^{T} W_\tau \left(x(\tau), P_d^*(\tau); \overline{P}_r(\tau \mid t) \right) \quad s.t. \sum_{\tau=t}^{T} x_i(\tau) \geq R_i(t)$$

where $R_i(t) = R_i(t-1) - x_i^*(t-1)$

Performance

- **Thm**: Algorithm 1 is optimal if
  - Cost functions and utility functions are quadratic
  - Constraint is changed to $\sum_{t=1}^{T} x_i(t) = R_i, \forall i$
  - Optimizations never hit non-negativity constraints

(Proof: An extension of Linear Quadratic Stochastic Control)

- **Thm**: Assume that utility functions are 0, $P_r(t)$ are independent. And

$$c_d(P) = c_b(P) = P^2/2, c_o(P) = 0.$$

Then the optimality gap is

$$\sum_{t=1}^{T} \frac{1}{T-t+1} \sigma^2(t)$$

Variance of $P_r(t)$

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Numerical results

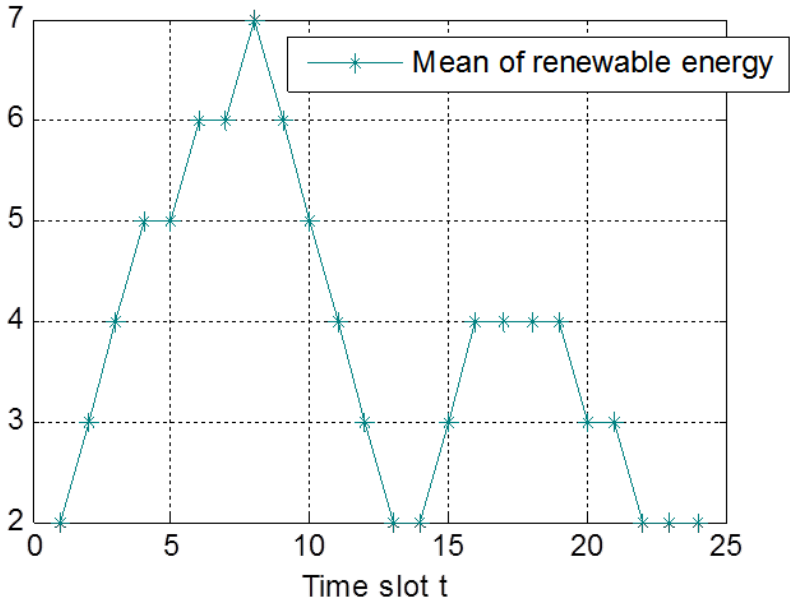
- T=24 slots, N=4 users
- Cost functions

$$c_d(P) = (P^2 + P)/2$$

$$c_o(P) = P/2$$

$$c_b(P) = (P^2 + 10P)/2$$

- Renewable energy uniformly distributed, with mean:



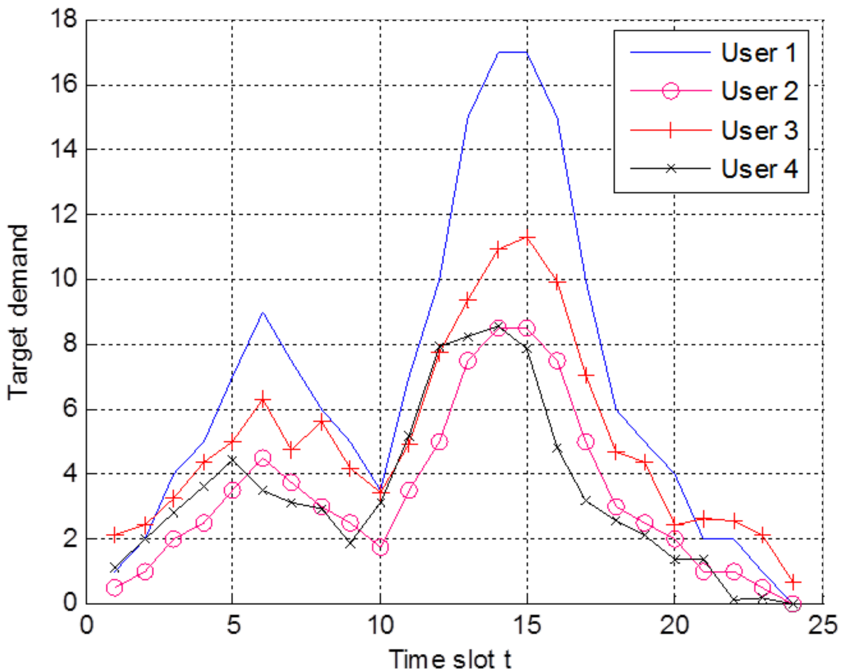
- Utility function

Target demand

$$U_i(x_i(t);t) = -[x_i(t) - z_i(t)]^2$$

- Consumption constraints

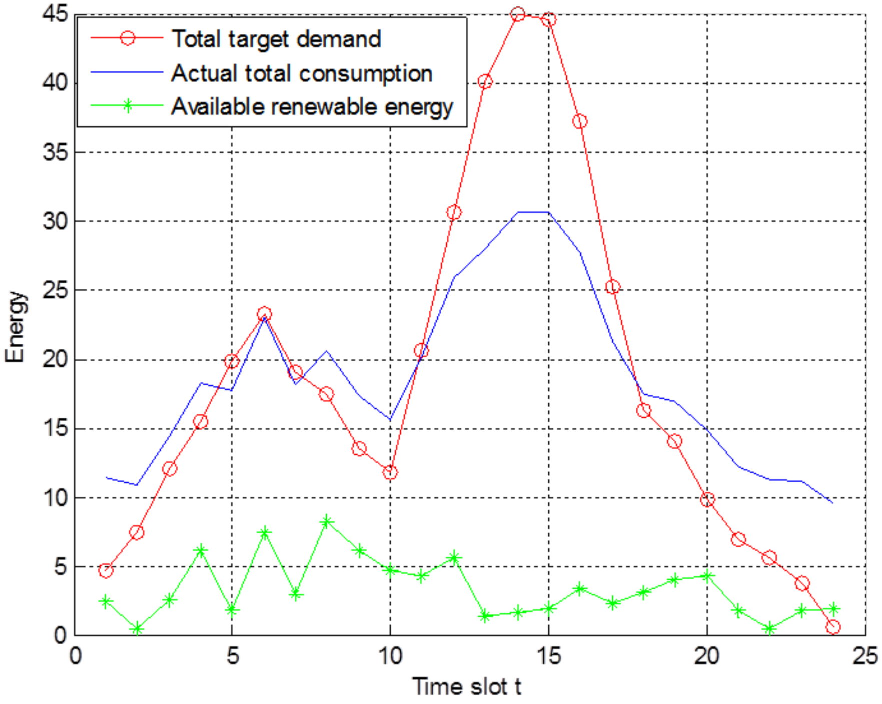
$$\sum_{t=1}^{T} x_i(t) \geq \sum_{t=1}^{T} z_i(t), \forall i$$



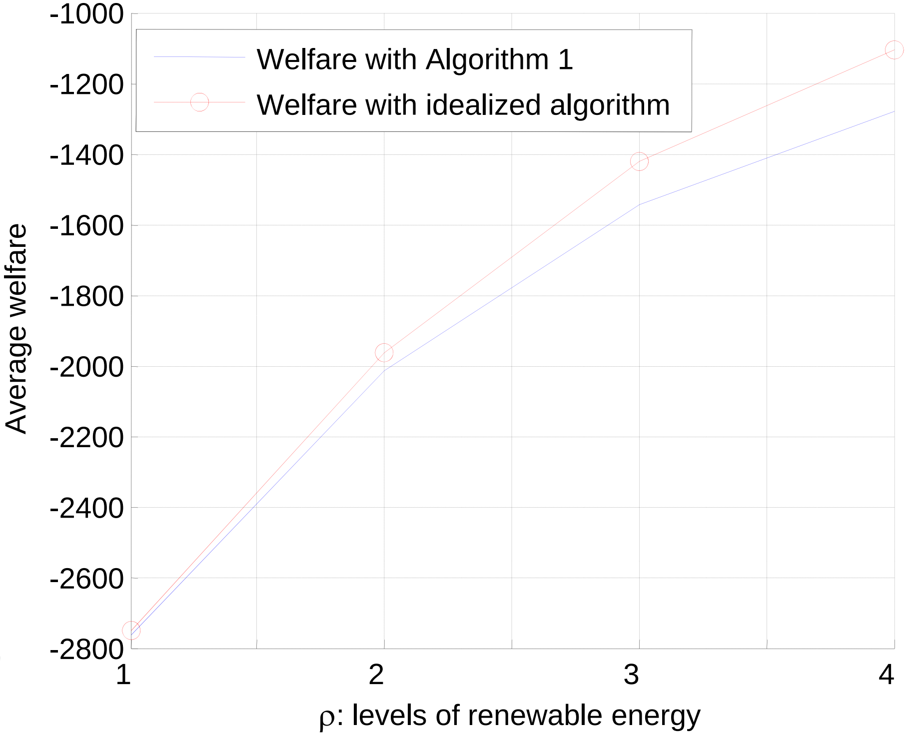
Numerical results

- With Algorithm 1:

Demand shaping



Welfare



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Effect of renewable on welfare

Renewable power:

$$P_r(t;a,b) := a \cdot \mu_r(t) + b \cdot V_r(t)$$

mean   zero-mean RV

Maximum welfare

$$J^*(a,b)$$

Theorem

- $J^*(a,b)$ increases in *a*, decreases in *b*
- $J^*(s,s)$ increases in *s*

Conclusion

- Energy procurement and real-time demand response with renewables
  - Multi-timescale optimization
  - Uncertainties of supply and demand
  - Decentralized computation
- Future work
  - Network constraints

Thank you!

Questions and comments?