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Frequency-dependent modeling of components in power electronics systems

Bjørn Gustavsen
SINTEF Energy Research
Trondheim, Norway

bjorn.gustavsen@sintef.no.

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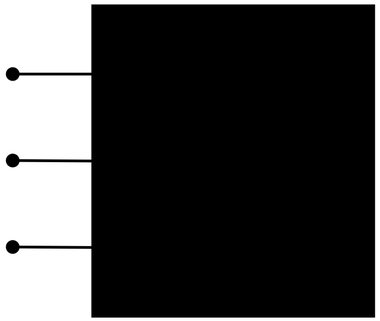
Outline of presentation

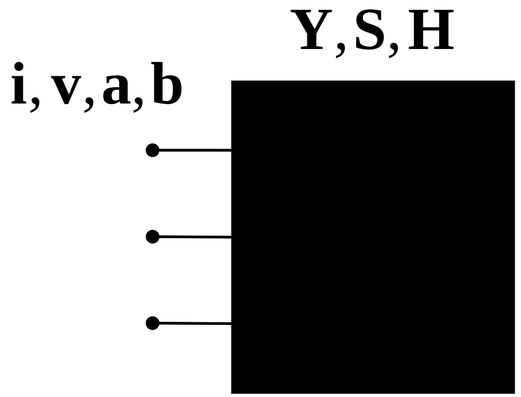
- Parameters for characterization of component behavior
- Extraction of rational function-based models
- Inclusion of models in circuit simulators
- Applications

Model classification

- Linear, time-invariant
- Frequency dependent behavior
- Black-box model (terminal equivalent)
  - No information about internal voltages/currents



Characterizing terminal behavior



- Admittance parameters $\mathbf{i}(\omega) = \mathbf{Y}(\omega)\mathbf{v}(\omega)$
- Scattering parameters $\mathbf{b}(\omega) = \mathbf{S}(\omega)\mathbf{a}(\omega)$

Interacting with system

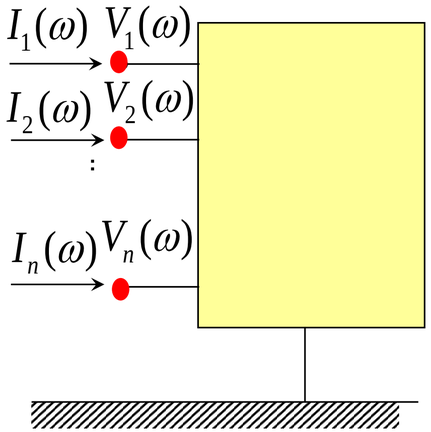
- Voltage transfer functions $\mathbf{v}_{\text{int}}(\omega) = \mathbf{H}(\omega)\mathbf{v}_{\text{ext}}(\omega)$

Non-interacting

Many other options…

Terminal characterization by admittance parameters

terminals



$$\mathbf{i} = \mathbf{Y}\,\mathbf{v}$$

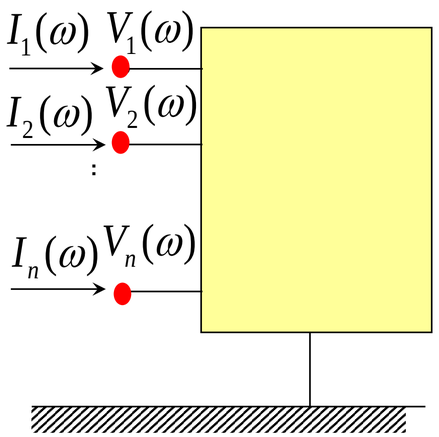
$$\begin{bmatrix} I_1(\omega) \\ I_2(\omega) \\ \vdots \\ I_n(\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) & & Y_{11}(\omega) \\ \vdots & & & \vdots \\ Y_{n1}(\omega) & Y_{n2}(\omega) & \cdots & Y_{nn}(\omega) \end{bmatrix} \begin{bmatrix} V_1(\omega) \\ V_2(\omega) \\ \vdots \\ V_n(\omega) \end{bmatrix}$$

Obtaining the admittance parameters

- Calculations
  - Using information about internal structure and material parameters
    Example: Cables
  - Reduced-order modeling of system of (known) linear components
    Example: network equivalents

- Measurements
  - Vector network Analyzer
    - Gain-phase, S-parameters
    - Impedance measurement
  - Other

Modeling using rational functions



Measurement

$$\begin{bmatrix} I_1(\omega) \\ I_2(\omega) \\ \vdots \\ I_n(\omega) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) & & Y_{11}(\omega) \\ \vdots & & & \vdots \\ Y_{n1}(\omega) & Y_{n2}(\omega) & \cdots & Y_{nn}(\omega) \end{bmatrix}}_{\mathbf{Y}} \begin{bmatrix} V_1(\omega) \\ V_2(\omega) \\ \vdots \\ V_n(\omega) \end{bmatrix}$$

Model extraction (fitting)

Non-linear least squares (NLLS)

$$\underbrace{\mathbf{Y}(\omega) \cong \sum_{m=1}^{N} \frac{\mathbf{R}_m}{j\omega - a_m} + \mathbf{D}}_{\text{Pole-residue}} \quad \underbrace{= \mathbf{C}(j\omega\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}}_{\text{State-space}}$$

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Why rational model?

- Analytical transformation from frequency domain to time domain (accurate!)

$$y(s) = \sum_m \frac{r_m}{s - a_m} \quad \Rightarrow \quad y(t) = \sum_m r_m e^{at}$$

- Recursive convolution in time domain (fast!)

$$i(t) = y(t) * v(t) \quad \Rightarrow \quad \begin{cases} \mathbf{x}_n = \boldsymbol{\alpha}\, \mathbf{x}_{n-1} + v_{n-1} \\ i_n = \mathbf{c}^T \mathbf{x}_n + \gamma\, v_n \end{cases}$$

Rational fitting via polynomials

$$f(s) \cong \frac{a_0 + a_1 s + a_2 s^2 + \ldots}{1 + b_1 s + b_2 s^2 + \ldots} \qquad s = j\omega$$

$$\Downarrow$$

$$f(s)(1 + b_1 s + b_2 s^2 + \ldots) \cong a_0 + a_1 s + a_2 s^2 + \ldots$$

$$\Downarrow \quad \textbf{Ax=b} \quad s_1, s_2, \ldots s_K$$

$$b_1, b_2, \ldots a_1, a_2, \ldots$$

$$\Downarrow$$

$$f(s) = \sum_m \frac{r_m}{s - a_m} + d$$

- Ill-conditioning
- Biased
- Unstable poles

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Vector Fitting (Pole Relocation)*

- Method for fitting a pole-residue model to frequency data

$$f(s) \cong \sum_{m=1}^{N} \frac{r_m}{s - a_m} + d \qquad s = j\omega$$

  - Robust, efficient, accurate
  - Guaranteed stable poles

- Freely available since 1999
- Widely applied for rational modeling of power systems, high-speed electronics, microwave structures,...

*B. Gustavsen and A. Semlyen,"Rational approximation of frequency domain responses by vector fitting", IEEE Trans. Power Del., 1999.

Vector fitting: Pole identification

$$\underbrace{(1+\sum\frac{\tilde{c}}{s-\bar{a}})}_{\sigma(s)} f(s) = \sum\frac{c}{s-\bar{a}}$$

$\bar{a}$:specified (initial) poles

Ax=b

$\rightarrow \tilde{c}, c$

$$f(s) = \frac{\sum\frac{c}{s-\bar{a}}}{\underbrace{(1+\sum\frac{\tilde{c}}{s-\bar{a}})}_{\sigma(s)}} = \frac{\frac{\prod(s-z)}{\prod(s-\bar{a})}}{\frac{\prod(s-\tilde{z})}{\prod(s-\bar{a})}} = \frac{\prod(s-z)}{\prod(s-\tilde{z})}$$

The poles of *f* are equal to the zeros of σ !

The new poles a re-used as initial poles → iterative procedure

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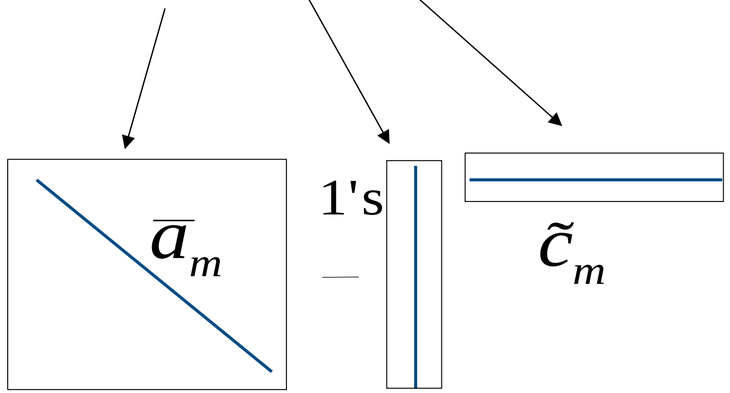
Vector fitting: Zeros calculation

$$\sigma(s) = \sum_{m=1}^{N} \frac{\tilde{c}_m}{s - \bar{a}_m} + 1$$

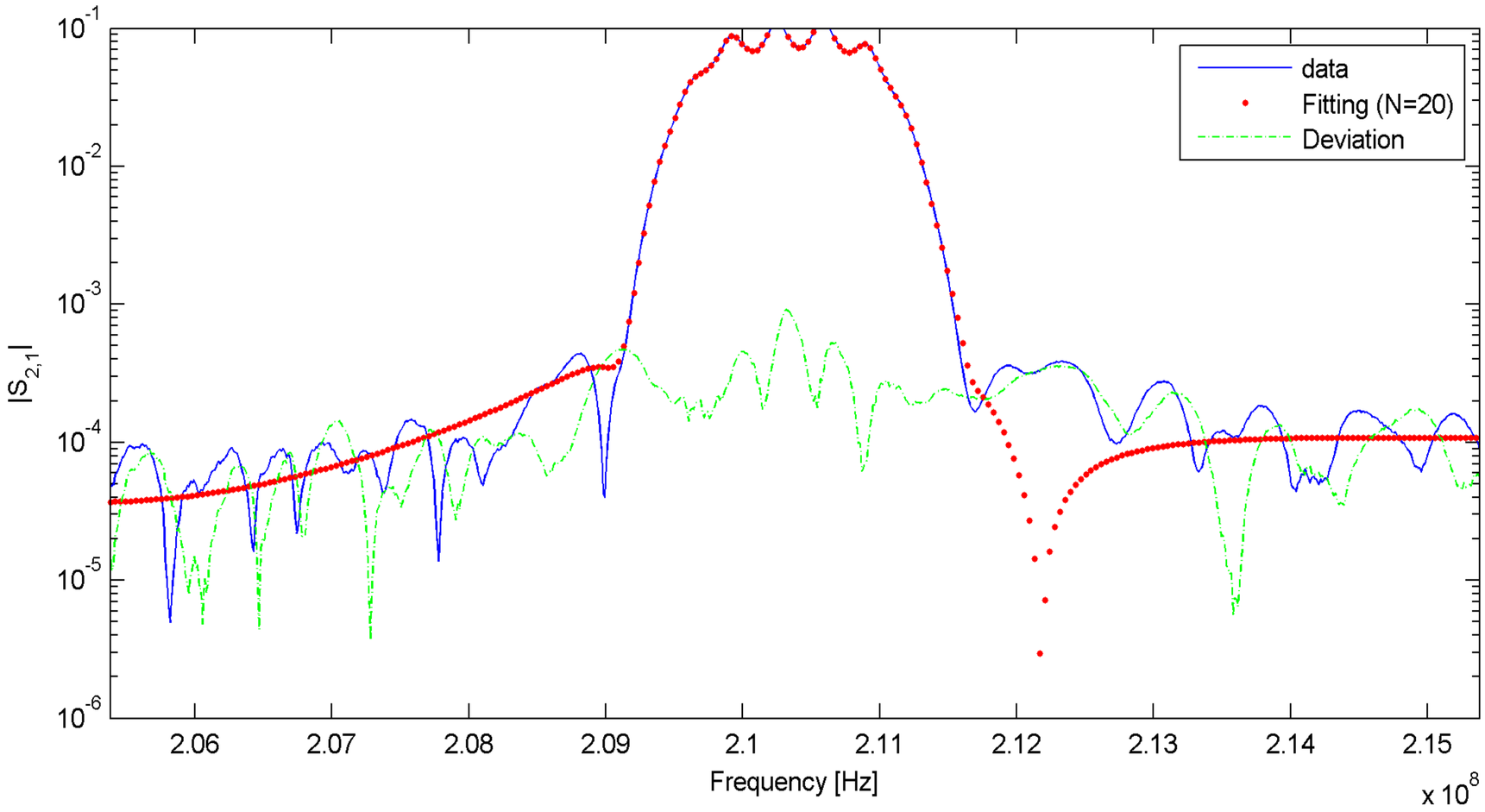
$$\{z_m\} = eig(\tilde{A} - b \cdot \tilde{c}^T)$$



Unstable poles are "flipped" into the left half-plane → guaranteed stable poles

Example SAW filter

20th order approximation



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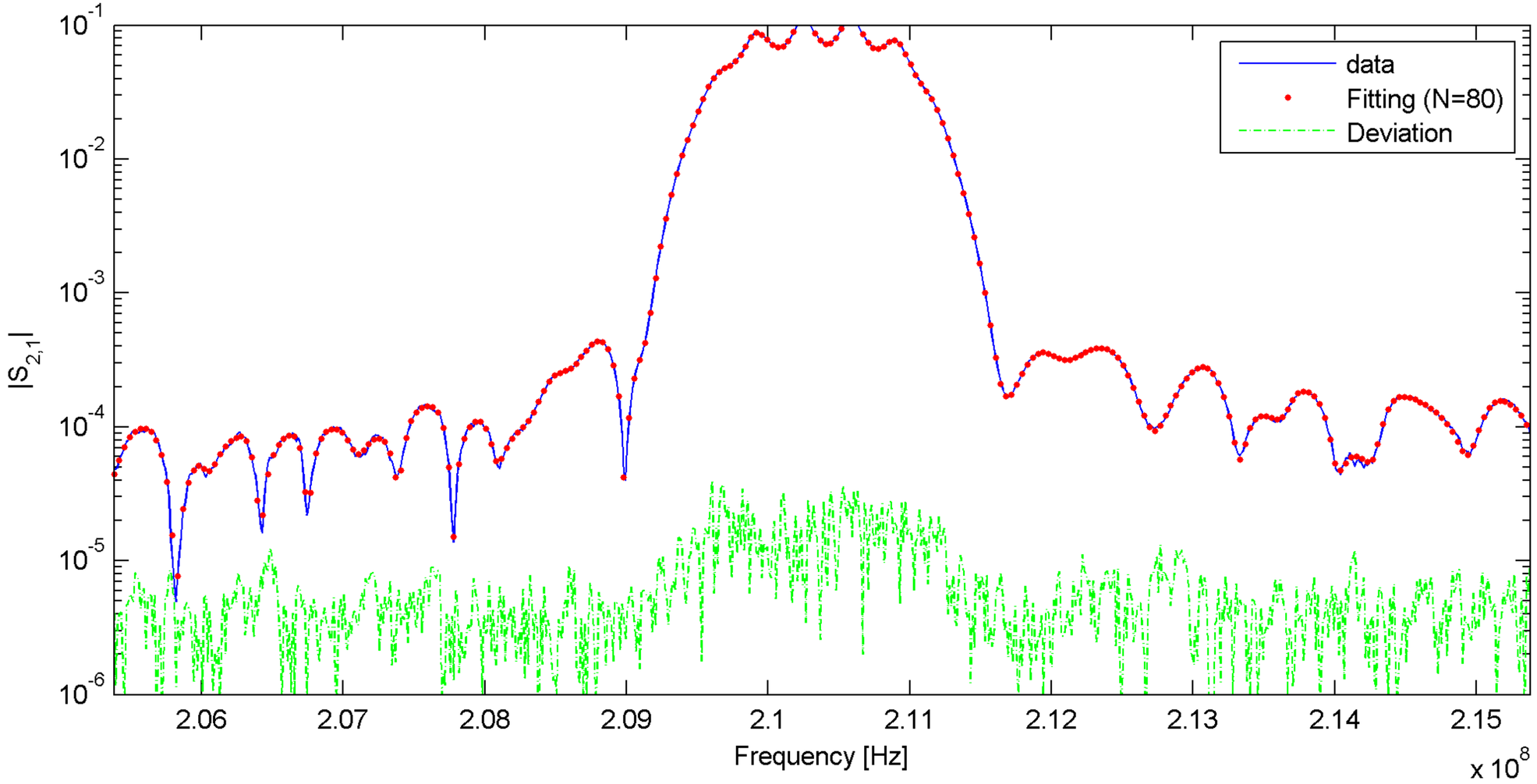
Example SAW filter

80th order approximation



Stability of time domain simulation

$$\mathbf{Y}(\omega) \cong \sum_{m=1}^{N} \frac{\mathbf{R}_m}{j\omega - a_m} + \mathbf{D}$$

Requirements:

1. Poles in left half plane: $\{a_m\} < 0$
2. eig(Re{**Y**(ω)}) > 0
3. eig(**D**)>0

} **Passivity condition** (symmetrical model)

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Passivity enforcement by perturbation

- Perturb the model's parameters so that the model becomes passive*
  - Perturb only elements in $\{\mathbf{R}_m\}$, $\mathbf{D}$
  - Make change to $\mathbf{Y}(s)$ minimal

LS problem

$$\Delta\mathbf{Y} = \sum_{m=1}^{N} \frac{\Delta\mathbf{R}_m}{s - a_m} + \Delta\mathbf{D} \cong \mathbf{0}$$

Constraint

$$eig\left(\mathrm{Re}\left\{\mathbf{Y} + \sum_{m=1}^{N} \frac{\Delta\mathbf{R}_m}{s - a_m}\right\}\right) > \mathbf{0}$$

$$eig(\mathbf{D} + \Delta\mathbf{D}) > \mathbf{0}$$

<u>Linearization</u> (eigenvalue perturbation)

$$\mathbf{A}_{sys}\,\Delta\mathbf{x} \cong 0$$

$$\mathbf{B}_{sys}\,\Delta\mathbf{x} < \mathbf{c}$$

Constrained LS problem (e.g. QP)

$\Delta\mathbf{x}$: elements of $\{\Delta\mathbf{R}_m\}$, $\Delta\mathbf{D}$

*B. Gustavsen, and A. Semlyen, "Enforcing passivity for admittance matrices approximated by rational functions", IEEE Trans. Power Systems, vol. 16, no. 1, pp. 97-104, Feb. 2001.

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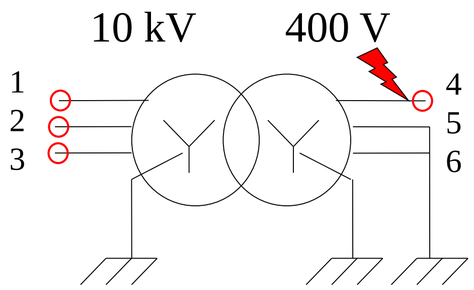
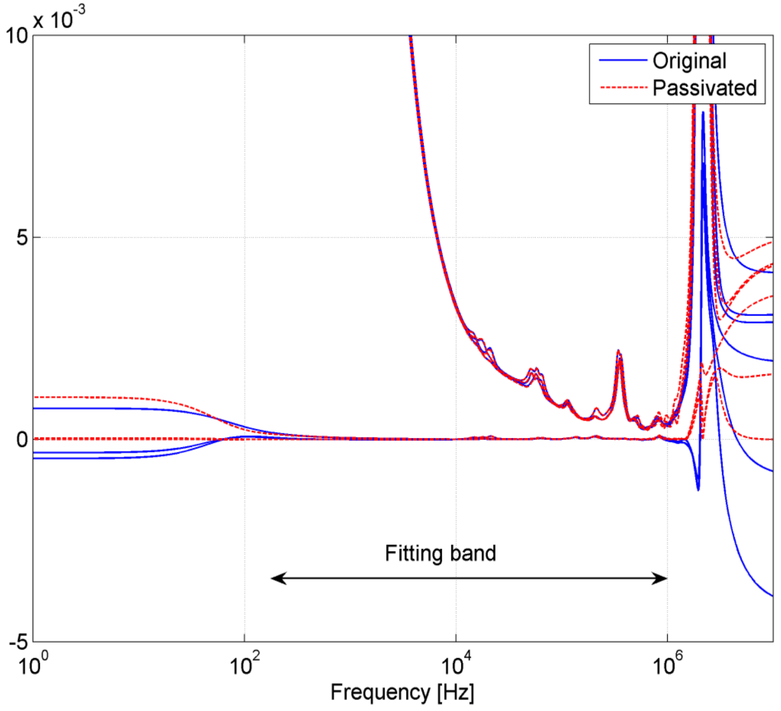
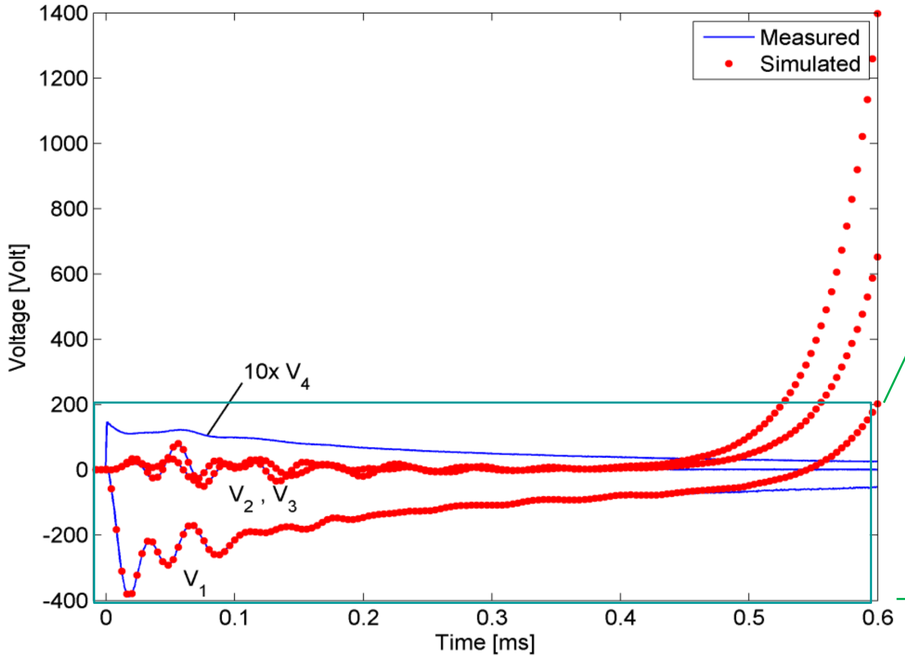
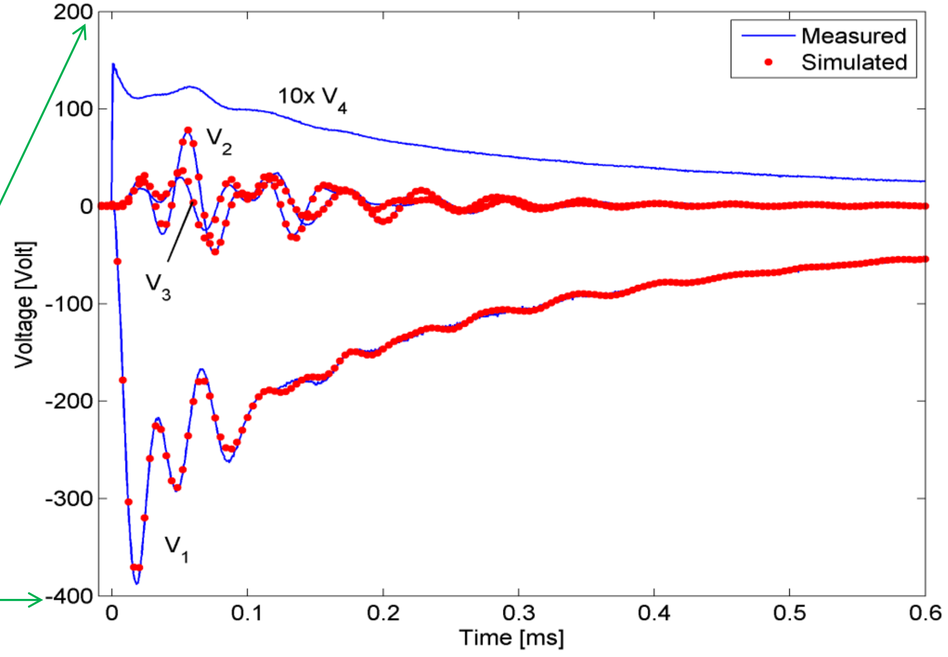
Example: transformer modeling



Pass. enf.







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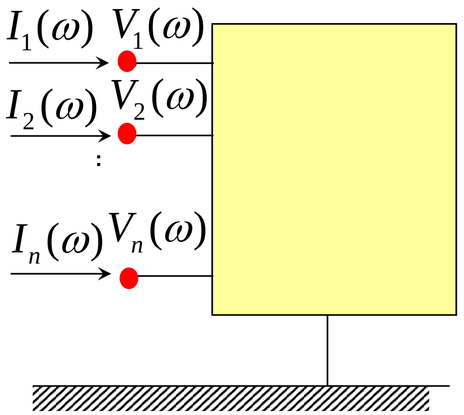
Including model in EMTP-type circuit solvers



Measurement

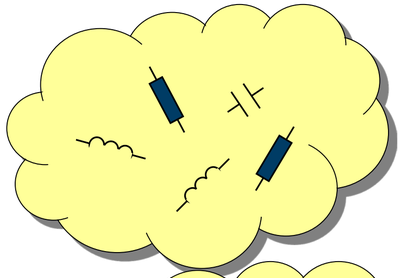
$$\begin{bmatrix} I_1(\omega) \\ I_2(\omega) \\ \vdots \\ I_n(\omega) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) & & Y_{11}(\omega) \\ \vdots & & & \vdots \\ Y_{n1}(\omega) & Y_{n2}(\omega) & \cdots & Y_{nn}(\omega) \end{bmatrix}}_{\mathbf{Y}} \begin{bmatrix} V_1(\omega) \\ V_2(\omega) \\ \vdots \\ V_n(\omega) \end{bmatrix}$$

Model extraction

(Vector Fitting +
Passivity enforcement)

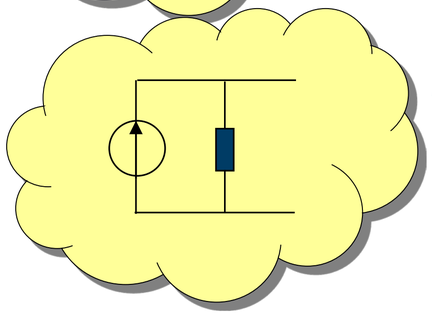
$$\mathbf{Y}(\omega) \cong \sum_{m=1}^{N} \frac{\mathbf{R}_m}{j\omega - a_m} + \mathbf{D}$$

Lumped circuit equivalent



or

Norton equivalent w/ controlled source



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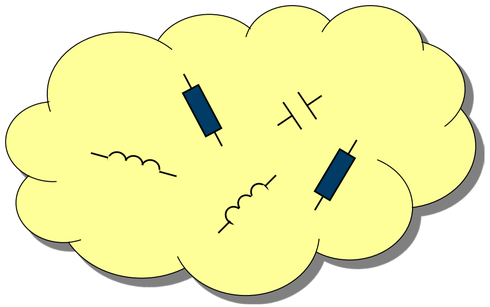
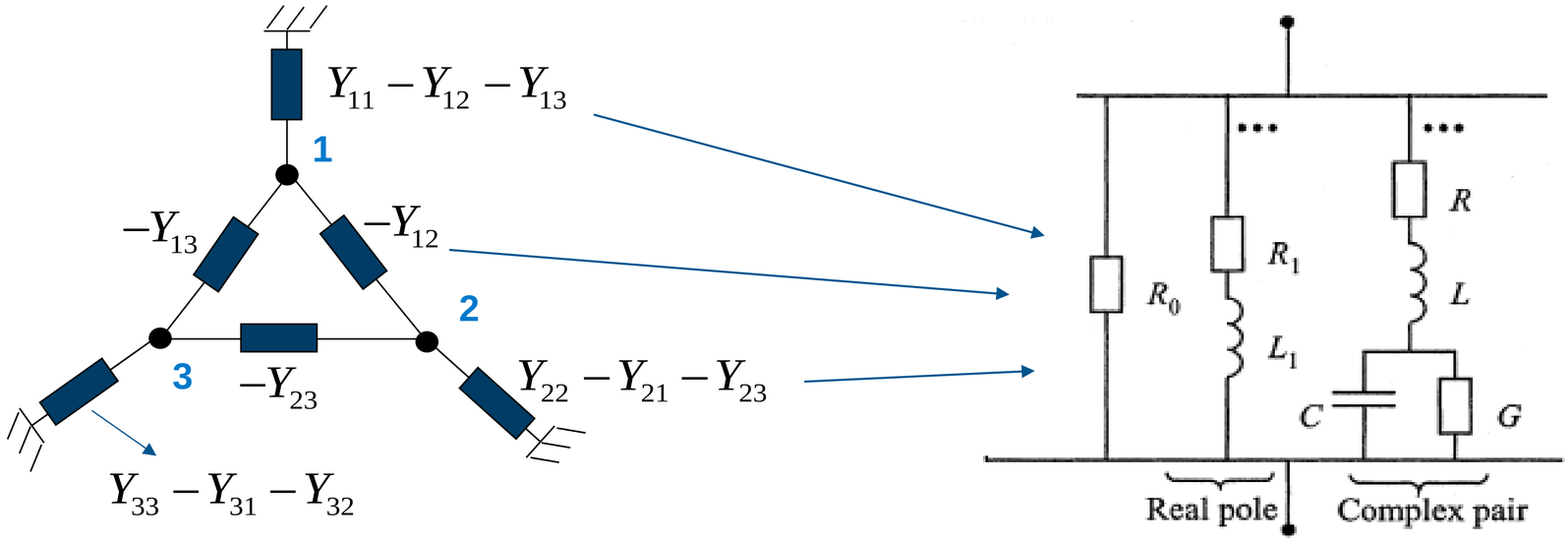
Lumped circuit equivalent



$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{ij} = \sum_{m=1}^{N} \frac{R_{ij,m}}{j\omega - a_m} + D_{ij}$$



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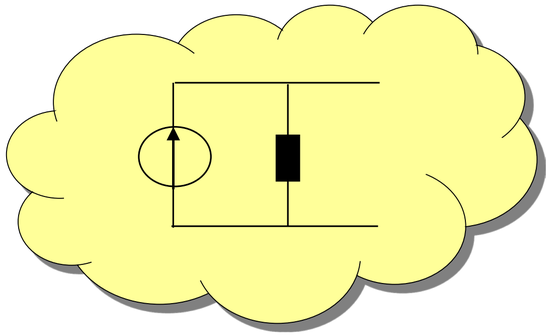
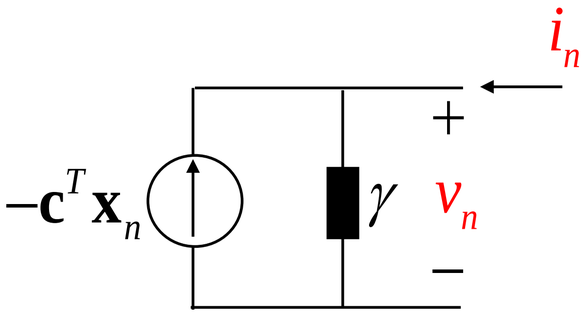
Norton equivalent w/ controlled current source



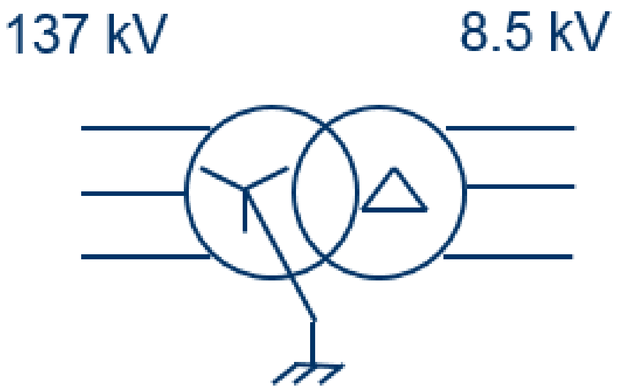
$$y(\omega) = \sum_m \frac{r_m}{j\omega - a_m} \quad \Rightarrow \quad y(t) = \sum_m r_m e^{at}$$

Impulse response

$$i(t) = y(t) * v(t) \quad \xrightarrow{\text{Trapezoidal integration}} \quad \begin{cases} \mathbf{x}_n = \boldsymbol{\alpha}\,\mathbf{x}_{n-1} + v_{n-1} \\ i_n = \mathbf{c}^T \mathbf{x}_n + \gamma\, v_n \end{cases} \quad \Rightarrow$$

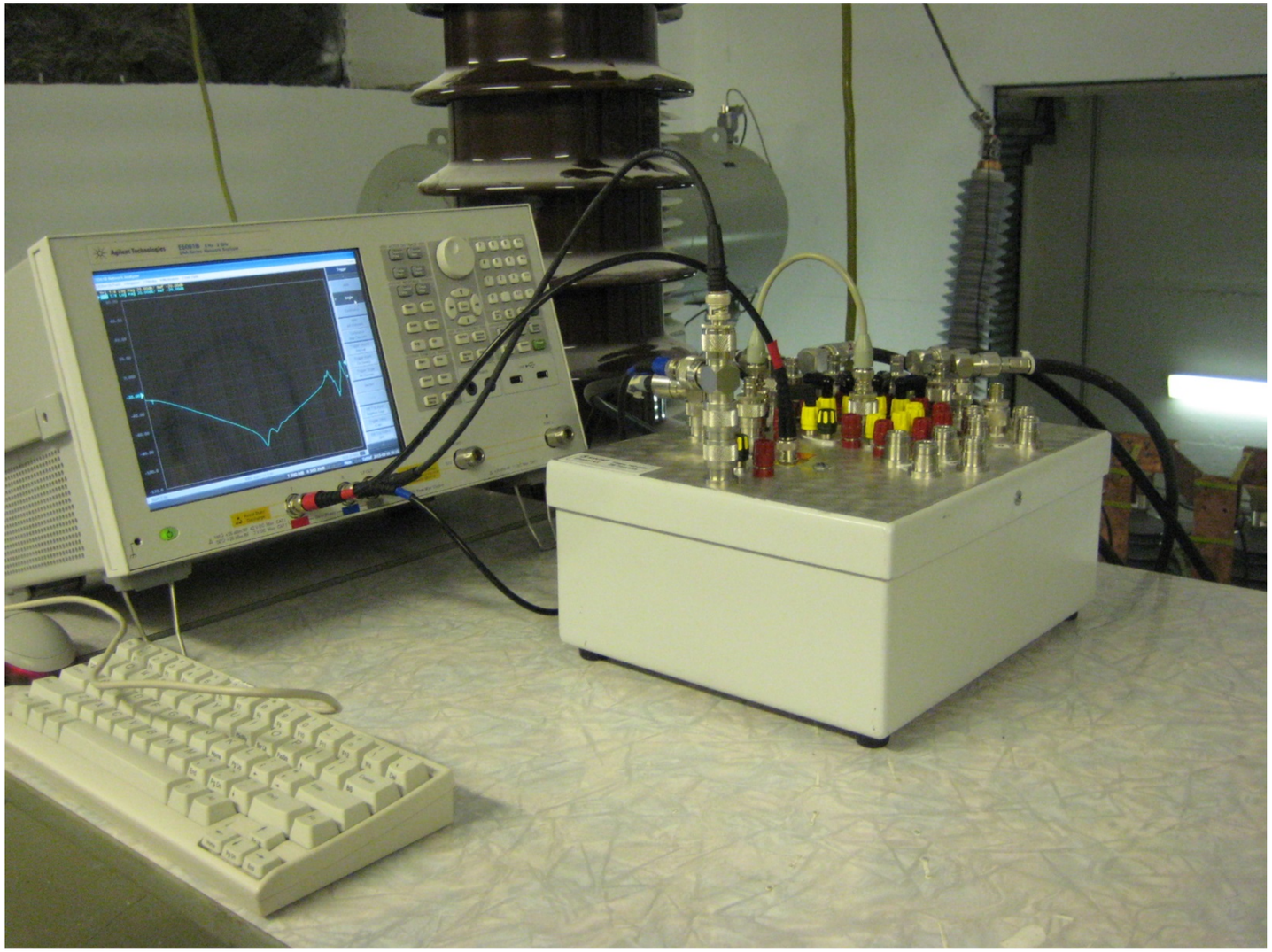


Example 1: High-frequency transformer modeling



45 MVA (three-phase),
137 kV / 8.5 kV
YNd

Admittance measurements



Gain-phase
Current sensor

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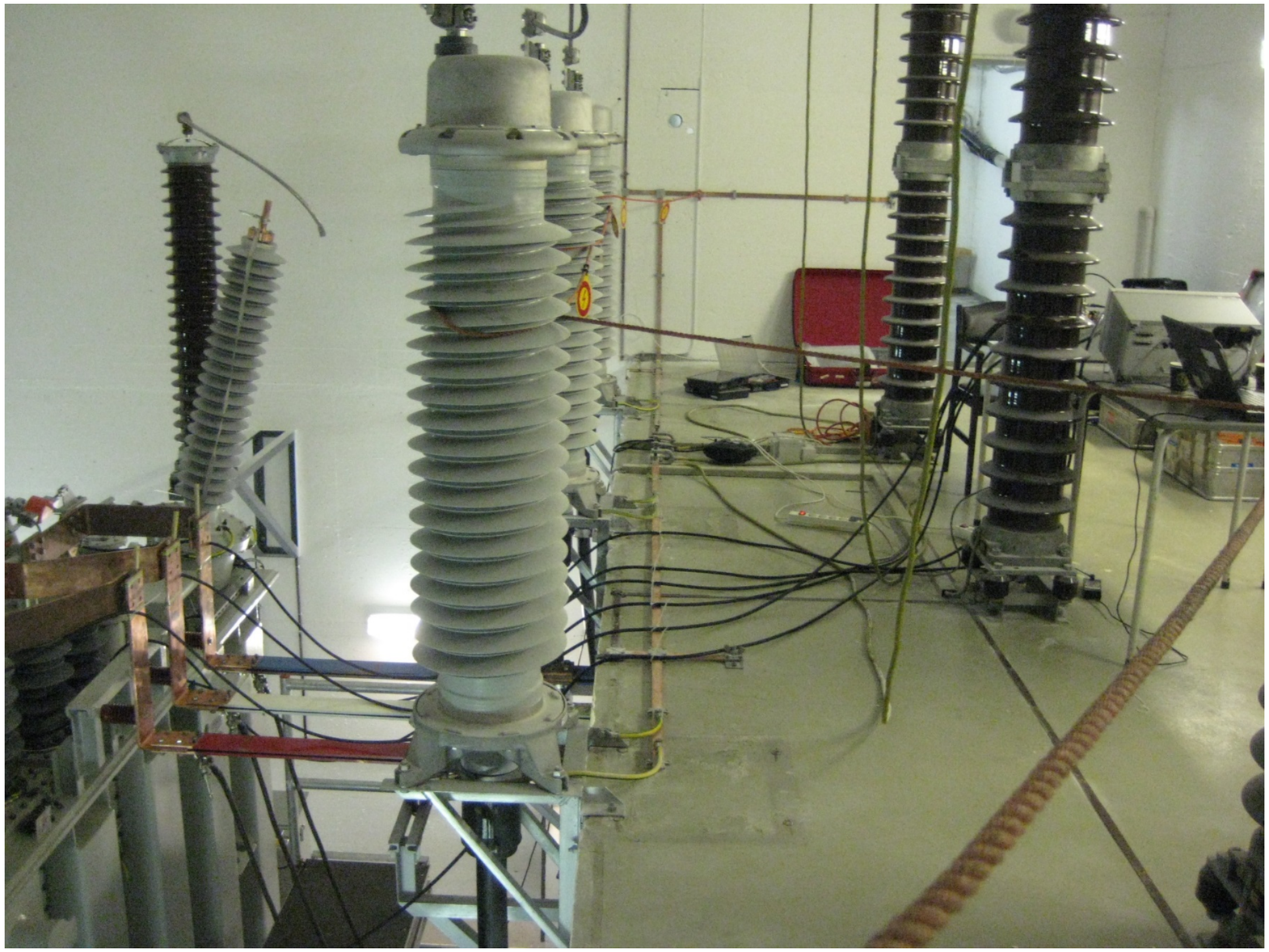

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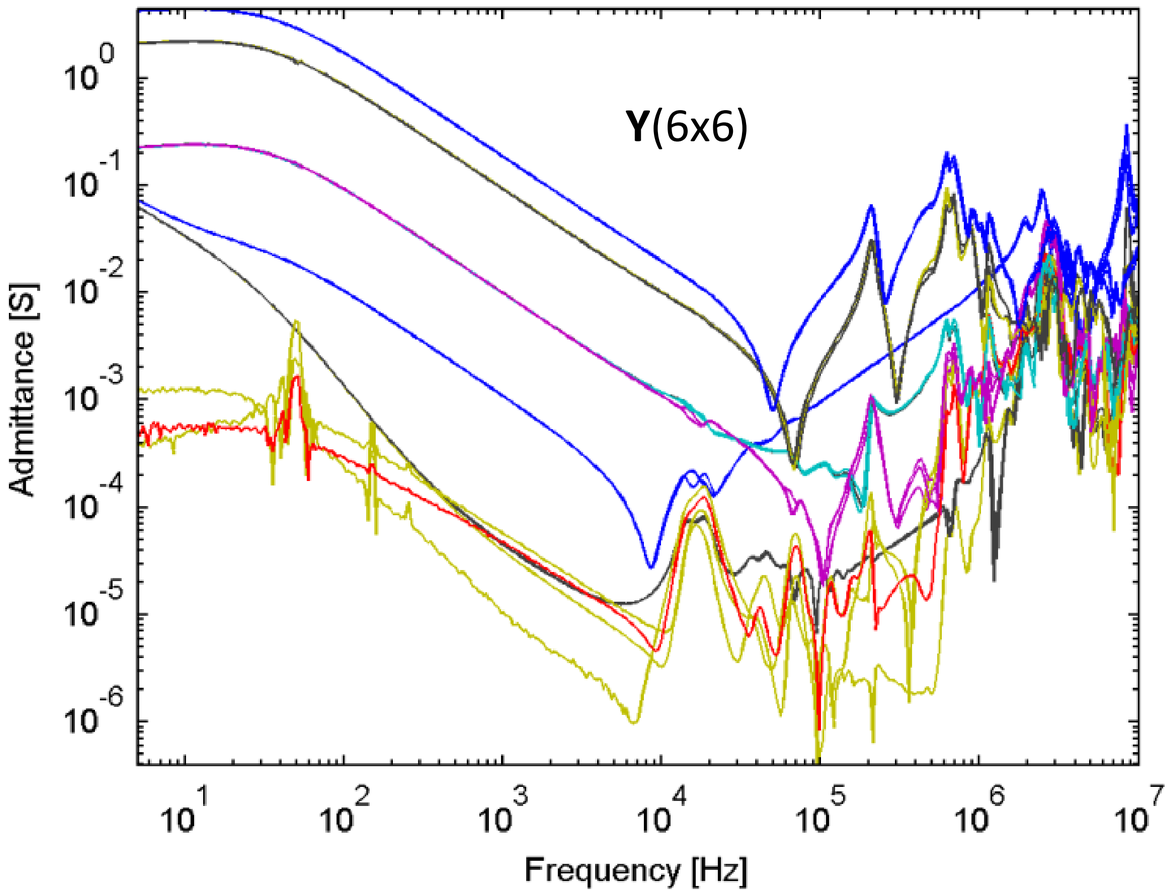
Admittance measurements

Calibration for

- Frequency variation in current sensor sensitivity (V/A)
- Insertion impedance effects
- Measurement cables



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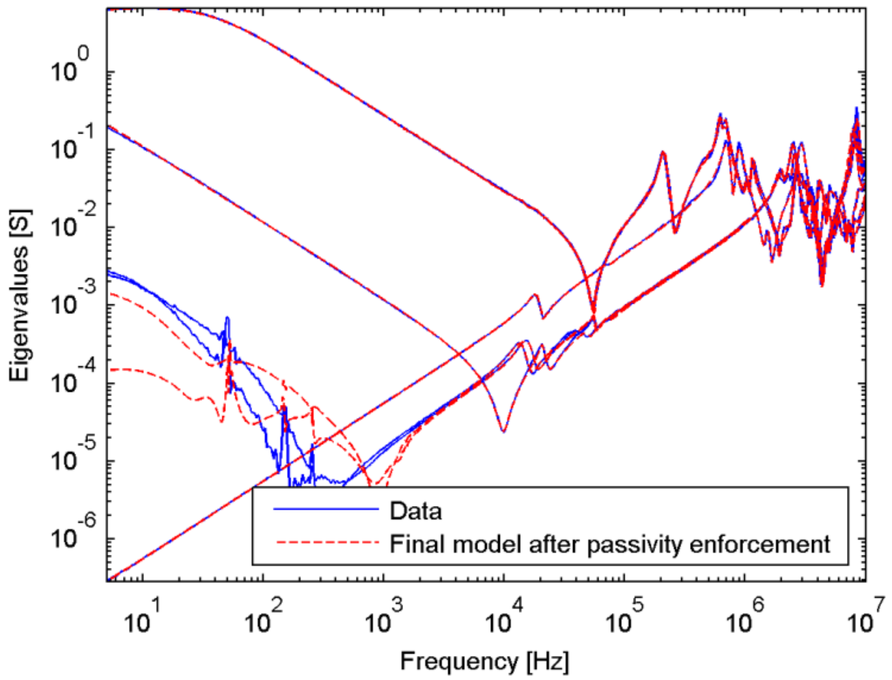
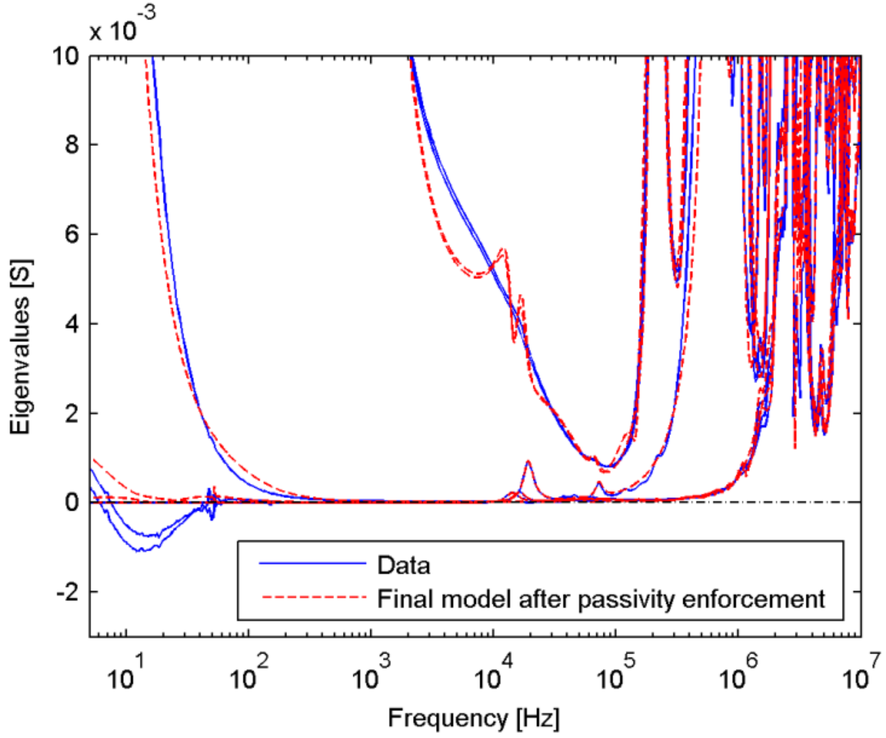
Model extraction (fitting + passivity enf.)

Eigenvalues of Y



Eigenvalues of G=Re{Y}



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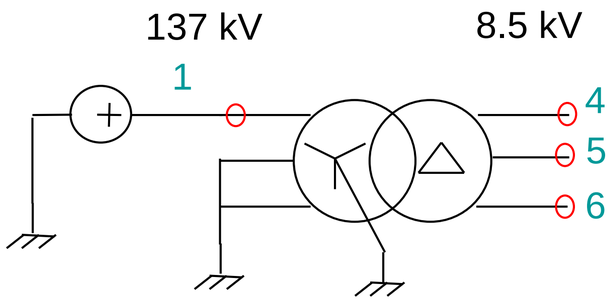
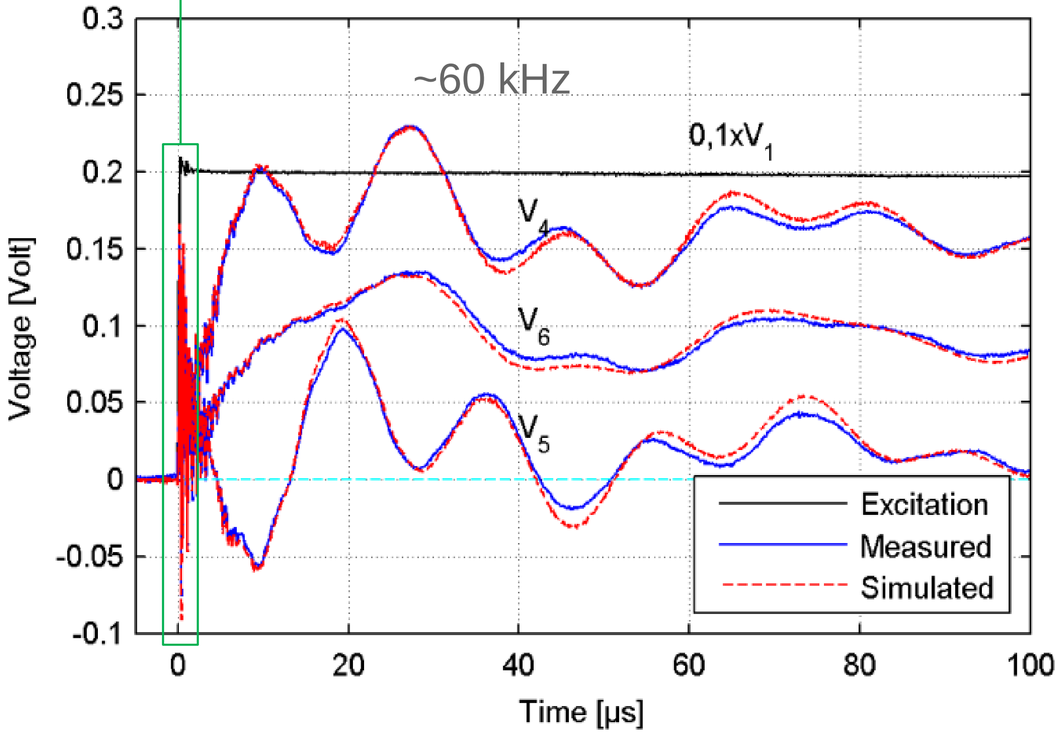
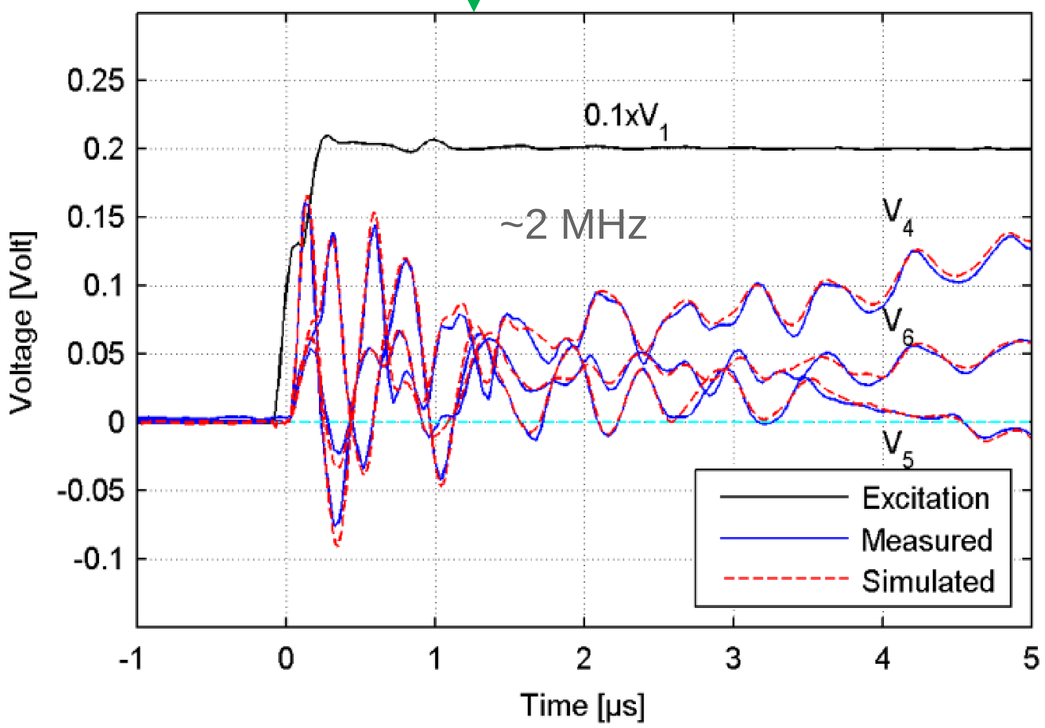
Time domain validation: Voltage transfer high→low







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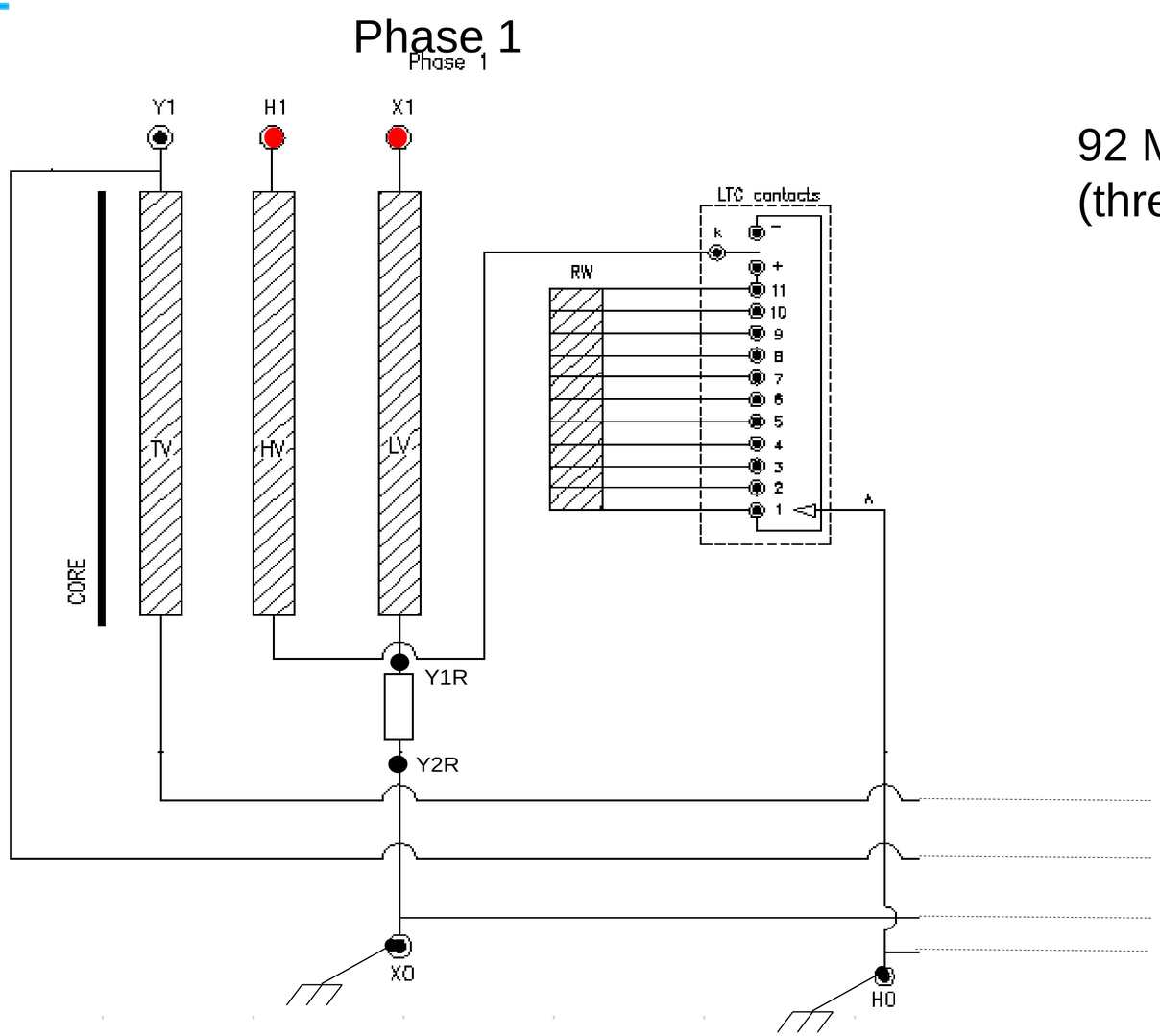
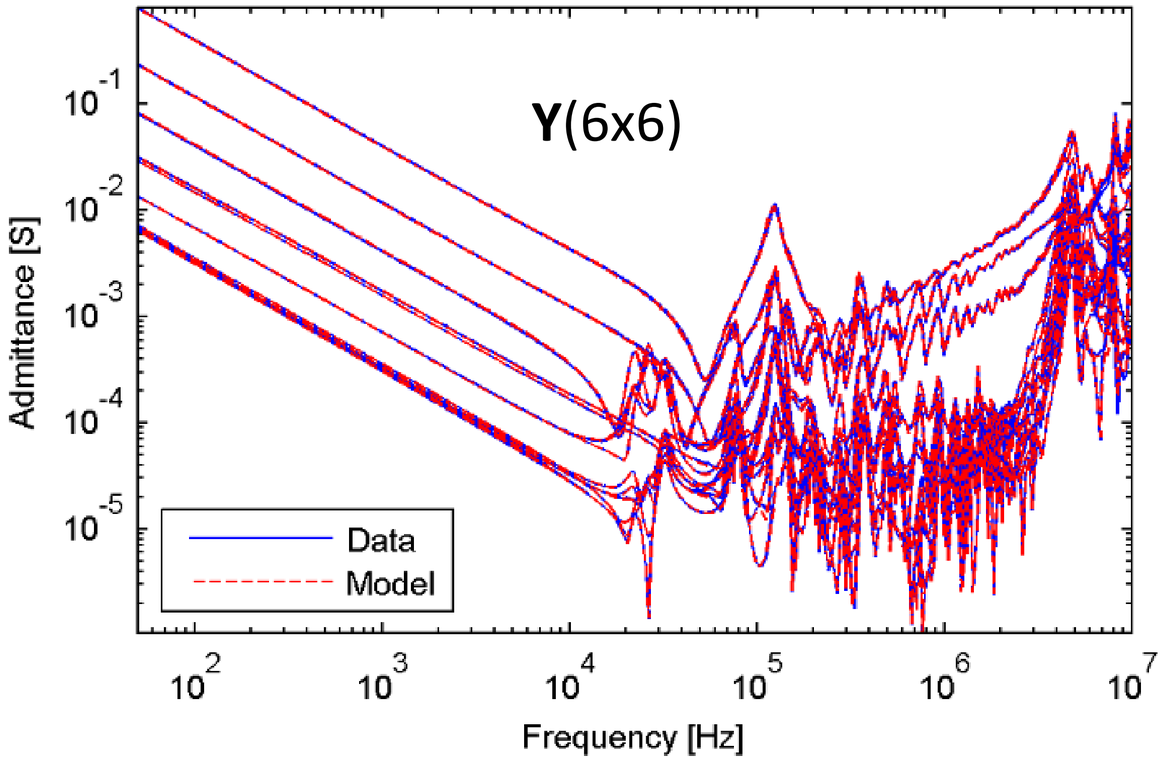
Example 2: High-frequency transformer modeling



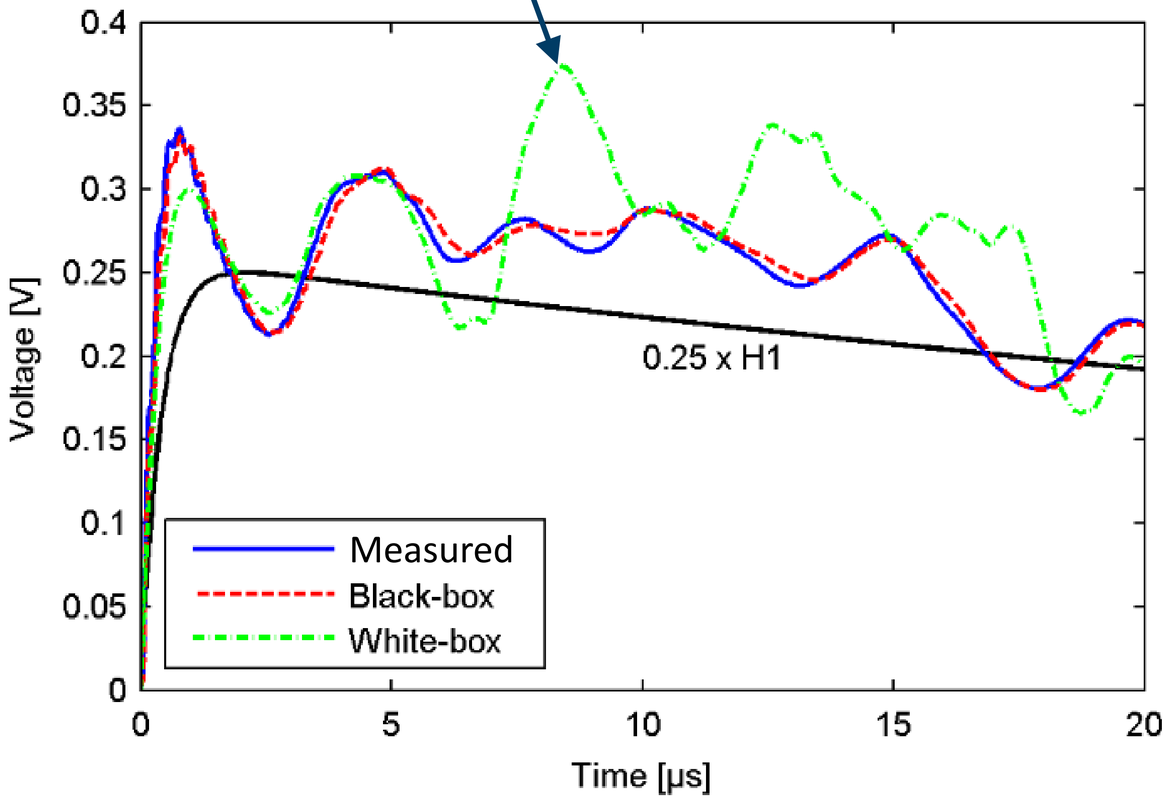
92 MVA
(three-phase, three-winding)

Admittance modeling w.r.t. H1, H2, H3, X1, X2, X3



Manufacturer's detailed model

Voltage transfer from H1 to X1



Measurement-based black-box modeling offers superior accuracy

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Final remarks

- Error magnification
  - Some admittance matrices have a combination of large and small eigenvalues (typically at low frequencies). Examples: cables, transformers, coils
  - The small eigenvalues tend to be lost in the measurement/modeling. Can lead to catastrophic error magnifications. Requires special handling.
- Passivity enforcement
  - The passivity enforcement may in some instances corrupt model behavior. May need a few trials before reaching a good model.
- Time delay effects
  - Systems with embedded delays may require excessive model orders
  - Delay-based rational modeling (e.g. traveling wave models)

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