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Relative Localization for Multi-Robot Systems: Theory, Scheme, and Platform

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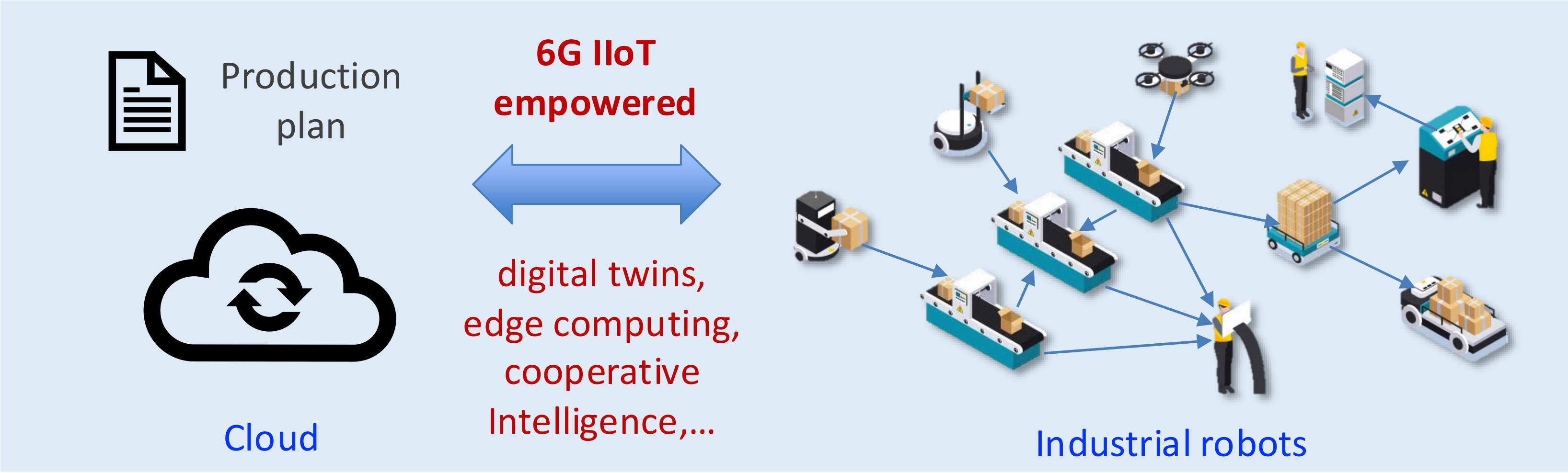
Background

- 6G Industrial Internet of Things (IIoT) for intelligent robots
  - Features of 6G IIoT: high data-rate, ultra-reliability, low-latency, massive access, energy-efficient, accurate localization and sensing, …
  - 6G IIoT for robotics: wirelessly connected multi-robot systems empowered by digital twins, edge computing, cooperative intelligence, …



Wirelessly connected multi-robot system for smart factory

[1] N. H. Mahmood, G. Berardinelli, E. J. Khatib, R. Hashemi, C. De Lima and M. Latva-aho, "A Functional Architecture for 6G Special-Purpose Industrial IoT Networks," in IEEE Transactions on Industrial Informatics, vol. 19, no. 3, pp. 2530-2540, March 2023,

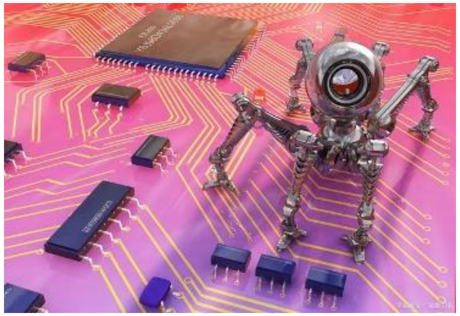
Background

- Applications of intelligent robots
  - Industry: logistics, sorting, construction, manufacturing, …
  - Service: public service, household duties, delivering, …
  - Special purpose: rescue, medical service, underwater tasks, …

Automobile production

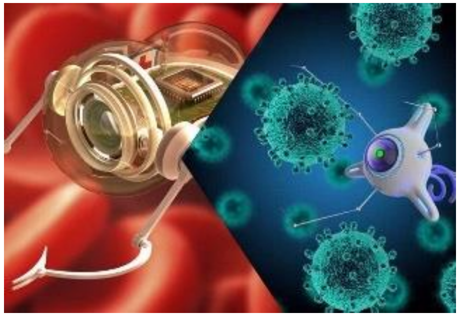
Household duties

Wild rescue



Precision manufacturing

Delivering



Medical treatment

Background

- From single-agent systems to multi-agent cooperative systems
  - Precise localization and sensing is the foundation of multi-agent cooperative tasks
  - Advantage of cooperative systems: accuracy, efficiency, robustness, flexibility, autonomy, …

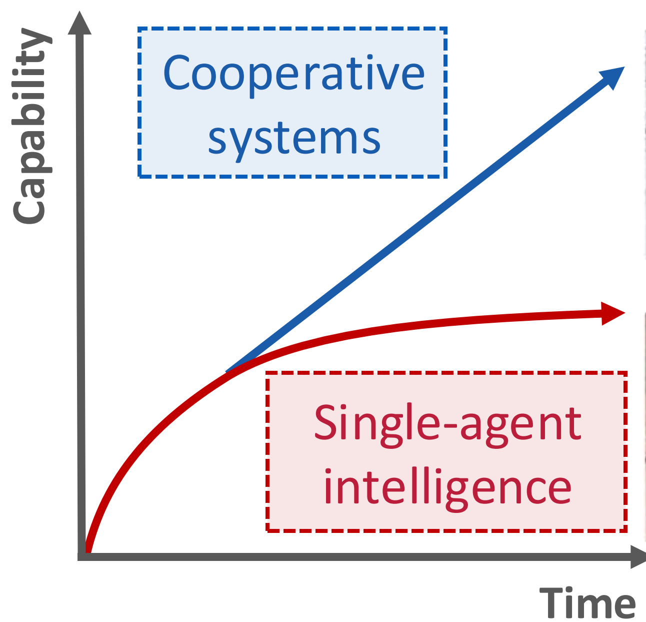
Applications of multi-agent cooperative systems

Autonomous driving

Search and rescue

Autonomous delivery

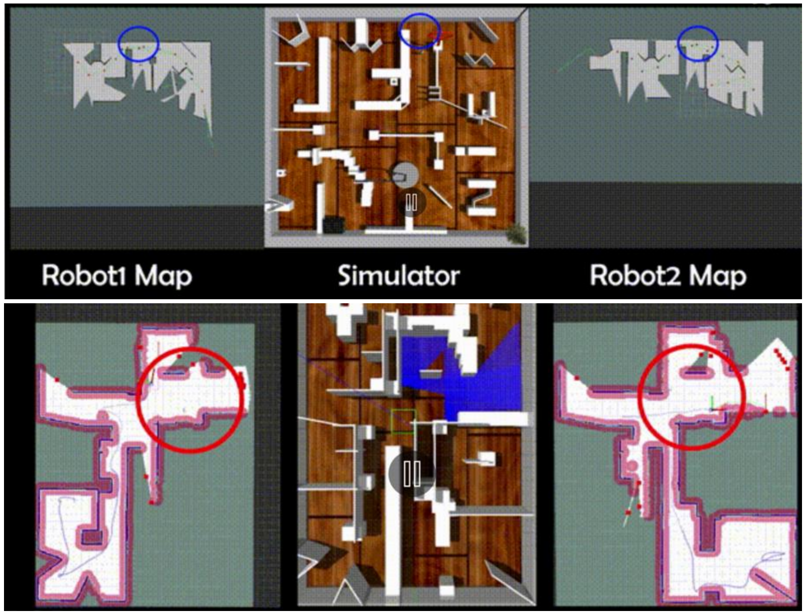
Intelligent manufacturing



Background

- From single-agent systems to multi-agent cooperative systems
  - Precise localization and sensing information is the foundation of multi-agent cooperative applications

Drone formation

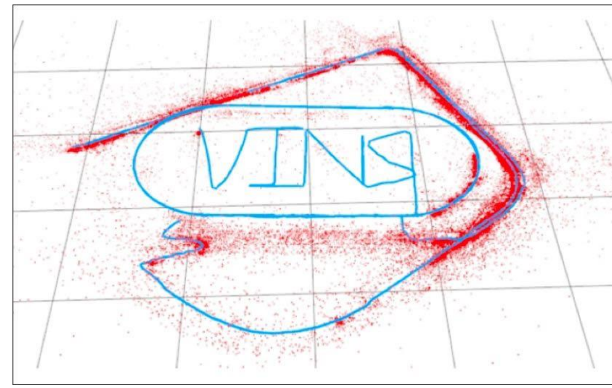


Multi-robot cooperative environment exploration

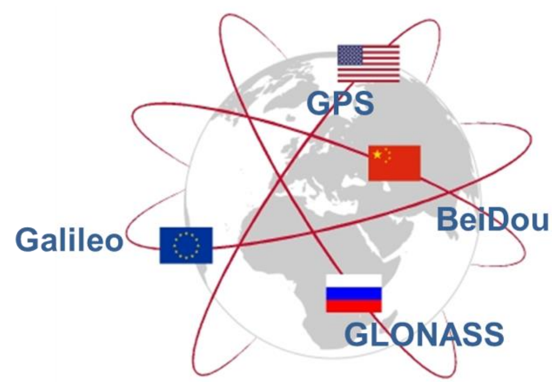
[1] Zhou X, Wen X, Wang Z, et al. Swarm of micro flying robots in the wild[J]. Science Robotics, 2022, 7(66): eabm5954.
[2] MR-TopoMap: Multi-Robot Exploration Based on Topological Map in Communication Restricted Environment, in IROS2022 / RAL.
[3] J. Yu, J. Tong, Y. Xu, et al, SMMR-Explore: SubMap-based Multi-Robot Exploration System with Multi-robot Multi-target Potential Field Exploration Method, in ICRA2021.

Background

- Localization and sensing technologies
  - Visual-inertial odometer (VIO)
    - Integrating visual and inertial information
    - Precise localization and mapping
    - Challenges: Accumulative error, illumination condition, …
  - Global Navigation Satellite System (GNSS)
    - Absolute position information
    - Poor in indoor/harsh environments
    - Challenges: Multipath, blockage, …



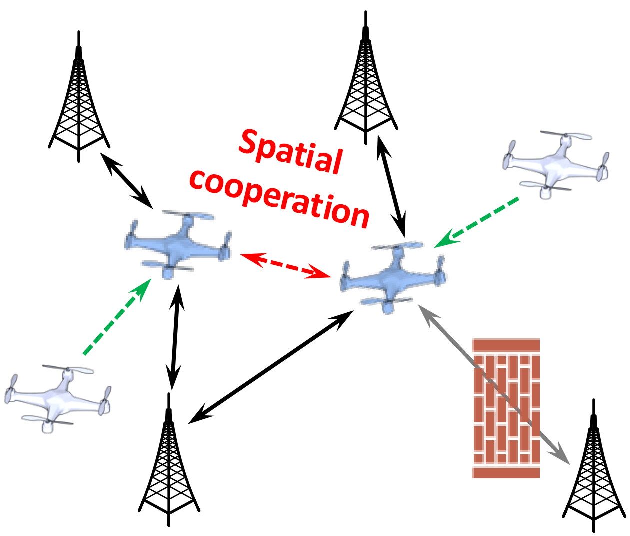
Visual-inertial odometer

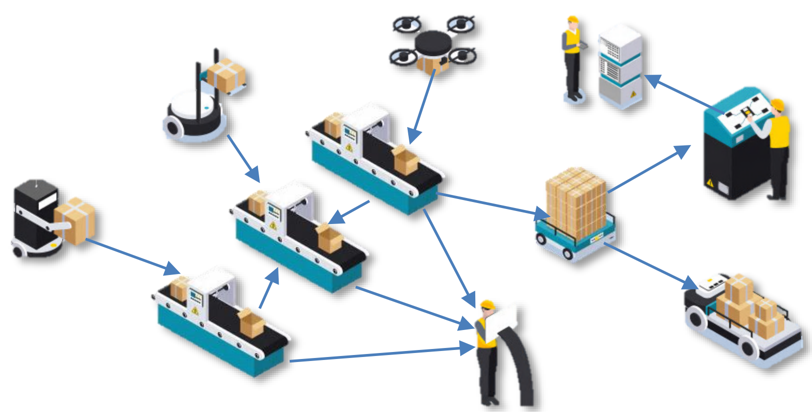


GNSS

Background

- Localization and sensing technologies
  - Wireless network localization
    - Precise and robust self-localization in GNSS-challenged scenarios
    - Measurements: Ranging and bearing based on wireless signals (UWB, Wi-Fi, BLE, 5G, etc.)
    - Cooperative localization: Information fusion in multi-agent networks



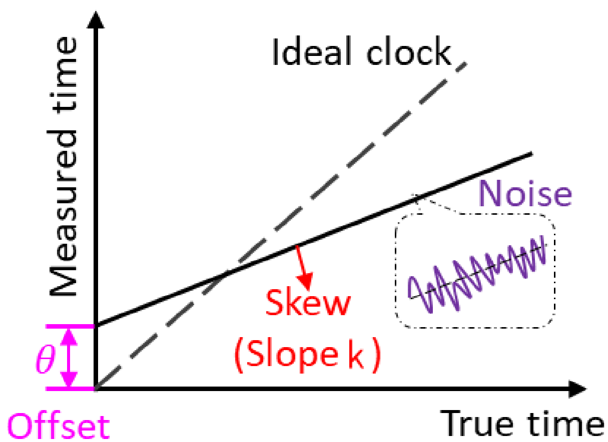


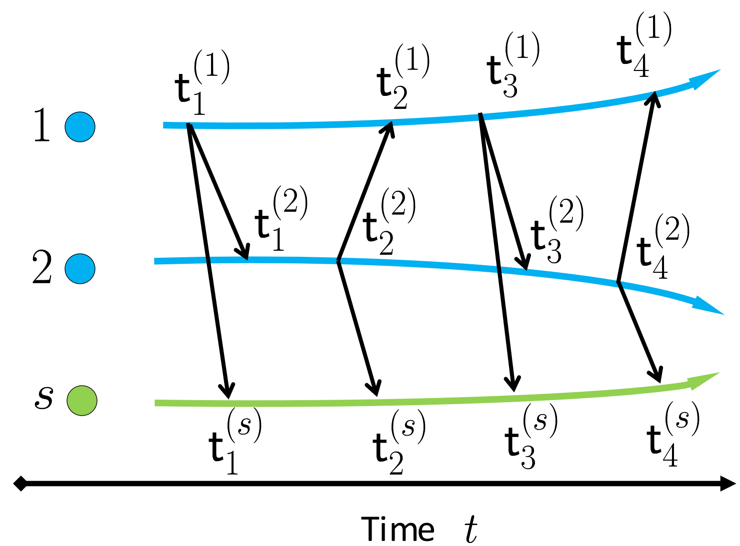
Motivation

- Challenges of network localization for multi-robot systems
  - Asynchronous networks
    - Asynchronous even with initial calibration: affected by varying voltage, ambient temperatures, hardware aging…
    - Require high measurement rates, especially in dynamic scenarios

$$\hat{\mathsf{t}} = (1+\mathsf{e})t + \theta + \mathsf{w} = \mathsf{k}t + \theta + \mathsf{w}$$

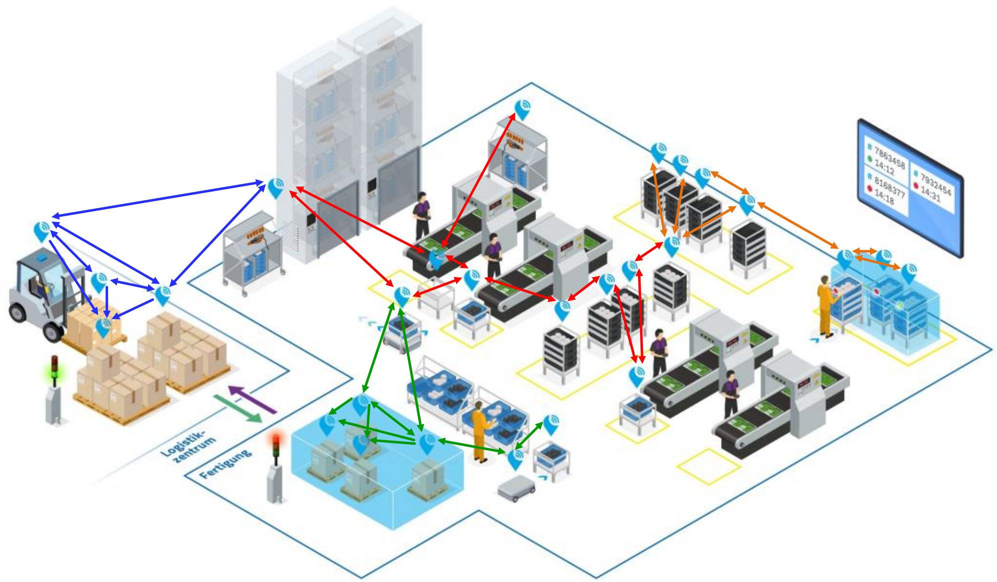
drift offset skew noise





Motivation

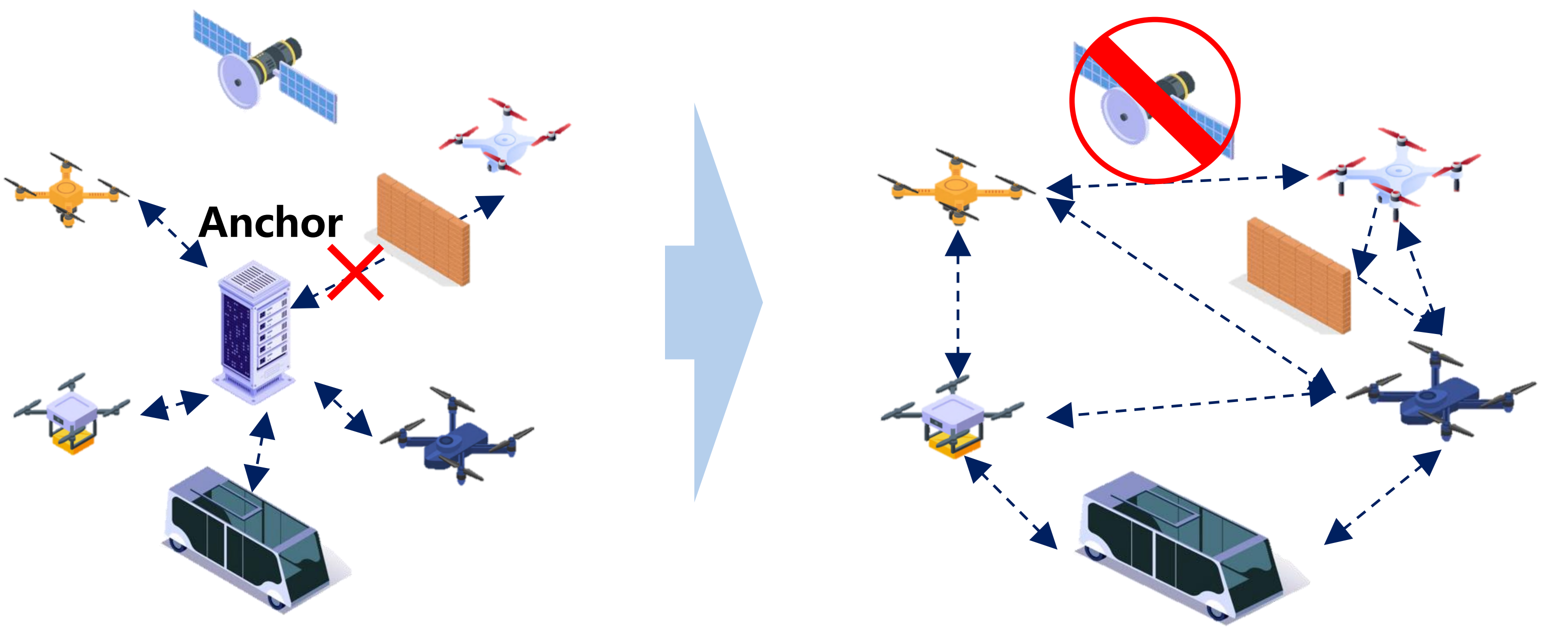
- Challenges of network localization for multi-robot systems
  - – Asynchronous networks
  - – Large network scale
    - Hundreds of mobile devices, sensors, and objects wait to connect for the foreseeable future



Large-scale networks

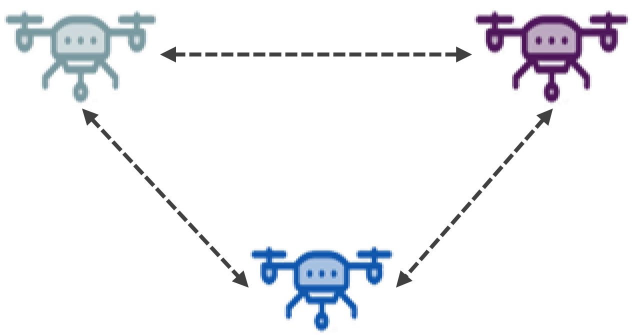
Motivation

- Challenges of network localization for multi-robot systems
  - Asynchronous networks
  - Large network scale
  - Infrastructure-free
    - Determination of the network geometry without absolute position information reference

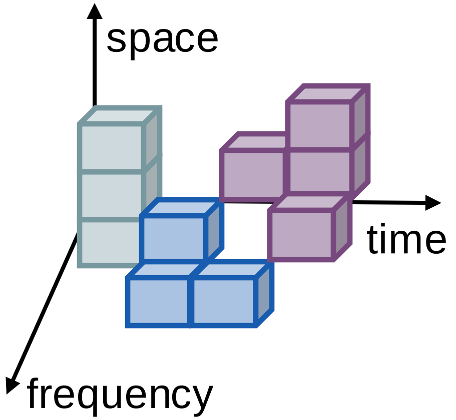


Motivation

- Challenges of network localization for multi-robot systems
  - Asynchronous networks
  - Large network scale
  - Infrastructure-free
  - Limited resources
    - Low-cost sensors with short battery life are preferred to expend coverage areas, and limited spectrum



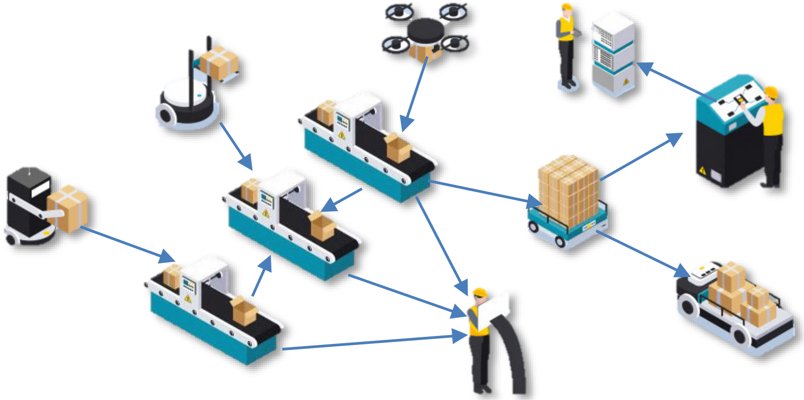
Low-cost nodes



Limited resources

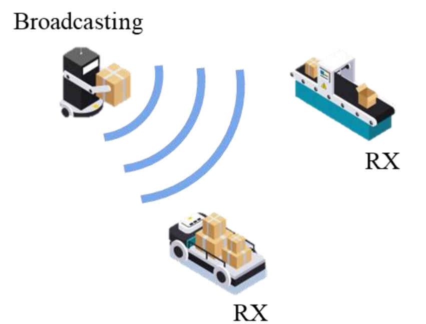
Motivation

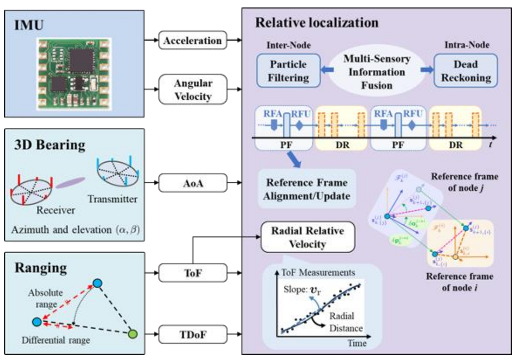
- Challenges of network localization for multi-robot systems
  – Asynchronous networks
  – Large network scale
  – Infrastructure-free
  – Limited resources
- Goal
  – Provide high-precision localization and sensing capability for large-scale mobile networks within acceptable resource consumptions

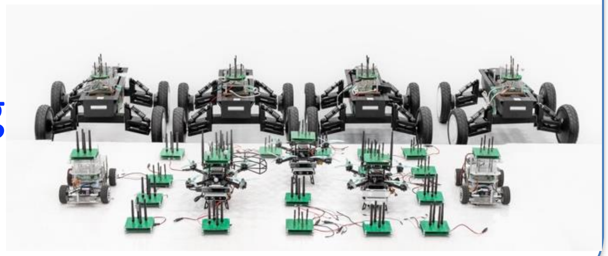


Contribution

- Relative localization theory
  - A unified theoretical framework to address the state estimation in relative localization networks
- Network measuring protocol
  - Signal-multiplexing network ranging (SM-MR) protocol, ranging and clock synchronization with minimal signal transmission
- Distributed relative localization algorithm
  - Infrastructure-free distributed localization
- Lightweight 3-D UWB array
  - Pairwise relative localization based on ranging and 3-D bearing using UWB antenna arrays



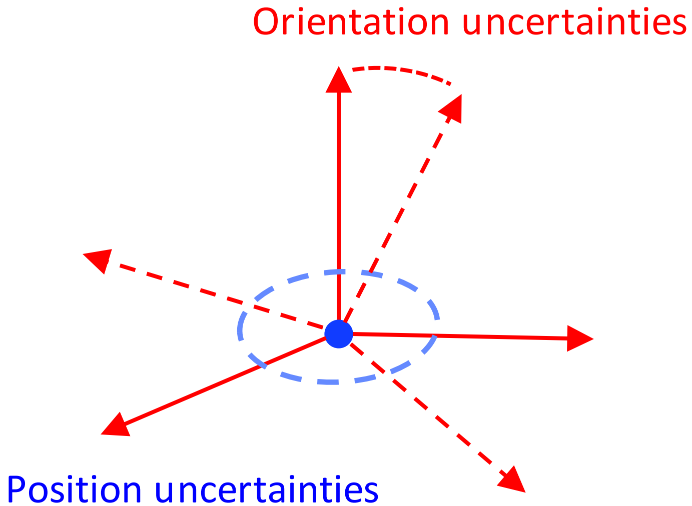


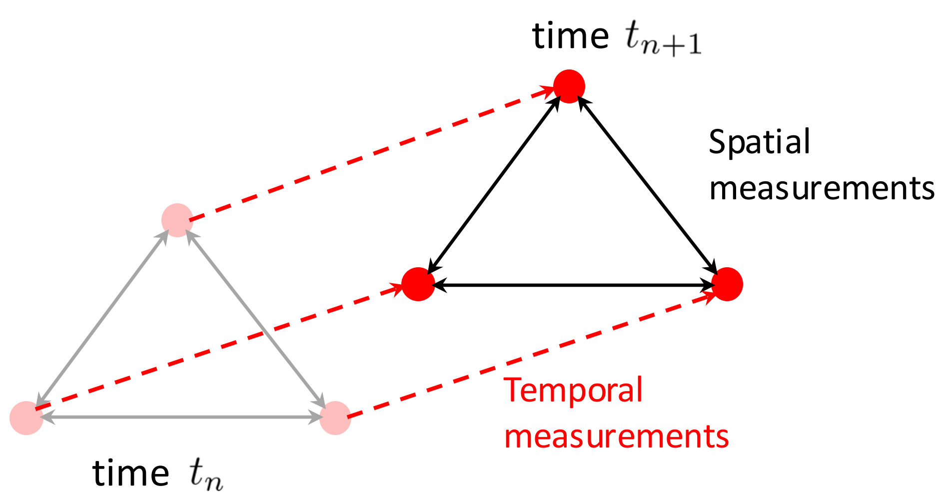


THEORY: RELATIVE LOCALIZATION FOR MULTI-ROBOT SYSTEMS

Relative Localization: Theory

- An Unified localization framework
  - From single-antenna to MIMO
    - MIMO system model, characterization of agent 3D orientations
  - State uncertainties
    - Effects of state uncertainties on the relative localization accuracy
  - Spatiotemporal measurements
    - Relative localization in spatiotemporal cooperative networks



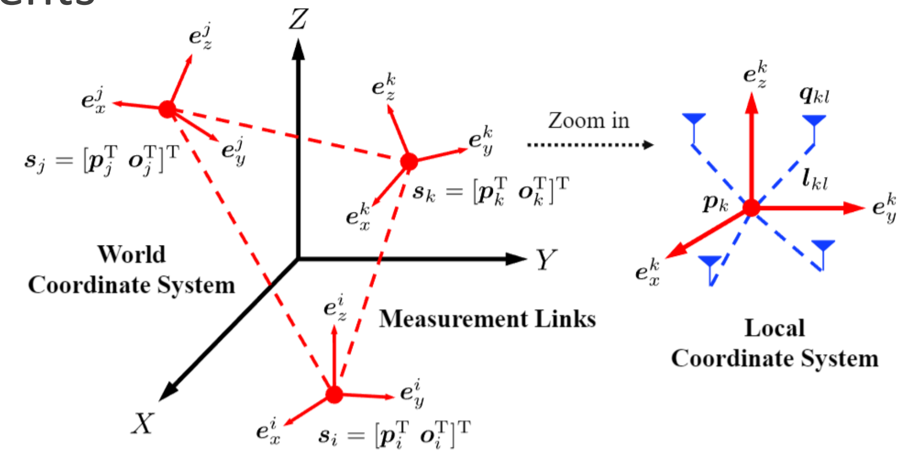


Relative Localization: Theory

- Network Model
  - 3D wireless network with N_a agents
  - Agent state vector

Positions Orientations

$$\boldsymbol{s}_k = [\boxed{\boldsymbol{p}_k^\mathrm{T}}, \boxed{\boldsymbol{o}_k^\mathrm{T}}]^\mathrm{T}$$



- Measurement Model
  - Graph representation: $\mathcal{G}(\mathcal{V}, \mathcal{E})$ (Agents: $\mathcal{V}$, Links: $\mathcal{E}$)
  - Neighbors of agent i : $\mathcal{N}_i = \{j \mid j \in \mathcal{N}_\mathrm{a} \backslash \{i\}, (i,j) \in \mathcal{E}\}$
  - Pairwise measurements:

$$\mathbf{z}_{kj} = \boxed{\boldsymbol{g}}(\boldsymbol{s}_k, \boldsymbol{s}_j) + \mathbf{n}_{kj}, \ \forall (k,j) \in \mathcal{E}$$

Measurement function, e.g., distance

Noise

Relative Localization: Theory

- State estimation in relative localization
  - State equivalent class

Definition

The equivalent class w.r.t. the network state vector $\boldsymbol{s}$ *is defined as the set*

$$\Gamma(\boldsymbol{s}) = \{\boldsymbol{s}' \in \mathcal{S} : \boldsymbol{g}(\boldsymbol{s}') = \boldsymbol{g}(\boldsymbol{s})\}$$

- $\Gamma(\boldsymbol{s})$ collects the states with the same measurements as $\boldsymbol{s}$

  - Relative error for the state estimation

Definition

Given a network state vector $\boldsymbol{s}$ *and its estimate* $\hat{\mathbf{s}}$, *denote* $\mathcal{I}$ *as the index set of the interested states. Then the relative error for states in* $\mathcal{I}$ *is defined as*

$$e_{\mathrm{r},\mathcal{I}} = \inf_{\tilde{\boldsymbol{s}} \in \Gamma(\hat{\mathbf{s}})} \|\mathbf{1}_{\mathcal{I}} \odot (\tilde{\boldsymbol{s}} - \boldsymbol{s})\|_2$$

- Example: relative error for entire states

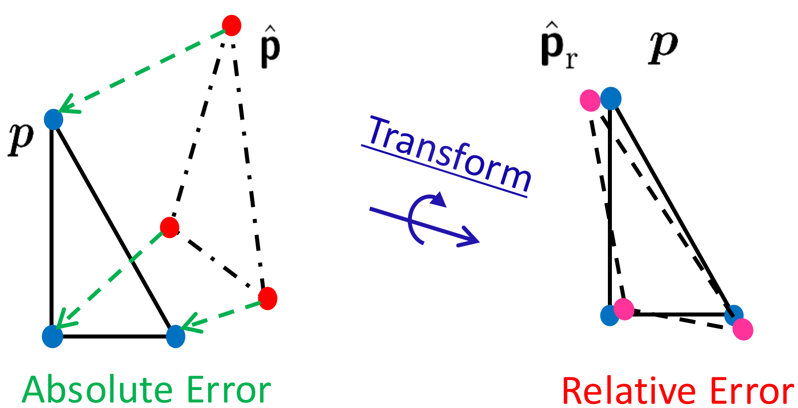
Relative Localization: Theory

- Example 1: Single-antenna case (no orientation)
  - Relative error for all agent positions

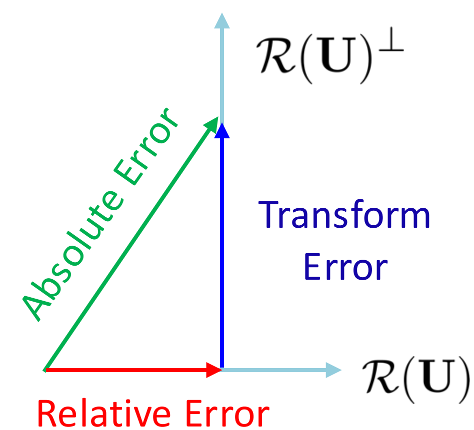
$$e_{\mathrm{r}} = \inf_{\tilde{\mathbf{p}} \in \Gamma(\hat{\mathbf{p}})} \|\tilde{\boldsymbol{p}} - \boldsymbol{p}\|_2$$

$$\Gamma(\hat{\mathbf{p}}) = \{\hat{\mathbf{p}}' : \hat{\mathbf{p}}' = \underbrace{(\boldsymbol{I}_{N_{\mathrm{a}}} \otimes \boldsymbol{R})}_{\text{rotation}} \cdot \boldsymbol{p} + \underbrace{\mathbf{1}_{N_{\mathrm{a}}} \otimes \boldsymbol{t}}_{\text{translation}}\}$$

  - Solved by *Procrustes coordinates*: closed-form solution



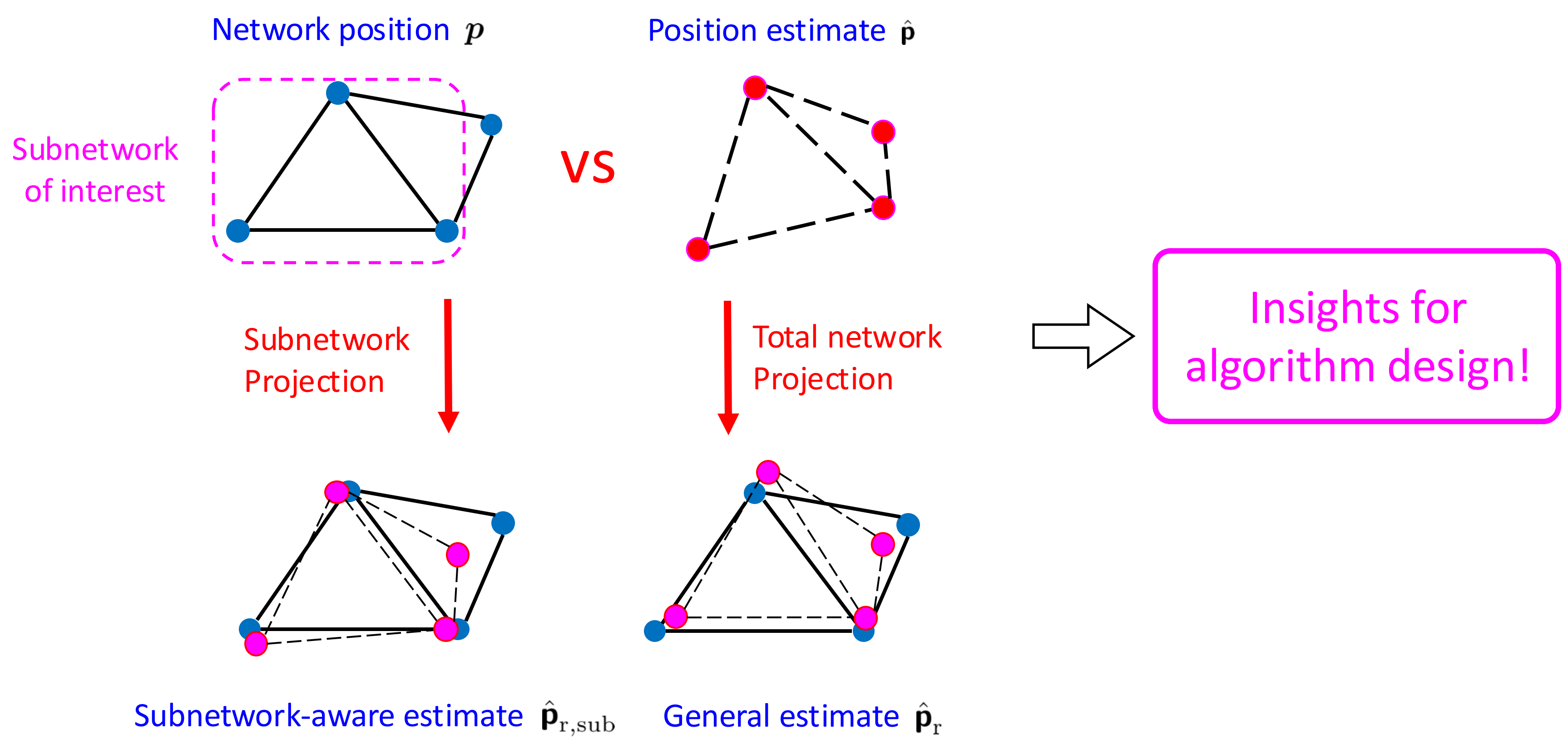
Geometric illustration



Error Decomposition

Relative Localization: Theory

- Example 2: Relative error for partial states
  - Subnetwork-aware relative localization



Relative Localization: Theory

- Performance analysis in relative localization
  - (Equivalent) Fisher information analysis

Definition

The Fisher information matrix (FIM) for the state vector $\boldsymbol{s}$ *is defined by*

$$\boldsymbol{J}(\boldsymbol{s}) = \mathbb{E}\left\{\left[\frac{\partial}{\partial \boldsymbol{s}}\ln f_{\mathbf{z}}(\boldsymbol{z};\boldsymbol{s})\right]\left[\frac{\partial}{\partial \boldsymbol{s}}\ln f_{\mathbf{z}}(\boldsymbol{z};\boldsymbol{s})\right]^{\mathrm{T}}\right\}$$

Partition $\boldsymbol{s} = [\boldsymbol{s}_1^{\mathrm{T}}\ \ \boldsymbol{s}_2^{\mathrm{T}}]^{\mathrm{T}}$ *and* $\boldsymbol{J}(\boldsymbol{s})$ *into*

$$\boldsymbol{J}(\boldsymbol{s}) = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^{\mathrm{T}} & \boldsymbol{C} \end{bmatrix}$$

Then the equivalent Fisher information matrix (EFIM) for partial states $\boldsymbol{s}_1$ *is*

$$\boldsymbol{J}_{\mathrm{e}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{C}^{-1}\boldsymbol{B}^{\mathrm{T}}$$

  - EFIM incorporates the effect of nuisance states
  - Singular FIM/EFIM due to rank-deficiency in relative localization

Relative Localization: Theory

- Performance analysis in relative localization
  - Performance bounds for the relative state error

Theorem 1

Given the actual state vector $\boldsymbol{s}$ and its estimate $\hat{\mathbf{s}}$, under mild conditions, the relative error for the entire states satisfies

Pseudo-inverse of the FIM

$$\mathbb{E}\{e_{\mathrm{r}}^2\} \geq \mathrm{tr}\{\boldsymbol{J}^{\dagger}(\boldsymbol{s})\}$$

Furthermore, the relative error for the partial states in $\mathcal{I}$ satisfies

$$\mathbb{E}\{e_{\mathrm{r},\mathcal{I}}^2\} \geq \mathrm{tr}\{\boldsymbol{J}_{\mathrm{e},\mathcal{I}}^{\dagger}\}$$

Pseudo-inverse of the EFIM

- Compare with absolute localization

$$\mathbb{E}\{e_{\mathrm{abs}}^2\} \geq \mathrm{tr}\{\boldsymbol{J}^{-1}(\boldsymbol{s})\}, \quad \mathbb{E}\{e_{\mathrm{abs},\mathcal{I}}^2\} \geq \mathrm{tr}\{\boldsymbol{J}_{\mathrm{e},\mathcal{I}}^{-1}\}$$

- Unified results for relative and absolute localization
  - Inverse replaced by Moore-Penrose pseudo-inverse

Relative Localization: Theory

- Performance analysis with state measurements
  - State measurement model

$$\mathbf{z}_{\mathcal{K}} = \boldsymbol{s}_{\mathcal{K}} + \mathbf{n}_{\mathcal{K}}$$

measurements actual states

  - Performance bounds with state measurements

Theorem 2

Given the actual state vector $\boldsymbol{s}$ and its estimate $\hat{\mathbf{s}}$, under mild conditions, the relative error for the partial states with state measurements in $\mathcal{I}$ satisfies

$$\mathbb{E}\{e^2_{\mathrm{r},\mathcal{I}}\} \geq \mathrm{tr}\{\boldsymbol{J}^{\dagger}_{\mathrm{e,u},\mathcal{I}}\}$$

where the EFIM is calculated with respect to

$$\boldsymbol{J}_{\mathrm{u}}(\boldsymbol{s}) = \boldsymbol{J}(\boldsymbol{s}) + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mathcal{K}} \end{bmatrix}$$

Information gain from state measurements

Relative Localization: Theory

- Performance analysis with state measurements
  - Unique property in relative localization
    - Extra state information may not contribute to the performance

Different from absolute localization!

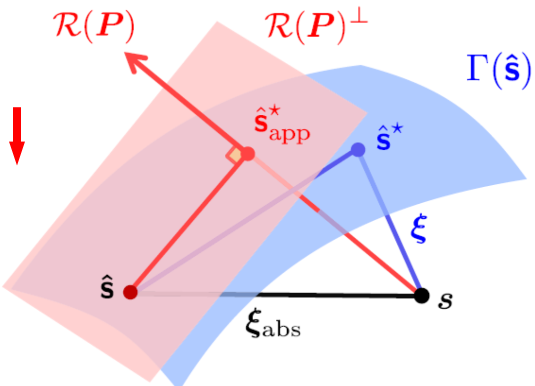
  - Interpretation based on error projection
    - Relative error is determined by absolute error e and projection space $\mathcal{R}(\boldsymbol{P})$
    - With state measurements

$$\boldsymbol{\xi}_{\mathrm{r}} = \boxed{\boldsymbol{P}\boldsymbol{P}^{\mathrm{T}}}\boxed{\boldsymbol{\xi}_{\mathrm{abs}}}$$

Larger projection space $\mathcal{R}(\boldsymbol{P})$ ↑ + **Smaller absolute error** ↓

⇓

The relative error
may not decrease

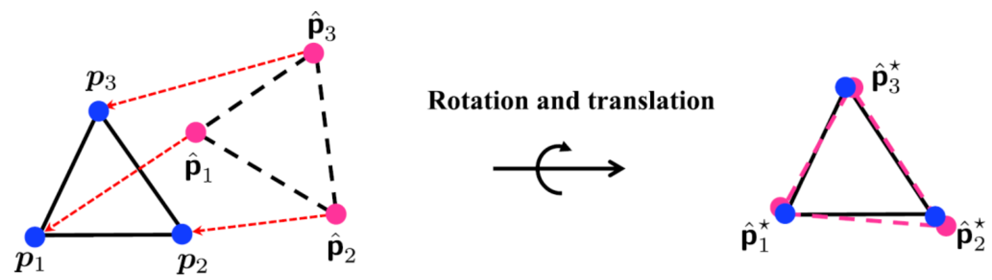


Relative Localization: Theory

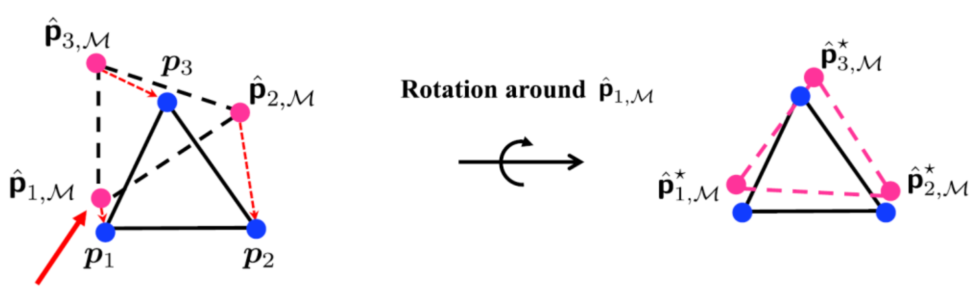
- Performance analysis with state measurements
  - Unique property in relative localization
    - Extra state information may not contribute to the performance

Different from absolute localization!

  - Geometric illustrations



No position measurements



Position measurements of p_1

Relative Localization: Theory

- Performance analysis in spatiotemporal cooperative networks
  - System model
    - Time instants: $t_1, \cdots, t_N$
    - Agent states at time t_n : $\boldsymbol{s}_k^{(n)}$
    - Known orientations
  - Intra-node measurement model

Displacement measurements

$$\mathbf{z}_{kk}^{(n)} = \boldsymbol{p}_k^{(n)} - \boldsymbol{p}_k^{(n-1)} + \mathbf{n}_k^{(n)}, \ \forall k \in \mathcal{N}_{\mathrm{a}}, n = 1, \cdots, N$$

  - Relative localization error with multiple time instants

Definition

Denote $\boldsymbol{p} = \boldsymbol{p}^{(1:N)}$ *and* $\hat{\mathbf{p}} = \hat{\mathbf{p}}^{(1:N)}$ *as the positions and estimates for all time instants, then the relative localization at current time* t_N *is*

$$e_{\mathrm{r},N} = \inf_{\tilde{\mathbf{p}} \in \Gamma(\hat{\mathbf{p}})} \|\tilde{\boldsymbol{p}}^{(N)} - \boldsymbol{p}^{(N)}\|_2$$

Minimizing the relative localization error at current time

Relative Localization: Theory

- Performance analysis in spatiotemporal cooperative networks
  - Performance bounds with temporal cooperation

Proposition 3

The relative localization error at current time t_N satisfies

$$\mathbb{E}\{e^2_{\mathrm{r},N}\} \geq \mathrm{tr}\{\boldsymbol{J}^{\dagger}_{\mathrm{e},N}(\boldsymbol{p})\}$$

where the EFIM $\boldsymbol{J}_{\mathrm{e},N}(\boldsymbol{p})$ can be calculated recursively

$$\boldsymbol{J}_{\mathrm{e},N}(\boldsymbol{p}) = \underbrace{\boxed{\boldsymbol{J}_N}}_{\text{Spatial information}} + \boldsymbol{T}_{N-1} - \boldsymbol{T}_{N-1}\boldsymbol{S}^{-1}_{N-1}\underbrace{\boxed{\boldsymbol{T}_{N-1}}}_{\text{Temporal information}}$$

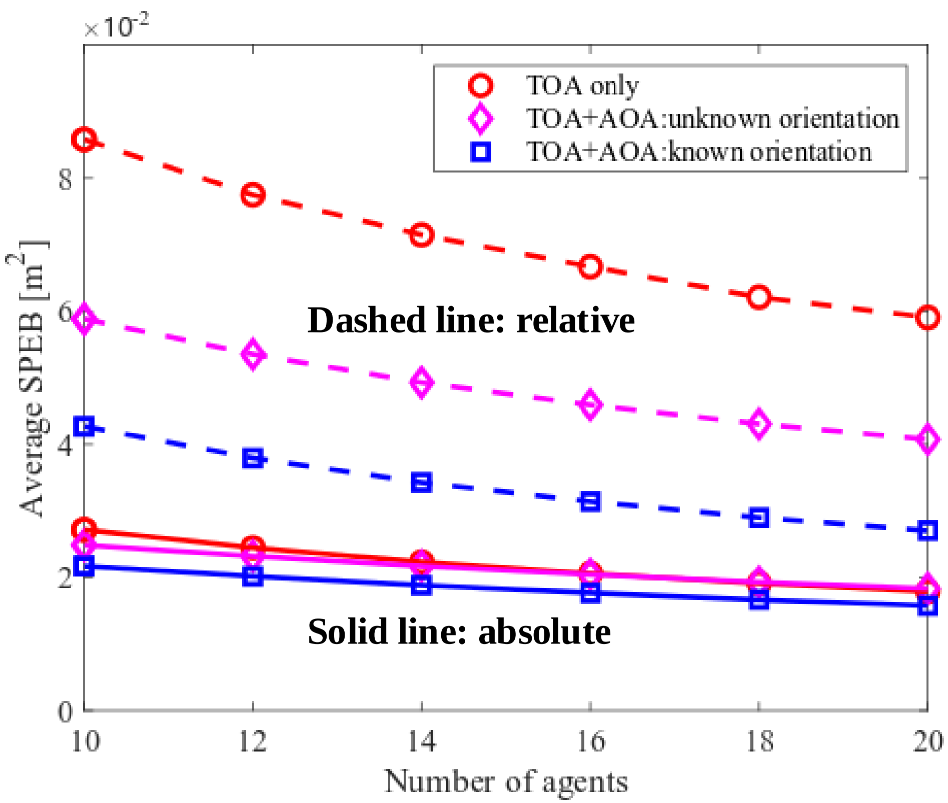
with

$$\underbrace{\boxed{\boldsymbol{S}_k}}_{\text{Carry-over information}} = \boldsymbol{J}_k + \boldsymbol{T}_k + \boldsymbol{T}_{k-1} - \boldsymbol{T}_{k-1}\boldsymbol{S}^{-1}_{k-1}\boldsymbol{T}_{k-1}$$

- $\boldsymbol{S}_k$ characterizes the effects of the information obtained previously on the current time.
- The information fusion process acts like Kalman-filtering

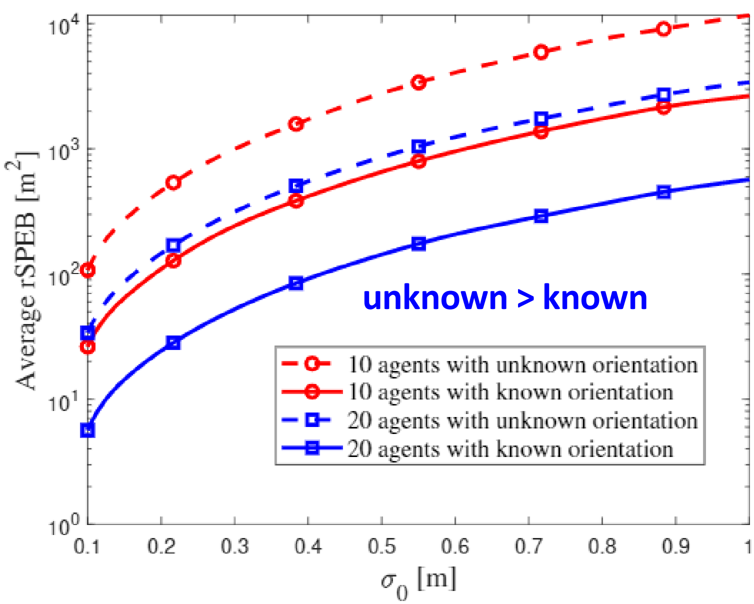
Relative Localization: Theory

- Comparison of absolute/relative localization
  - The relative localization error decays with the number of agents in both relative and absolute localization

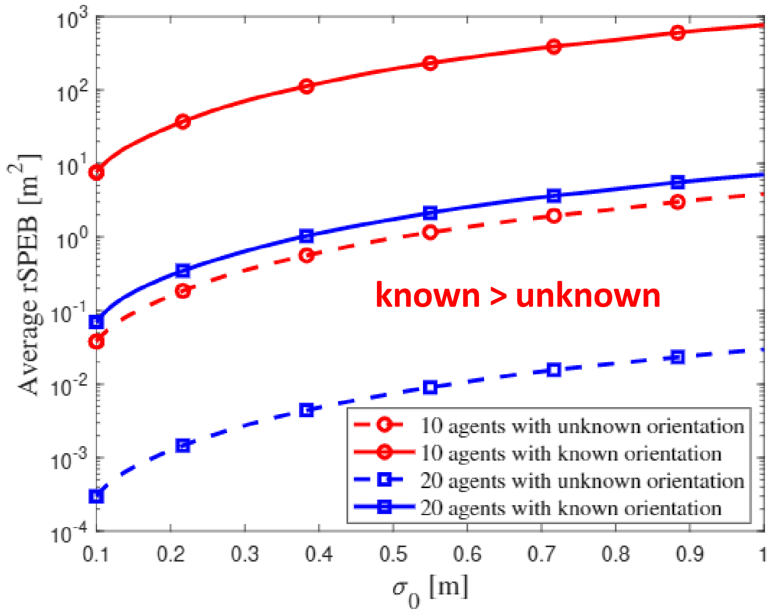


Relative Localization: Theory

- Effects of agent orientations with different communication ranges
  - Remark: whether the knowledge of agent orientation decreases the error depends on the network topologies



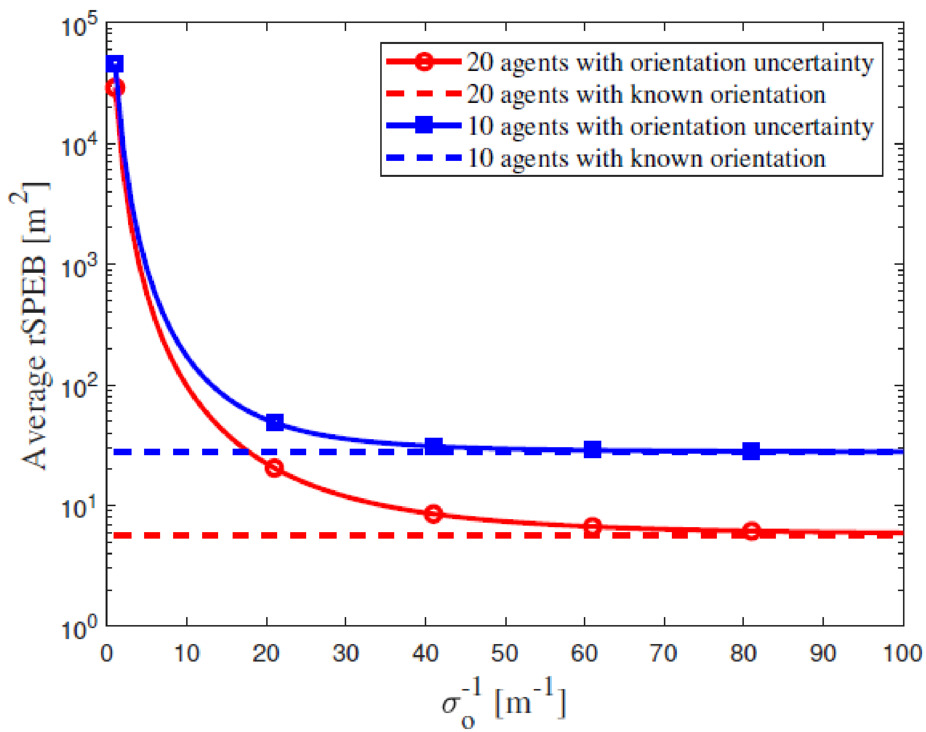
Network Topology 1



Network Topology 2

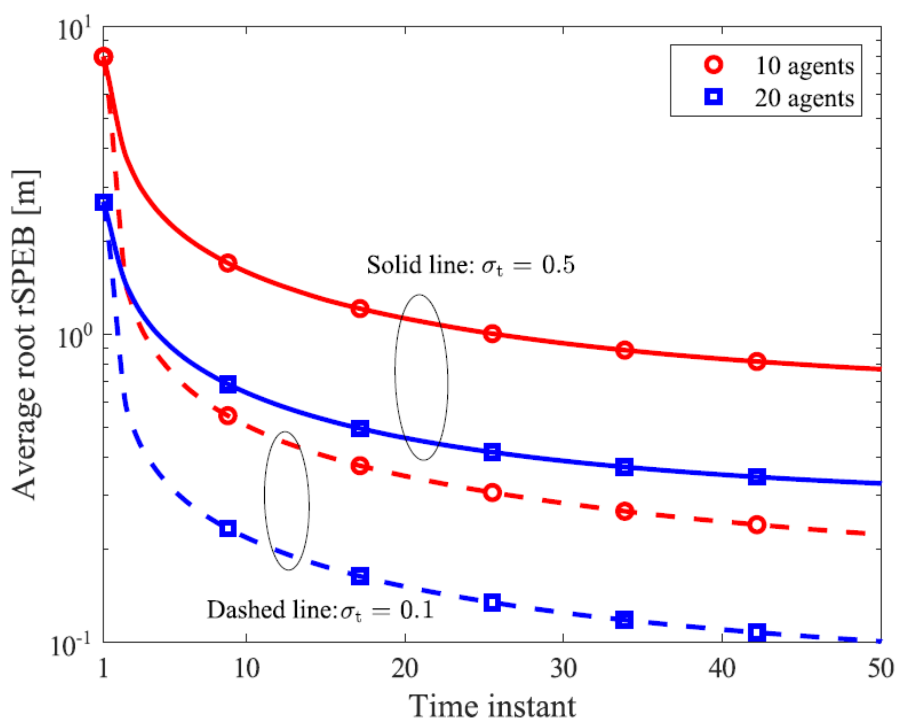
Relative Localization: Theory

- Effects of orientation uncertainties
  - As the uncertainty vanishes, the localization performance approaches the orientation-known case



Relative Localization: Theory

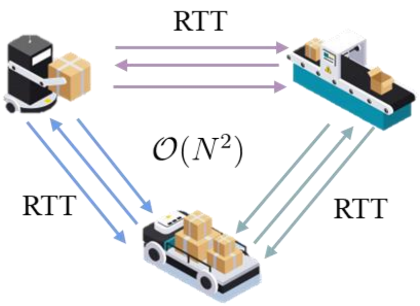
- Effects of spatiotemporal cooperation
  - The relative localization error decays exponentially with time and finally converges to a stable value



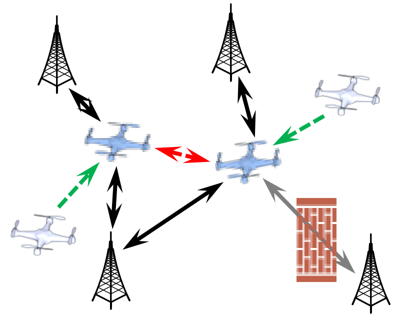
SCHEME: NETWORK MEASUREMENT FOR MULTI-ROBOT SYSTEMS

Challenge

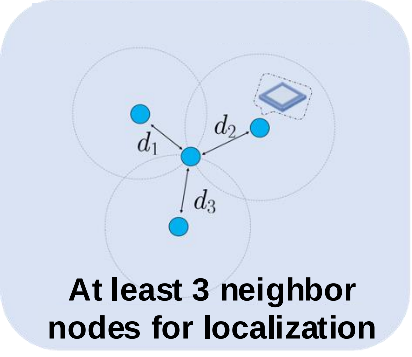
- Asynchronous networks
  - Asynchronous even with initial calibration: need joint clock synchronization and ranging for precise ranging



- Large network scale
  - Hundreds of mobile devices, sensors, and objects to be connected: leading to excessive signal overhead

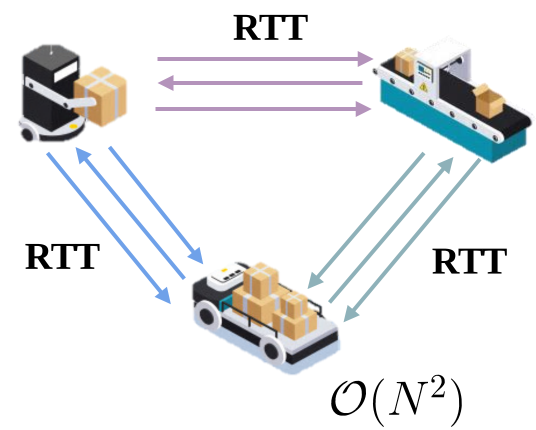


- Efficiency of measurements
  - Ranging-based localization: fail to utilize bearing information and hence inefficient



Signal-Multiplexing Network Ranging

- Current clock synchronization and ranging methods
  - Mass transmission, unsuitable to large-scale networks
    - Long latency, large energy, heavy hardware resource occupation…
  - Does not consider the nodes' mobility



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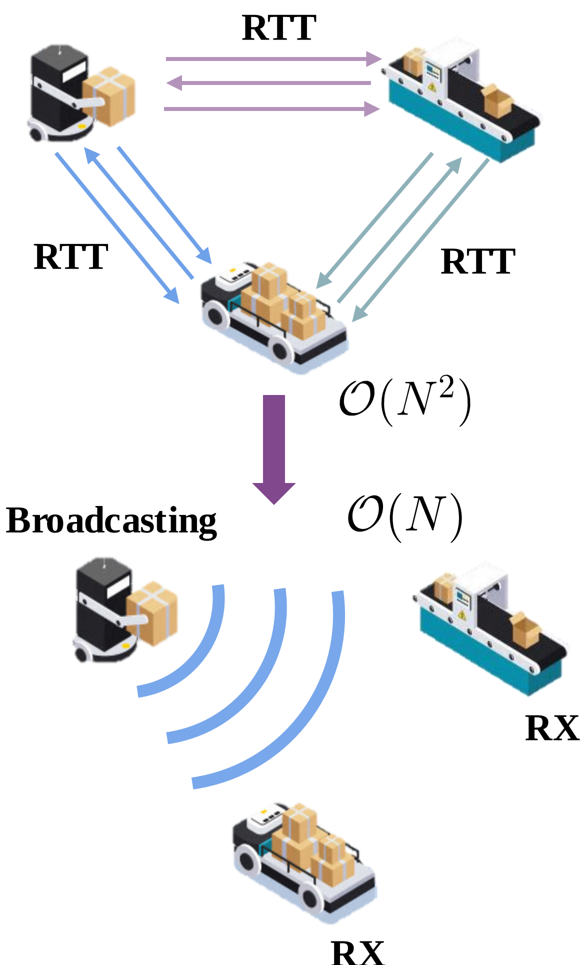
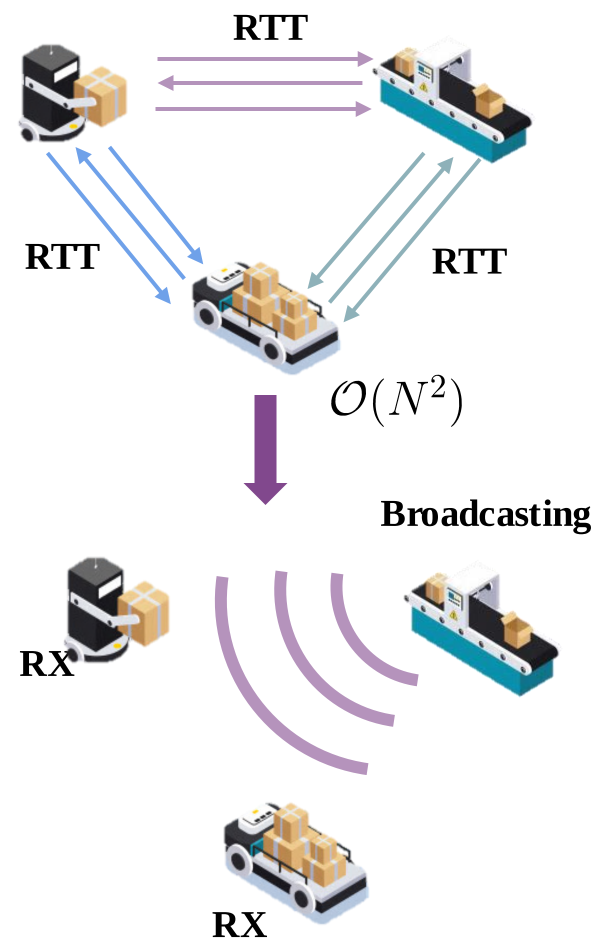
Signal-Multiplexing Network Ranging

- Current clock synchronization and ranging methods
  - Mass transmission, unsuitable to large-scale networks
    - Long latency, large energy, heavy hardware resource occupation…
  - Does not consider the nodes' mobility
- Proposed *SM-NR*
  - Multiplexing signals for network measuring
    - Reduce the number of signal transmissions to the minimum
  - High-accuracy range estimation
    - Clock errors, user mobility
    - High measurement update rates, small accumulated errors



Signal-Multiplexing Network Ranging

- Current clock synchronization and ranging methods
  - Mass transmission, unsuitable to large-scale networks
    - Long latency, large energy, heavy hardware resource occupation…
  - Does not consider the nodes' mobility



- Proposed *SM-NR*
  - Multiplexing signals for network measuring
    - Reduce the number of signal transmissions to the minimum
  - High-accuracy range estimation
    - Clock errors, user mobility
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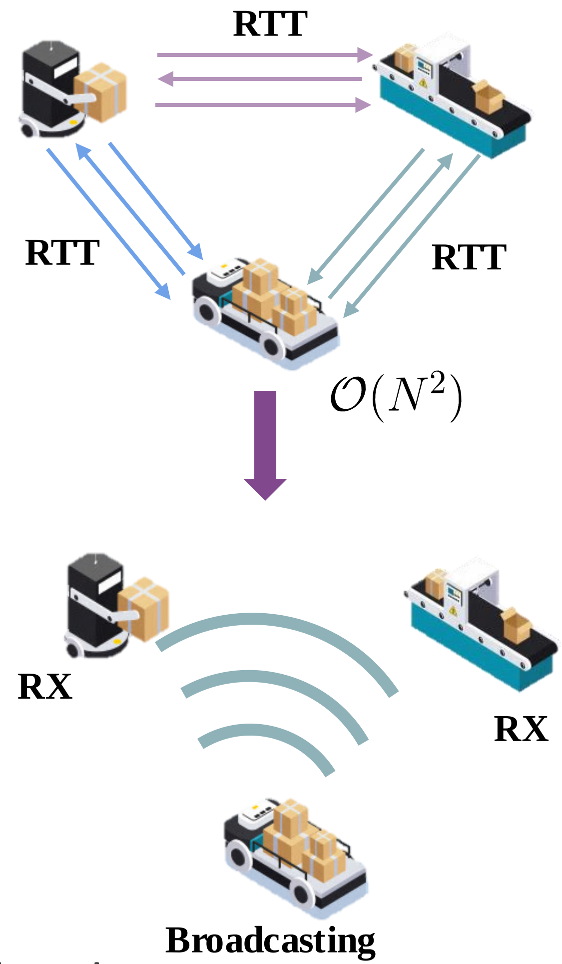

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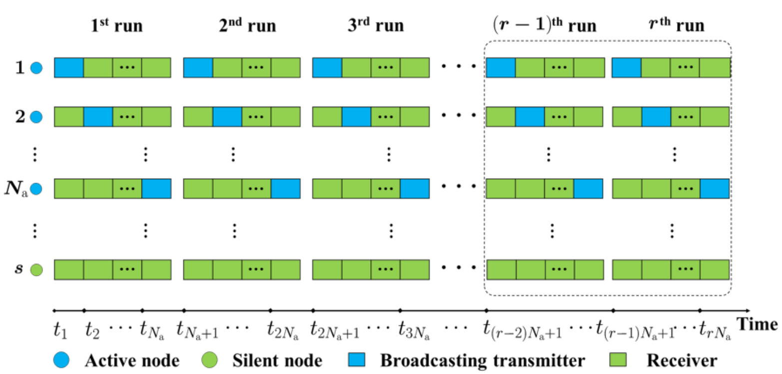
  - Multiplexing signals for network measuring
    - Reduce the number of signal transmissions to the minimum
  - High-accuracy range estimation
    - Clock errors, user mobility
    - High measurement update rates, small accumulated errors

Signal-Multiplexing Network Ranging

- New method for network ranging and clock synchronization
  - Protocol
    - Reduce the signal overhead from $\mathcal{O}(N^2)$ to $\mathcal{O}(N)$, boost scalability
    - Ensure access to the freshest measurement data, diminish the ranging/localization errors incurred by outdated measurements
  - Ranging method
    - Estimate time-varying distances without the knowledge of velocity
    - Robust to the coupling effects of clock errors and node mobility



$$\mathcal{D}_k^\star, \mathcal{R}_k^\star, \mathbf{e}^\star, \mathbf{v}_k^\star := \underset{\mathcal{D}_k, \mathcal{R}_k, \mathbf{e}, \mathbf{v}_\mathbf{k}}{\operatorname{argmax}} \; \ell_\mathbf{e}(\mathcal{O}_k \,; \mathcal{D}_k, \mathcal{R}_k, \mathbf{e}, \mathbf{v}_\mathbf{k})$$

Decouple the effects of node mobility and clock errors → Establish a global average time as reference time for clock synchronization → ToF/TDoF estimation

Signal-Multiplexing Network Ranging

- ToF estimation against clock errors and node mobility

$$\widehat{\mathsf{T}}^{\star}_{\mathrm{oF}} = \underset{\boldsymbol{T}_{\mathrm{oF}}}{\operatorname{argmax}} \; \ell_{\mathbf{e}}\big(\hat{\mathbf{t}}\,;\boldsymbol{T}_{\mathrm{oF}}\big)$$

likelihood fun. *w.r.t* the clock drifts   Timestamps

Proposition 1

The ToF estimation between active nodes i and j at time instant k when active node i broadcasts is given by

$$\widehat{\mathsf{T}}^{\star}_{\mathrm{oF}}(i,j)[k] = \frac{\mathsf{R}_1(2\mathsf{t}^{(j)}_{k_4} - \mathsf{t}^{(j)}_{k_3} - \mathsf{t}^{(j)}_{k_2}) - \mathsf{R}_2(\mathsf{t}^{(j)}_{k_4} - \mathsf{t}^{(j)}_{k_1})}{2(\mathsf{t}^{(j)}_{k_4} - \mathsf{t}^{(j)}_{k_2} + \mathsf{t}^{(j)}_{k_1} - \mathsf{t}^{(j)}_{k_3})}$$

where

$$\mathsf{R}_1 = \frac{\widehat{\mathsf{T}}^{\star}}{\mathsf{T}^{(j)}}(\mathsf{t}^{(j)}_{k_4} - \mathsf{t}^{(j)}_{k_1}) - \frac{\widehat{\mathsf{T}}^{\star}}{\mathsf{T}^{(i)}}(\mathsf{t}^{(i)}_{k_4} - \mathsf{t}^{(i)}_{k_1}) \qquad \mathsf{R}_2 = \frac{\widehat{\mathsf{T}}^{\star}}{\mathsf{T}^{(i)}}(\mathsf{t}^{(i)}_{k_3} - \mathsf{t}^{(i)}_{k_2}) - \frac{\widehat{\mathsf{T}}^{\star}}{\mathsf{T}^{(j)}}(\mathsf{t}^{(j)}_{k_3} - \mathsf{t}^{(j)}_{k_2})$$

in which the synchronization is fabricated as

$$\mathsf{T}^{(x)} \triangleq \mathsf{t}^{(x)}_{k_4} - \mathsf{t}^{(x)}_{k_2} + \mathsf{t}^{(x)}_{k_3} - \mathsf{t}^{(x)}_{k_1}, \quad \text{for } x = i,j$$

with estimation $\hat{\mathsf{T}}^{\star} = \mathrm{argmax}_{T}\,\ell_{\mathbf{e}}(\mathbf{t};T)$

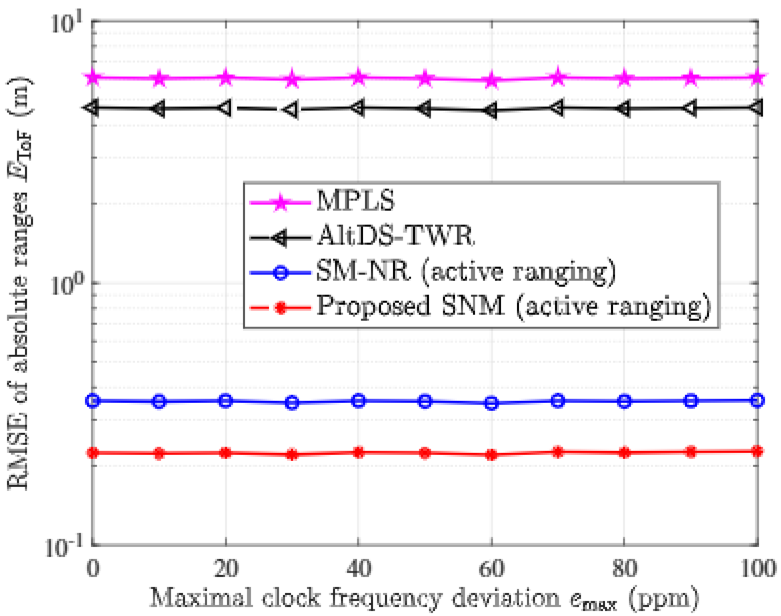
Signal-Multiplexing Network Ranging

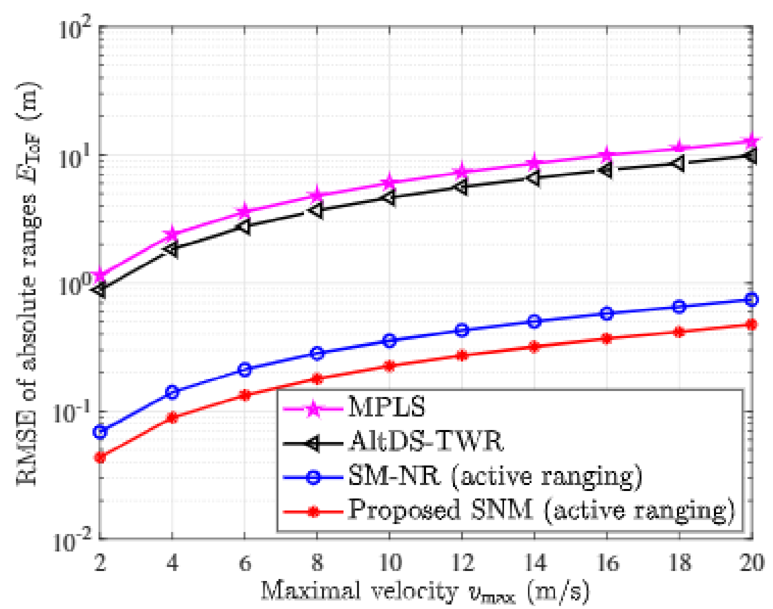
- SM-NR achieves higher ranging resolution with fewer measuring signals

Method	Measurement update rates	Mobility	ToF	TDoF	Worst-case errors
AltDS-TWR	$1/3$		✓		$e_{\max}d_{\max} + O(N_{\mathrm{a}}^2 I_{\max} v_{\max})$ Estimation error Error induced by outdated measurements
MPLS	$1/M$	✓	✓		$O(e_{\max}d_{\max}) + O(N_{\mathrm{a}}^2 I_{\max} v_{\max})$
PER	$N_{\mathrm{s}}/3$			✓	$\mathcal{O}(e_{\max}c\left\|\delta_{i,j}^{(s)} - I_{\max}\right\|) + \mathcal{O}(N_{\mathrm{a}}^2 I_{\max} v_{\max})$
Our method	$(N_{\mathrm{a}}-1)(N_{\mathrm{s}}+1)$	✓	✓	✓	$2e_{\max}d_{\max} + O(N_{\mathrm{a}} I_{\max} v_{\max})$

Numerical Results

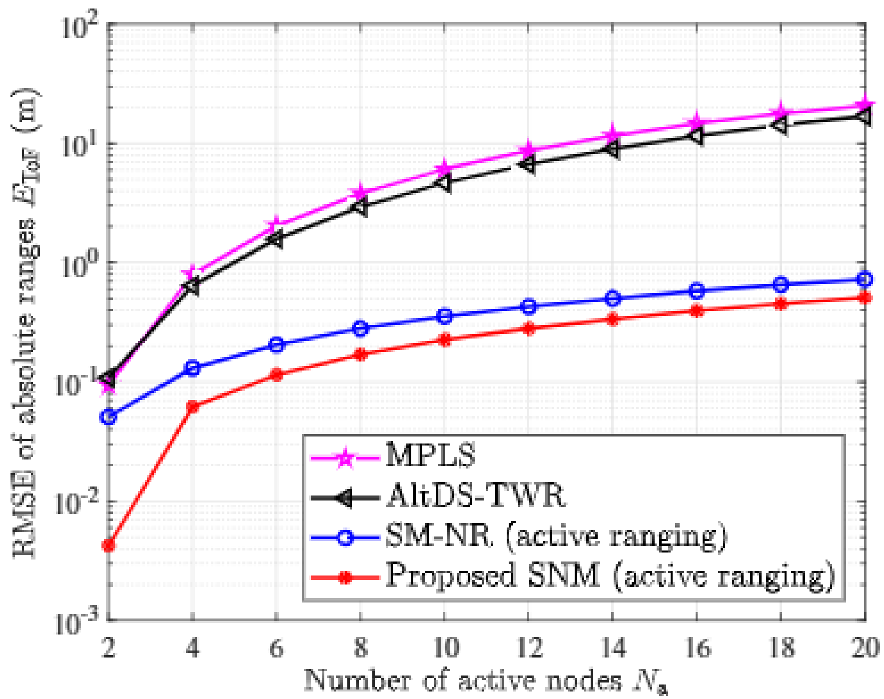
- RMSEs vs clock errors and node mobility
  - The low measurement update rate replaces clock errors, becoming the dominating factor in range errors
  - SM-NR effectively eliminates clock errors, significantly improving the ranging accuracy by boosting the efficiency, achieving about one order of magnitude lower error than those of the others





Numerical Results

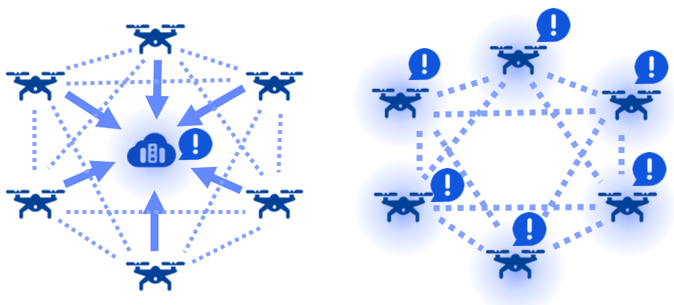
- RMSEs vs network scale
  - Ranging errors increases with the number of active nodes, and the growth rates reflect the scalability of the ranging methods
  - SM-NR exhibit a smaller growth rate and lower errors, manifesting its adaptability to large-scale networks



SCHEME: RELATIVE LOCALIZATION FOR MULTI-ROBOT SYSTEMS

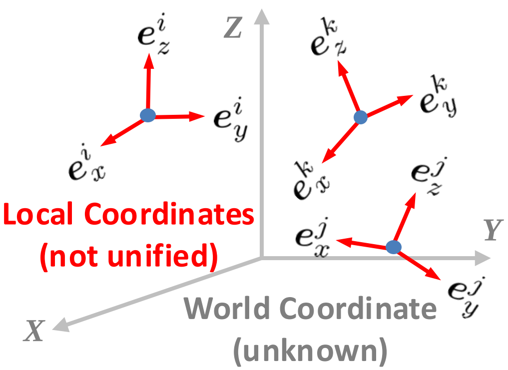
Challenge

- Distributed networks
  - **Centralized**: global optimization, high latency, high complexity, vulnerable
  - **Distributed**: local optimization, low latency, low complexity, high robustness

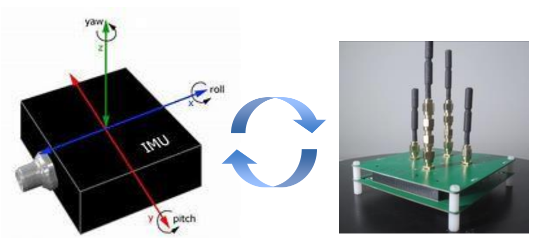


Centralized Distributed

- Infrastructure-free demands
  - Determination of the network geometry without absolute position information reference

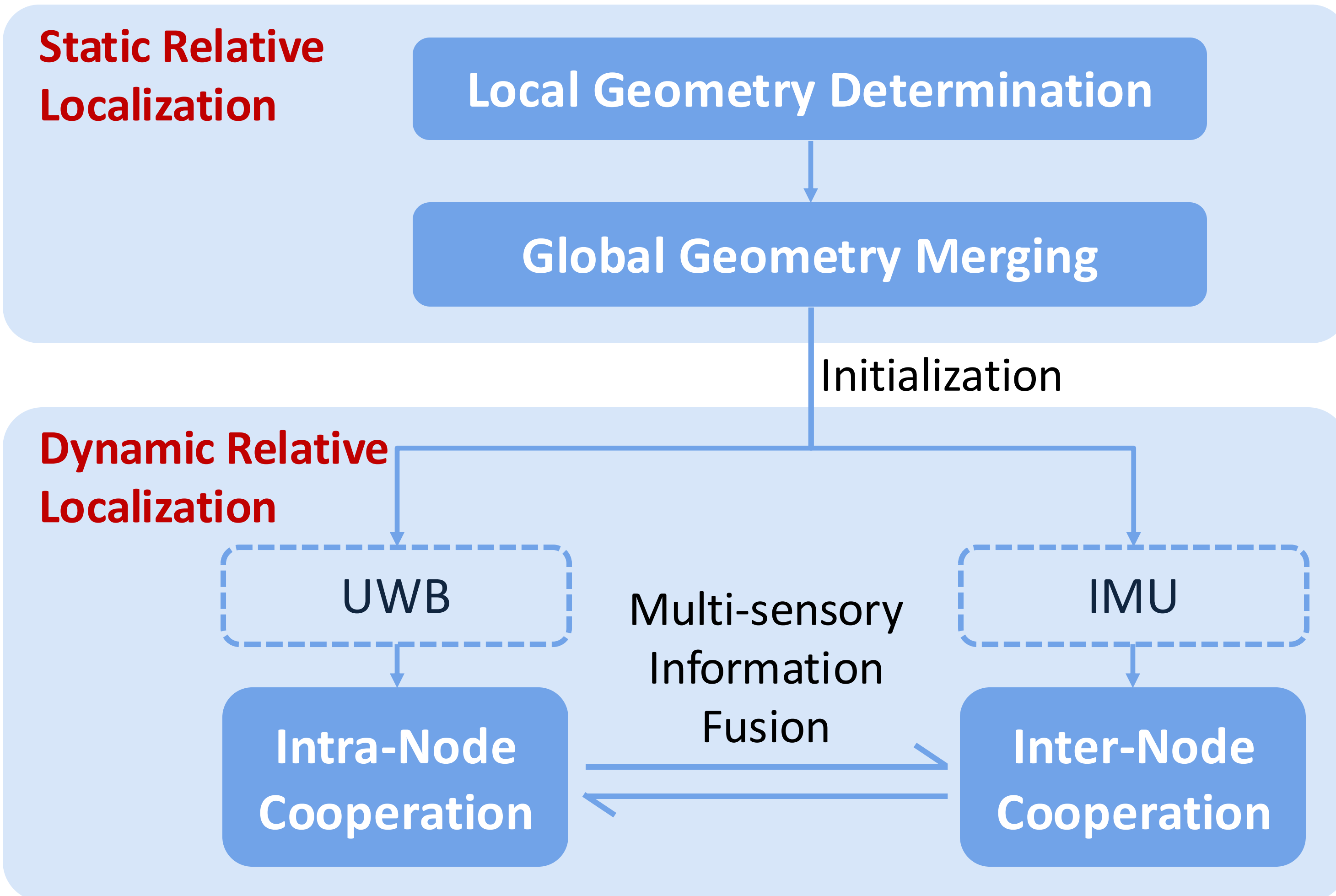


- Information fusion
  - The local networks share no unified coordinate system in decentralized networks
  - In networks with multi-sensors, high-effiecient information fusion scheme is required for spatiotemporal cooperation



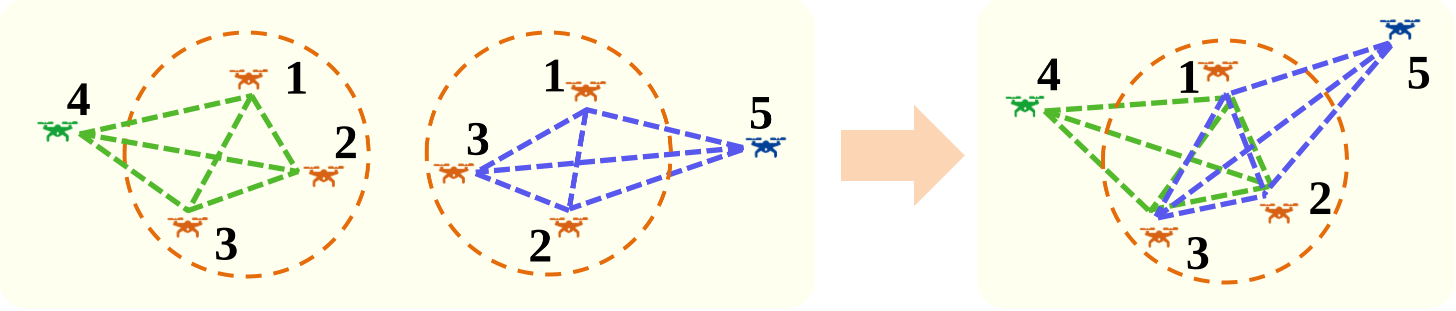
Relative Localization Algorithm

- Distributed Relative Localization framework



Relative Localization Algorithm

- Distributed Relative Localization framework
  - Static relative localization
    - Local Geometry Determination: Estimate the geometric shape of the network without any absolute position information
    - Global Geometry Merging: Fuse the information of different geometries under different reference frames
  - Dynamic relative localization
    - Exploit the spatio-temporal gain by inter-intra node cooperation with multi-sensory information fusion
    - Achieve distributed localization via dynamic reference frame alignment



Relative Localization Algorithm

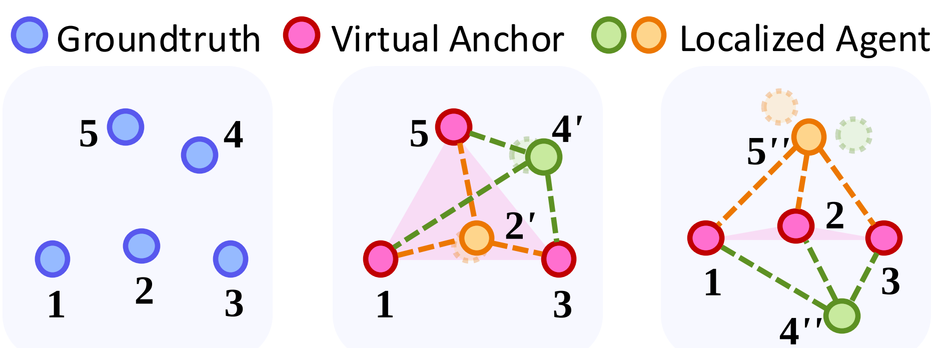
- Local Geometry Determination
  - Without absolute position reference, the high-efficient estimation of the shape of the local geometry is achieved by a dynamic selection scheme of reference frames

Joint Optimization of Reference Frame & Relative Position

$$\min_{\hat{\mathbf{p}}} \min_{\Gamma} \varepsilon_{\mathrm{r}}(\boldsymbol{p}, \Gamma(\hat{\mathbf{p}}))$$

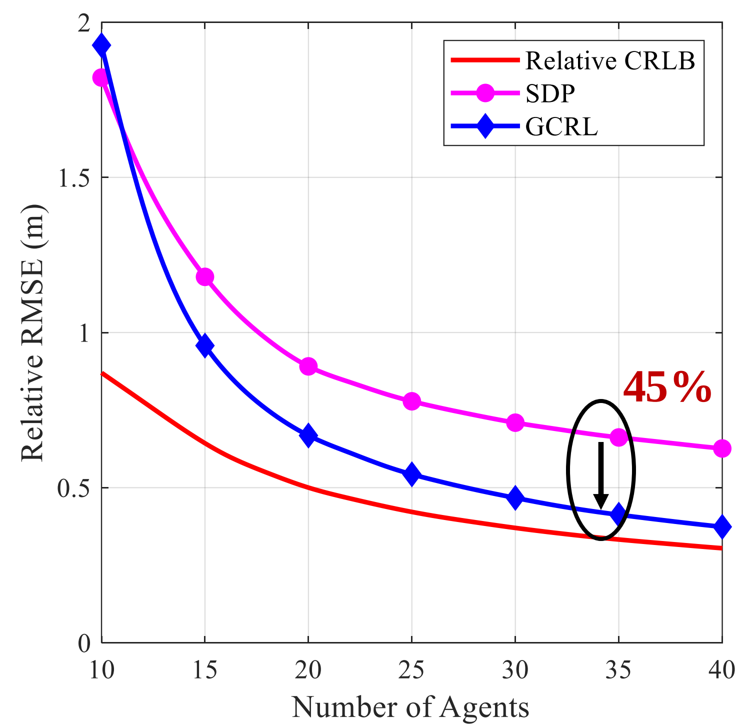
Given the relative position estimate, optimize the selection of reference frame

Given the reference frame, optimize the relative position estimate



Groundtruth **Well-conditioned** **Ill-conditioned**

Localization Performance



Relative Localization Algorithm

- Global Geometry Merging
  - Distributed information fusion under different reference frames is achieved by minimizing relative error within the equivalent state class with a merging priority evaluation criterion

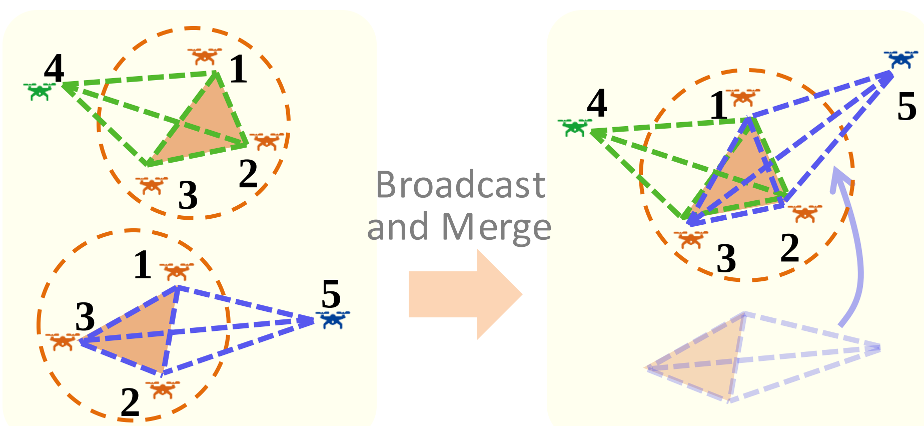
Scale of Common Geometry

Confidence of Local Geometry

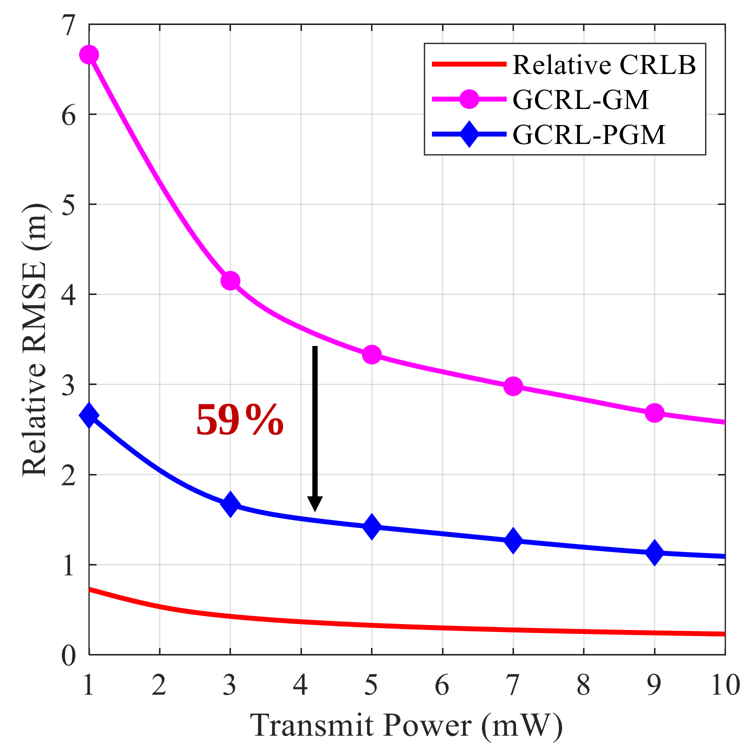
Merging Confidence

$$\gamma_{ij} = \frac{N_{\mathrm{c}} \left(\beta_i + \beta_j - 2\beta_0\right)}{\mathrm{Cond}\left(\mathbf{C}_i, \mathbf{C}_j\right)}$$

Common Geometry Condition

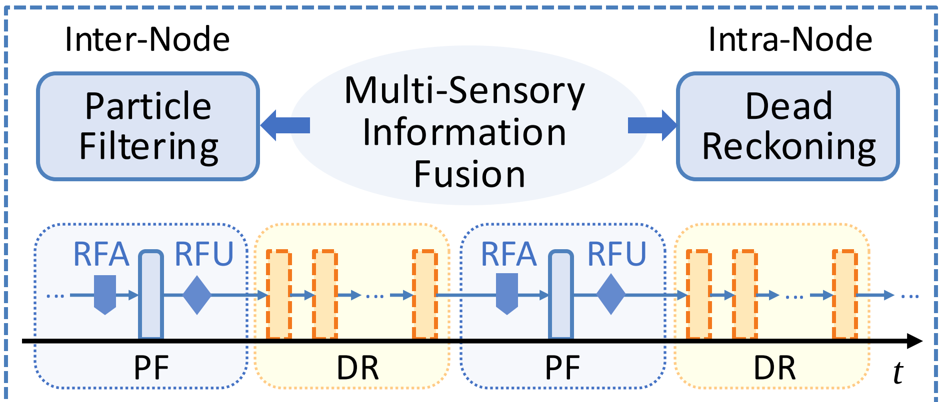


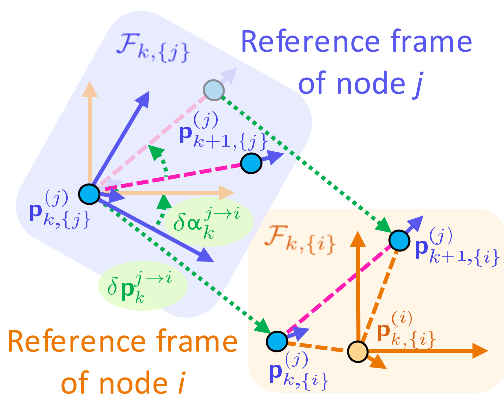
Localization Performance



Relative Localization Algorithm

- Dynamic Relative Localization
  - In dynamic scenarios, real-time distributed relative localization is achieved via dynamic reference frame alignment and multi-sensory information fusion

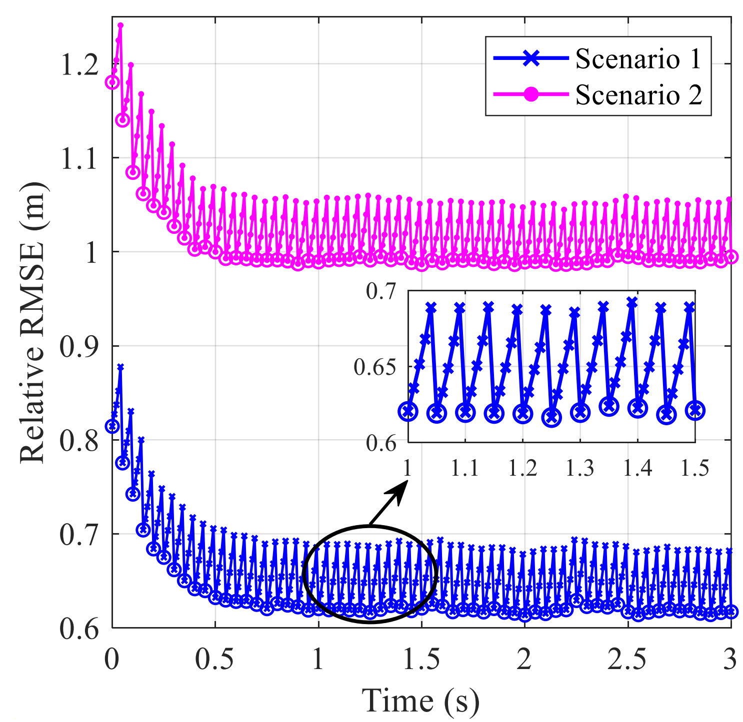




$$\mathbf{T}^{j\to i}_k = \begin{bmatrix} \mathbf{R}^{j\to i}_k & \delta\mathbf{p}^{j\to i}_k & \delta\mathbf{v}^{j\to i}_k \\ \mathbf{0} & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{bmatrix}$$

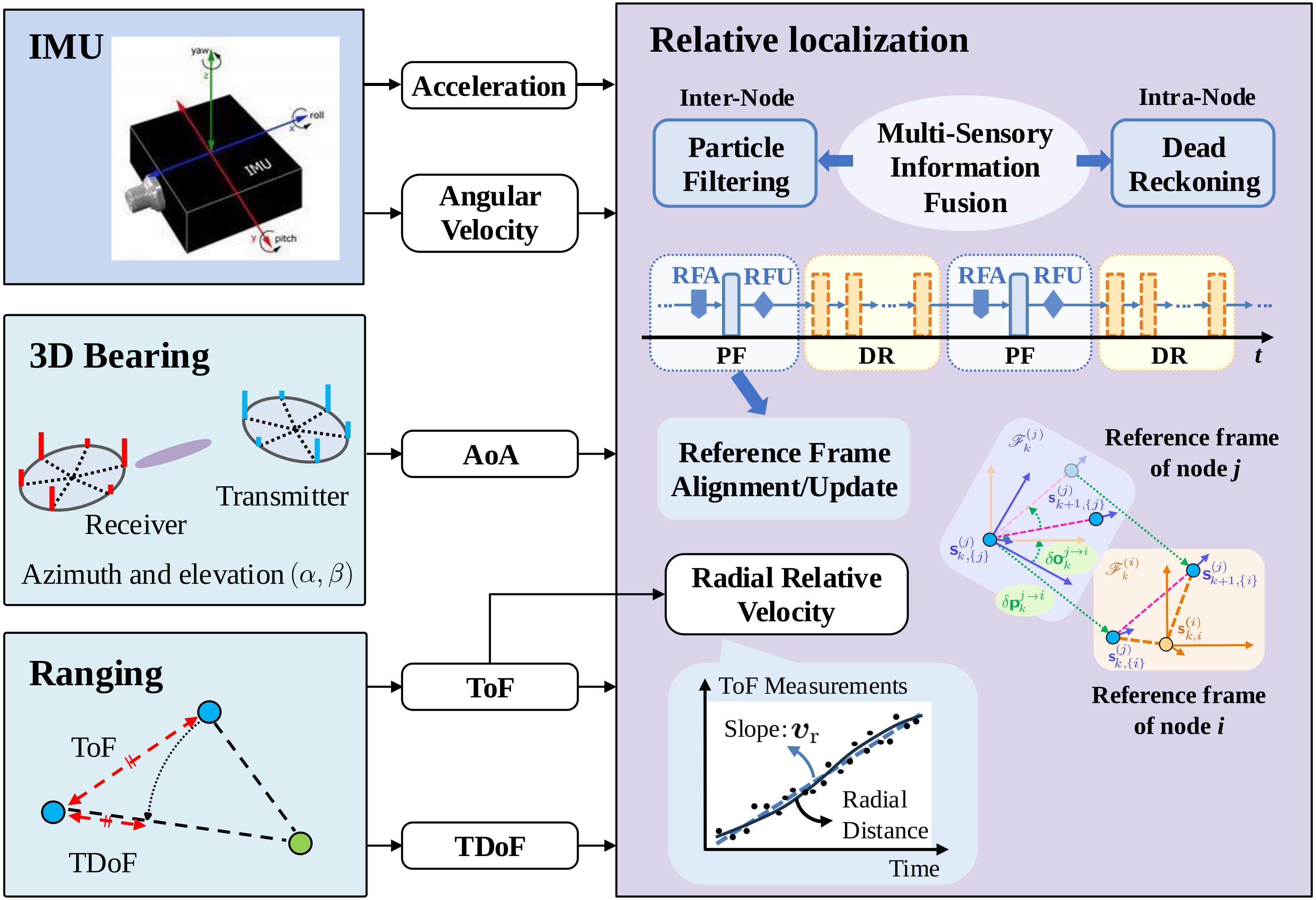
Homogeneous State Transformation:
Alignment of origins, axes and translational velocities

Localization Performance



PLATFORM: RELATIVE LOCALIZATION FOR MULTI-ROBOT SYSTEMS

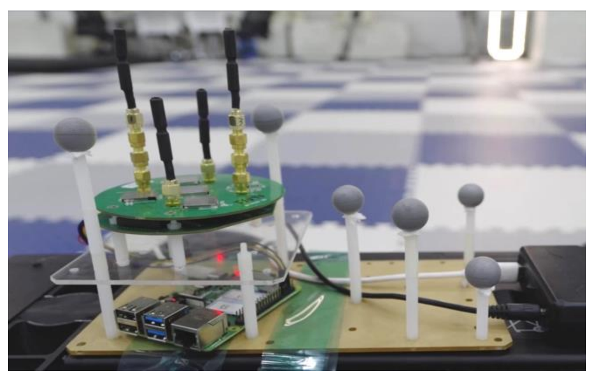
System and Module



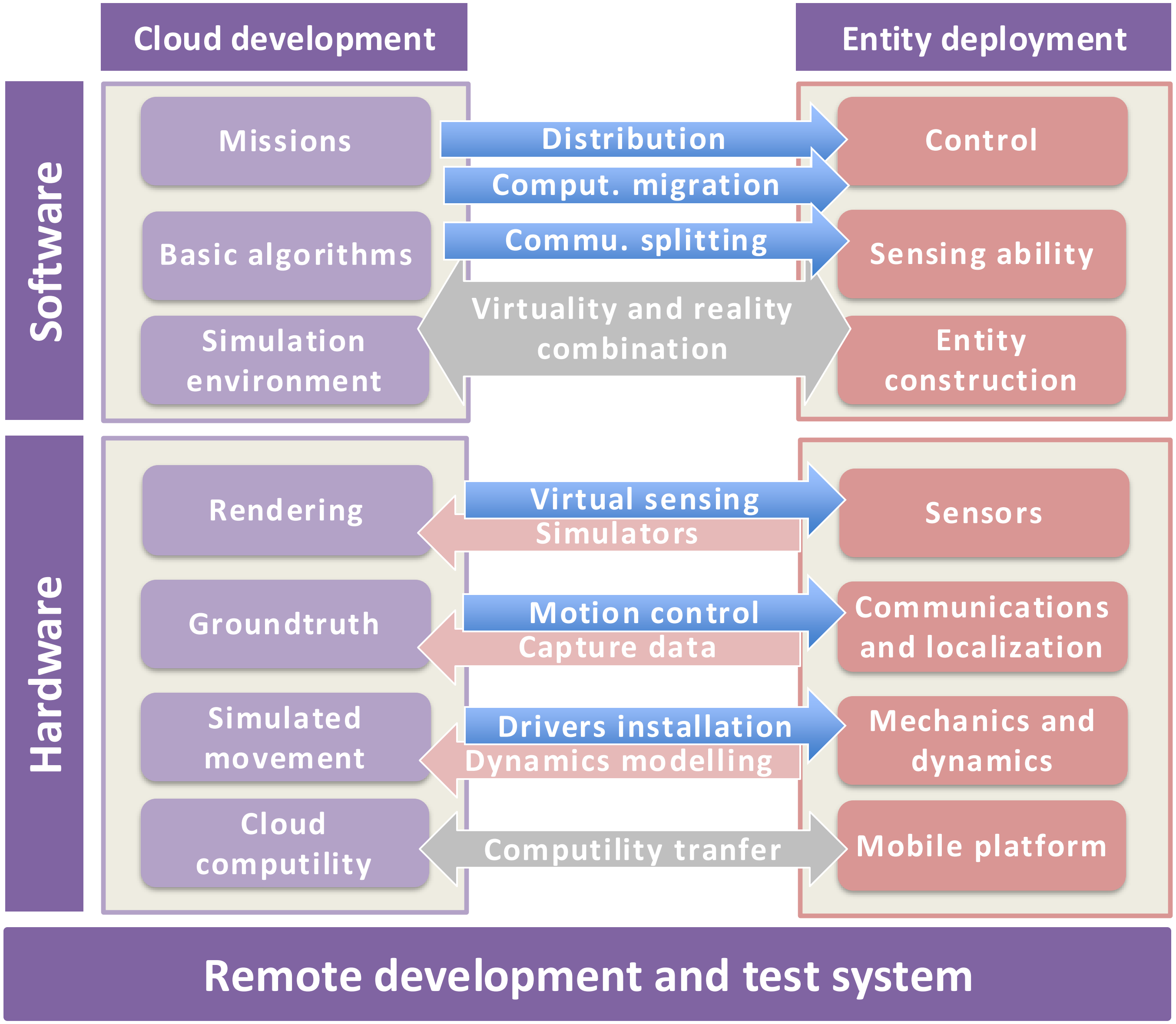
System and Module

- Cloud test field
  - **Motion capture system:** 50m*25m*12m, 144 cameras, reconstruction error <0.5mm, update frequency 100Hz+
  - **Remote test:** Cloud computing and simulation



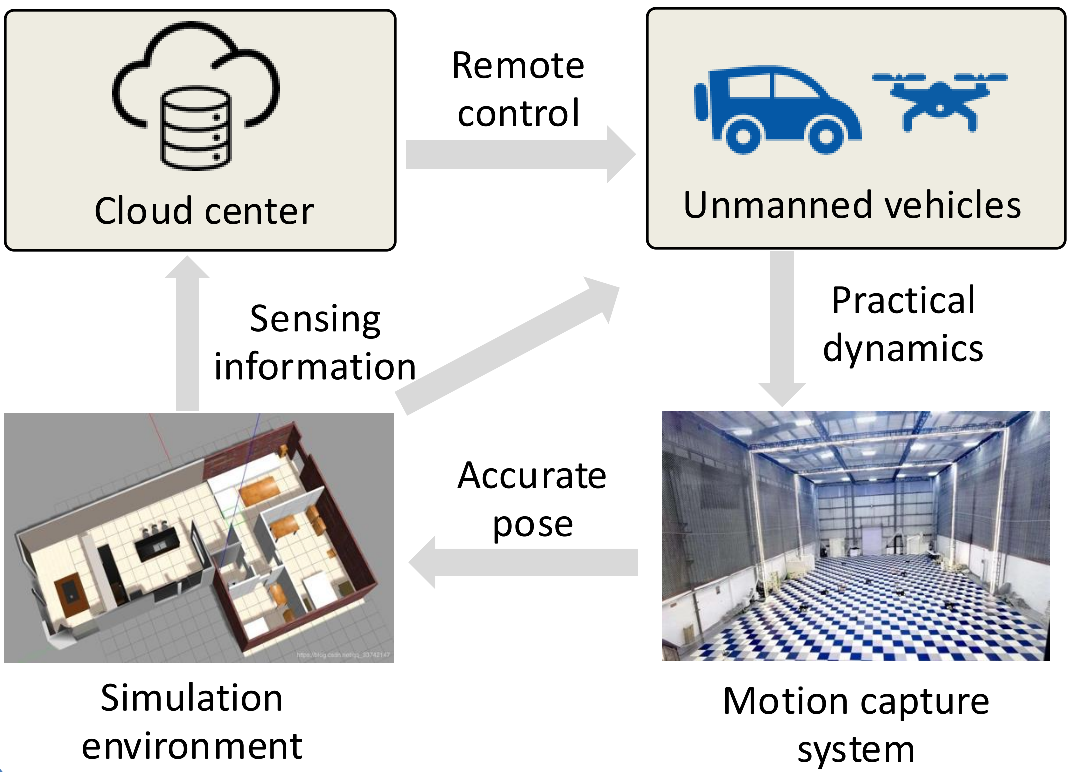



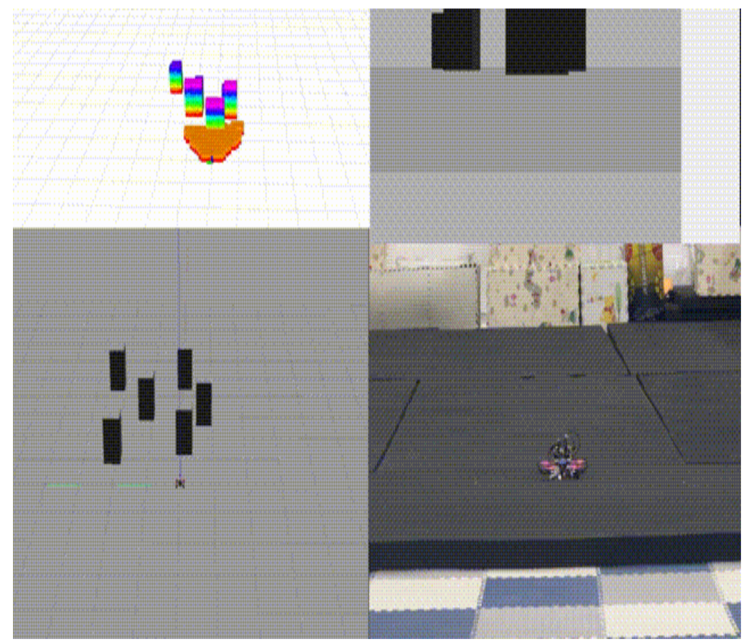

System and Module



System and Module

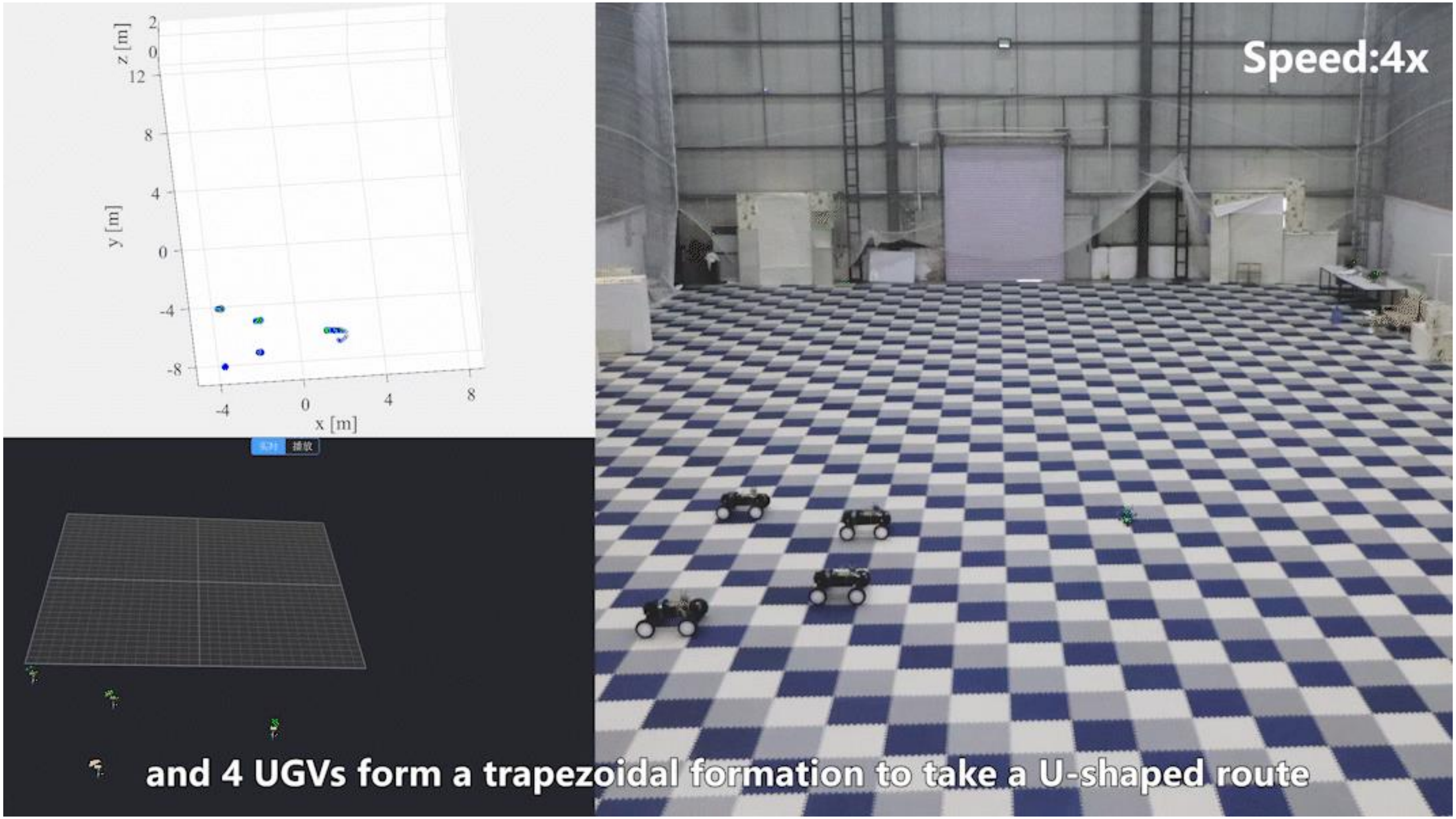
- Cloud test field
  - Virtuality and reality combination
  - Sensing in simulation, control and motion in reality





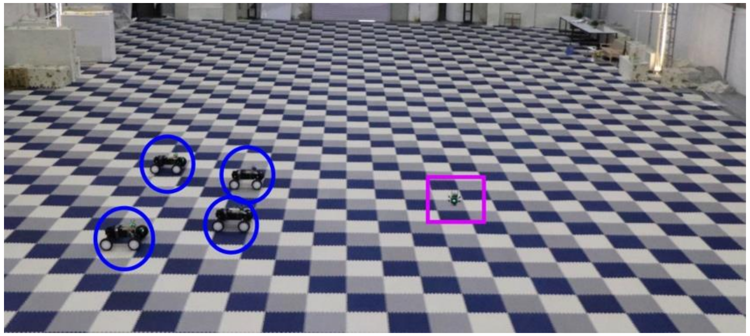
Real-world Experiment

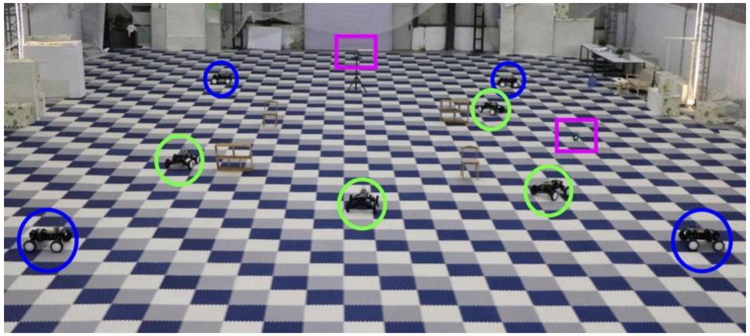
- Three application scenarios

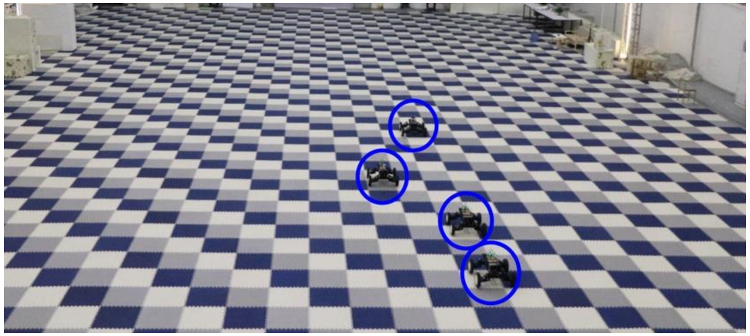


Real-world Experiment

- Three application scenarios
  - Scenario 1: Formation
    - 4UGV(active) + 1UAV(active)
    - U-shape formation route
    - Involve high-speed (accel.) movement
  - Scenario 2: Search and Rescue
    - 4UGV(active) + 4UGV(silent) + 2UAV(active)
    - Involve a few obstacles
    - Involve high-speed (accel.) movement
  - Scenario 3: Overtaking
    - 4UGV(active)
    - One UGV overtakes the other 3 on the two-lane road

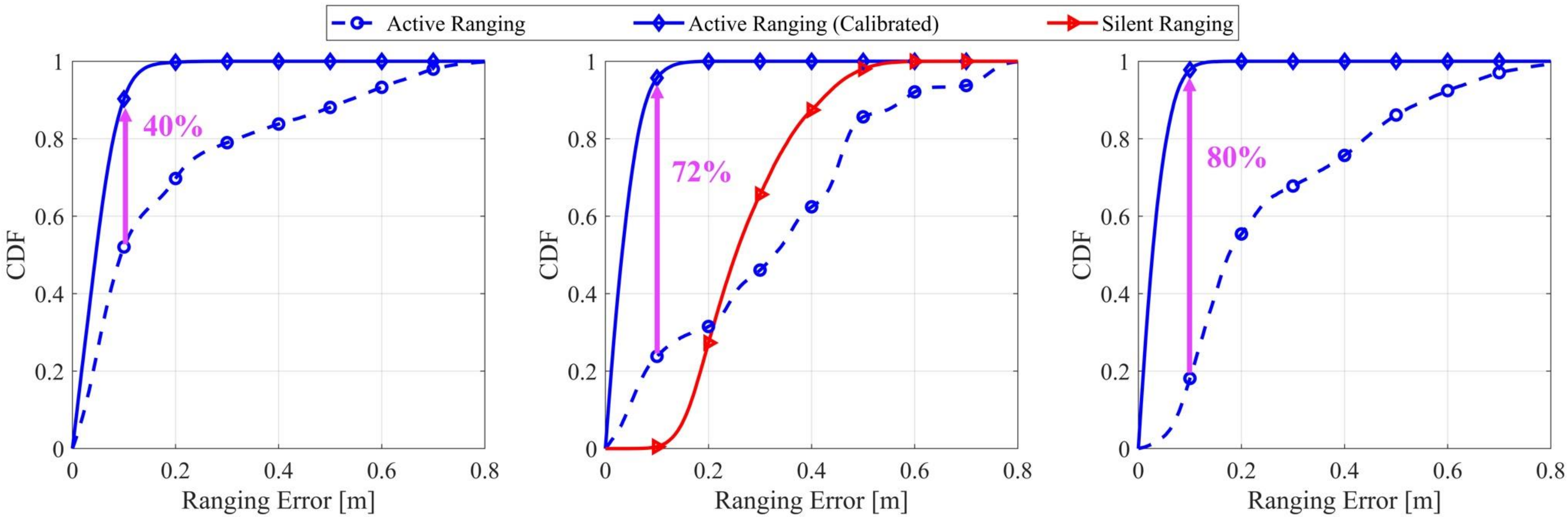






Real-world Experiment

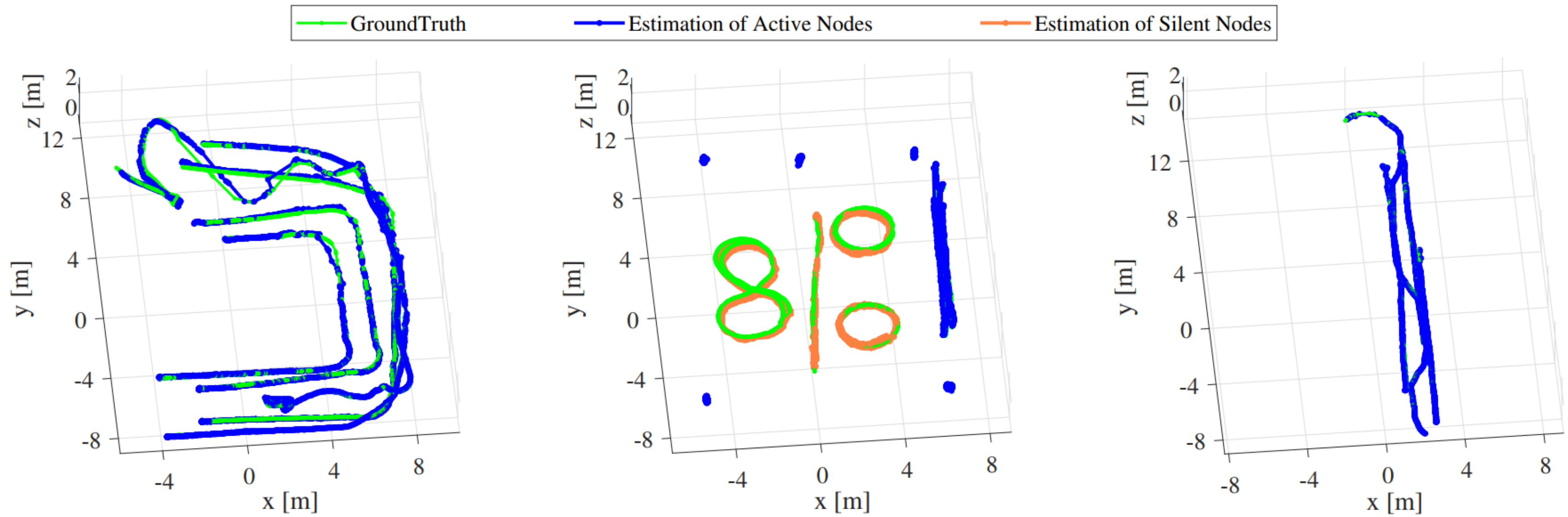
- Network ranging performance
  - CDF of ranging estimation errors



RMSE	Scenario 1	Scenario 2		Scenario 3
	Active	Active	Silent	Active
Ranging	26.90 cm	37.54 cm	29.21 cm	32.06 cm
Ranging (calibrated)	6.22 cm	4.90 cm	–	4.26 cm

Real-world Experiment

- **Relative localization performance**
  - Localization results



RMSE	Scenario 1	Scenario 2		Scenario 3
	Active	Active	Silent	Active
Localization	16.84 cm	19.20 cm	30.71 cm	6.23 cm

Conclusion

- Theory
  - Deriving the performance bounds of the relative error, investigating the effects of state measurements, clock asynchronization and temporal cooperation
- Scheme
  - Network measurement protocol: The proposed SM-NR minimizes the signal overhead, ensuring scalability and timeliness for accommodating larger swarm sizes
  - Localization algorithm: The proposed relative localization scheme exploits multi-sensory information fusion and achieves real-time alignment for local reference frames, ensuring distributed localization with high efficiency, accuracy, and robustness
- Platform
  - Leveraging SM-NR on our UWB arrays, where an implementation attains centimeter-level accuracy at an update rate over 100 Hz, solely utilizing UWB

Reference

- Theory
  - X. Shen, L. Xu, Y. Liu, and Y. Shen, “A theoretical framework for relative localization,” *IEEE T-IT*, 2024.
  - Y. Liu, Y. Wang, J. Wang, and Y. Shen, “Distributed 3D relative localization of UAVs,” *IEEE T-VT*, 2020.
  - M. Z. Win, Y. Shen, and W. Dai, “A theoretical foundation of network localization and navigation,” *Proc. IEEE*, 2018.
- Scheme
  - H. Zhao, Z. Zhang, L. Xu, Y. Wang, and Y. Shen, “Enhancing timeliness in asynchronous vehicle localization: A signal-multiplexing network measuring approach,” *IEEE T-ITS,* 2024.
  - Z. Zhang, H. Zhao, J. Wang, and Y. Shen, “Signal-multiplexing ranging for network localization,” *IEEE T-WC, 2022.*
- Platform
  - X. Li, Y. Wang, K. Ma, L. Xu, Z. Zhang, J. Wang, Y. Wang, and Y. Shen, “A cooperative relative localization system for distributed multi-agent networks,” *IEEE T-VT*, Nov. 2023.

Q & A

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